The sound transmission loss of windows at very low frequencies cannot be interpreted by the theory of infinitely large partitions. Here, the bending wave resonances are essential but not the only contributors. The low sound radiation coefficient and “niche effect” are also important. The goal is to illustrate such phenomena via simplified theoretical models and experimental results. In particular, the role of low sound radiation is interpreted using the sound radiation model of a piston inside the aperture cut in a rigid baffle between the source and receiving rooms. The “niche effect” is compared to the low-frequency sound transmission via symmetric triple windows (where the middle element is the real window and the role of the edge leaves is played by the air masses entrained at the aperture edges). In addition to the previously published results, the new experimental data are provided to support this theory in the “double-niche” case: the aperture consists of two in-series halves with different areas, and the window is installed in the smaller one. Such simple models can also be utilized to interpret the experimental sound transmission data for relatively small partitions mounted in the aperture made in a thick wall between two rooms. The paper may be helpful for students and engineers engaged in measurements of the sound transmission loss.

1. Introduction

The first engineering theories of airborne sound transmission via single and double walls were developed for uniform partitions of infinite area by Cremer in 1942 [1] and London in 1950 [2]. For single partitions at low frequencies, the theoretical sound transmission loss was described by the Mass Law as a function of the surface density, frequency, and incident sound field (normal or diffuse); the normal-incidence Mass Law values are 5 – 6 dB over the diffuse-incidence those at the same frequencies.

However, in 1954 Peutz published a paper [3] with his paradoxical experimental findings: the transmission loss of thin plates at low frequencies notably (up to 10-15 dB) exceeded the Mass Law predictions, even those for normal incidence.

The elevated sound transmission loss could be affected and partially masked (to some extent) by the bending vibration resonances. But if the fundamental resonance frequency of bending vibration is relatively low, the effect of such resonances in the important frequency range is limited.

The other important phenomenon (the niche or tunnelling effect [4-7]) is mostly reported at low frequencies but some researchers observed it at the medium frequencies too. As was found, the airborne sound transmission loss of a thin partition filling the opening in a thick wall between two reverberation rooms gets low if the partition is installed in the centre of the opening.

The goal of this paper is twofold:
1. Using simplified mathematical models, to interpret the elevated transmission loss at very low frequencies.
2. To illustrate experimentally and interpret the “double-niche” effect.

2. Sound transmission loss increase caused by the reduced radiation coefficient at very low frequencies

At very low frequencies, the sound radiation coefficient of small-area partitions can be much lower than for large-area partitions. This should be the main reason why the sound transmission loss of
small-area partitions at very low frequencies can be notably over the values calculated for the infinite-area models.

2.1 Sound transmission via a circular rigid piston freely moving in the congruent aperture made in an infinite sound-proof wall

Suppose that a circular, flat, and absolutely rigid piston of radius $R$ can freely move in the congruent aperture made in a sound-proof wall of infinite area separating two semi-infinite spaces: here, the noise source is on the left (Fig. 1A). Consider also that the sound wavelength is much bigger than the piston radius: $\lambda \gg R$. In this case, the acoustic pressure is about equally distributed over the piston area, so, the incident sound field can be approximately considered like normal. Suppose also that the wall and piston thicknesses are about similar (in order to neglect the niche effect effects).

Since the amplitude of the incident and reflected waves in the source semi-space are about equal, the equation of vibration motion is expressed in the form

$$2 p_{inc} S \approx Z V$$

where $Z = -i m \omega$ is the piston mechanical impedance, $V$ is the piston velocity amplitude, $p_{inc}$ is the acoustic pressure amplitude of the incident wave, $i = \sqrt{-1}$ is the imaginary unit, $m$ and $S = \pi R^2$ are respectively the piston mass and area, $\omega$ is the angular sound frequency; the time-oscillating factor $\exp(-i \omega t)$ is omitted.

Since the incident sound power $W_{inc} = \frac{|p_{inc}|^2 S}{2 \rho c_0}$ and the radiated sound power is

$$W_{rad} = \frac{1}{2} |V|^2 \Re \{Z_{rad}\}$$

where $Z_{rad}$ is the piston radiation impedance [8], using Eq. (1), calculate the sound transmission coefficient

$$\tau = \tau_{inf} X.$$  

Here $\tau_{inf} = \left| \frac{2 \rho c_0}{Z} \right|^2 = \left( \frac{2 \rho c_0}{M \omega} \right)^2$ is the coefficient of normal-incident sound transmission via an infinite single partition with the surface density $M = \frac{m}{S}$, and the real part of the dimensionless radiation factor is given by the equation

$$X = 1 - \frac{J_1(2kR)}{kR}$$

where the wave number $k = \omega / c_0$ and $J_1(u)$ is the Bessel function of the first kind, order one. The transmission loss is calculated as

$$\text{TL} = 10 \log \left( \frac{1}{\tau} \right) = \text{TL}_{inf} + 10 \log \left( \frac{1}{X} \right)$$

where the value

$$\Delta L = 10 \log \left( \frac{1}{X} \right)$$
is the excess of the transmission loss over the transmission loss of infinite single partition at normal incidence $\text{TL}_\text{inf} = 10 \log \left( \frac{1}{\tau_{\text{inf}}} \right)$.

