A Solution of the Fermat's Last Theorem

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It is obtained a solution of the Fermat's last theorem.

Key words: Fermat's last theorem.

Theorem: for non-zero positive integers numbers x, y, z and n, the so called Fermat's last theorem says that the equation

$$x^n + y^n = z^n \tag{1}$$

is false for $n > 2$.

Proof: from (1), $z^n > x^n$ and $z^n > y^n$, then $z > x$ and $z > y$, and for $n \geq 2$, from (1) and from the binomial formula, $(x + y)^n = x^n + y^n + \textit{other non-zero positive integers values} > x^n + y^n = z^n$, then $x + y > z$. Also from (1) and for $n \geq 2$, it is for $y = x$, $2x^n = z^n$, then $2^{1/n}x = z$, which is false for the non-zero positive integer number z, then $y \neq x$. We choose $x < y$, then $x < y < z$. And as $(x/z)^k < 1$ and $(y/z)^k < 1$, k being a non-zero positive integer number, then, from (1), $x^{n-k} + y^{n-k} > (x/z)^k x^{n-k} + (y/z)^k y^{n-k} = (x^n + y^n)/z^k = z^n/z^k = z^{n-k}$, that is

$$x^{n-k} + y^{n-k} > z^{n-k} \tag{2}$$

for $n > k \geq 1$. Hence, if $x^n + y^n = z^n$ were true, then from (1) and (2), for these values of x, y and z