As known [8], at relatively high frequencies ($ka \rightarrow \infty$), the coefficient $X \rightarrow 1$ and therefore $\text{TL} \approx \text{TL}_\text{inf}$ that fits the normal-incidence Mass Law.

But at very low frequencies ($kR \ll 1$), the coefficient $X \approx (kR)^2 / 2 \ll 1$. In this case, $\Delta L \gg 1$, so, the low-frequency sound transmission loss can notably exceed the normal-incidence Mass-Low.

Figure 1. Mathematical models: A – for the sound transmission via a circular piston in the congruent aperture made in an infinite wall, B – for the sound radiation by a square rigid piston freely moving in the congruent aperture made in the centre of the edge wall of a semi-infinite tunnel.

A similar conclusion can be deduced for a double piston system consisting of two single pistons installed in the same opening and separated with an air gap. In this case, at low frequencies the sound transmission loss of such double piston system should exceed the sound transmission loss for the appropriate infinite double wall.

2.2 Sound radiation by a square rigid piston freely moving in the congruent aperture made in the edge wall of a semi-infinite tunnel

As shown above, the low-frequency sound transmission loss notably depends on the sound radiation coefficient of the panels tested. Now consider a more feasible mathematical model: a square rigid piston is installed in the congruent aperture made in the square edge wall of a semi-infinite tunnel with absolutely rigid side walls (Fig. 1B). The goal is to calculate its sound radiation coefficient.
The side lengths of the piston and edge wall are denoted by \( b \) and \( a \), respectively. Such a study was done by the author earlier [9-11].

At frequencies where only the sound wave with plane fronts (zero modes, or mode-0) is evanescent (can travel in the tunnel), the dimensionless radiation coefficient equals

\[
(6) \quad r = S / S_w = (b/a)^2.
\]

Here, \( S_w = a^2 \) is the wall area, \( S_w = b^2 \) is the piston (aperture) area.

The other modes can travel in the channel if the sound frequency exceeds their cut-off frequencies. The modes are symmetric (mode-0, mode-2, mode-4, etc.) or asymmetric (mode-1, mode-3, etc.) relative to the axial line of the channel (Fig. 2). If the aperture with the rigid piston is arranged symmetrically in the centre of the wall, only the symmetric evanescent modes can be generated in the channel. If the aperture is in the corner of the wall (as shown in Fig. 3), both asymmetric and symmetric evanescent modes are in play. The cut-off frequencies for the mode-1 and mode-2 are

\[
f_{m1} = c_0 / (2a) \quad \text{and} \quad f_{m2} = c_0 / a = 2f_{m2},
\]

respectively. At frequencies over the appropriate cut-off frequency, the sound radiation coefficient goes up from the value given by Eq. (6) to 1. That is, the sound transmission loss increase reduces from \( \Delta L = 20 \log (a/b) \) to zero as shown in Fig. 3 for the particular case \( a / b = 4 \).

Since the cut-off frequency for the mode-2 is twice that for the mode-1, the frequency range of lower sound radiation has a higher upper bound for the central arrangement. In this frequency range, the sound transmission loss of a real partition is expected to be relatively high.

Commonly, the transverse dimensions of reverberation rooms are at least 3 m in each rectangular direction, so, the cut-off frequency \( f_{m2} = c_0 / a > 340 \text{ m/s} / 3 \text{ m} \approx 110 \text{ Hz} \), so, a notable increase in the sound transmission loss is not much drastic in the common frequency range 100 – 5000 Hz. However if the sound wavelength remains to be notably larger than the partition dimensions (that is, at frequencies below \( f_1 \approx c_0 / (2b) \)), the sound transmission loss of single partitions can rather fit the normal-incidence Mass Law and exceed the diffuse-incidence Mass Law by 5-6 dB.

### 2.3 The effect of bending vibration resonances in windows

At the natural frequencies of bending vibration of thin plates, the sound transmission loss may be well below the values calculated the rigid pistons of the same surface density. But according to the Asymptotic Shoch’s Law [12], at relatively high frequencies a finite thin plate, excited by the normal-incidence sound waves, vibrates as the solid piston of the same surface density. In practice, this holds approximately true if the measurements are performed in relatively wide (in particular, one-third octave) frequency bands if their central frequencies are 2-3 times the fundamental natural frequency of the plate. The fundamental natural frequencies of ordinary window panes, standard gypsum boards, and many other single partitions are commonly below 30 - 40 Hz, so, the solid piston model may be approximately applicable at frequencies over 60 - 80 Hz.
Figure 2. The mode shapes in the channel.

Figure 3. The potential increase of the sound transmission loss at very low frequencies if the aperture is made: in the centre (common case) and in the corner of the wall.
3. Niche effect

In the “double-niche” design, the aperture consists of two in-series halves with different areas, and the window is installed in the smaller one. In this case, the niche effect can be alleviated even if the window is centred in the aperture.

3.1 Simplified physics of the niche effect

Currently, there is no undisputed theory of the niche effect. The author’s interpretation is based on the model of an infinite triple window with a massive inner pane and two lightweight outer panes separated with air gaps [7, 9, 13]. Certainly, the niche effect can also be controlled by sound-absorbing material installed inside the niche [14] but such an arrangement is not typical, in particular for windows.

In this interpretation, the inner pane stands for the partition tested. The role of the outer panes is played by adjacent air masses at the aperture edges (Fig. 4). At low frequencies such a partition is simulated by a lumped mass-spring-mass-spring-mass system with two non-zero natural frequencies. If the system is fully symmetric (the outer masses are similar and both springs are equivalent) and the central mass is much over the outer masses, the natural frequencies get close together. Such a phenomenon can notably increase the resonance effects and therefore the transmissibility of the system.

On the other hand, if one of the springs is much stiffer than the other spring, one of the two natural frequencies gets high, and resonance effects at low frequencies are not as harsh as in case of the similar springs. In triple windows (or partitions), such an effect occurs if the middle pane (leaf) is close together to one of the outer panes (leaves) as sketched in Figs 3B and 3C. The same holds true for the partition arrangements in a single niche (4-7, etc.).

3.2 When the pane is centred in the aperture consisting of two in-series halves with different areas

The other way is to reduce one of the outer masses in the lumped mechanical model. As a result, one of the two natural frequencies gets high even if the springs remain similar. For windows, such a situation can be achieved in the “double-niche” sketched in Fig. 4D. Here, the niche cavity facing the receiving room is made of larger area than the other cavity. The surface density of the adjacent air layer reduces with its area. Certainly, the coefficient of the sound radiation into the receiving room increases with the cavity area but this unfavourable effect is compensated by reduction of the sound velocity with the area expansion.

![Figure 4. Window in the aperture made in a massive wall between two reverberation rooms.](image-url)
Figure 5. Sound transmission loss spectra for the window installed as shown in Fig. 4.

In Fig. 5, the transmission loss spectra are plotted for the same sound insulating glass unit in all the four cases sketched in Fig. 4. The glass unit configuration is coded as 10+20+5+2+6; here, 10, 5, and 6 mm are the glass pane thicknesses, mm, and 20 and 2 are the air gap thicknesses, mm). The total thickness of the brick wall between the source and receiving rooms is 0.6 m. The unit is 1.3 m high and 0.9 m wide. It is installed in the narrow semi-niche of the same area $1.3 \times 0.9 \approx 1.2 \text{ m}^2$. The wide half-niche was proportionally increased to the twofold area: $1.8 \times 1.3 \approx 2.4 \text{ m}^2$.

As seen, the niche effect occurs between 160 and 400 Hz where the sound transmission loss for the configurations B, C, and D is 4 dB higher on average than that for the configuration A.

It is noteworthy that sound transmission loss at frequencies below 100 Hz goes up with the frequency reduction. This may occur due to the mass-spring-mass-spring resonance at the frequency of 114 Hz.

But the other important cause can be the effect of low sound radiation at very low frequencies described in Chapter 2 because a similar trend was observed for single panes (6, 8, and 10 mm thick) installed as in Fig. 3A.

4. Conclusions

The simplified interpretations of two important phenomena affecting the sound transmission loss of windows and other lightweight partitions installed in the congruent aperture made in a thick wall were discussed and illustrated with the experimental data.

The first interpretation explains the effect of relatively high sound transmission loss at very low frequencies because of the low sound radiation coefficient. Two simplified models were discussed: (1) a circular rigid piston that can freely move in the aperture made in an infinite rigid wall, (2) a square rigid piston vibrating in the aperture made in the edge rigid wall of a semi-infinite rectangular channel.

The second simplified interpretation is related to the “double-niche” effect (for relatively thick walls). As shown, if the niche cavity facing the receiving room is made of larger area than the other cavity, the unfavourable niche effect can be notably reduced even if the window is centred in the aperture.

The results may be interesting and helpful for students and practical engineers.
REFERENCES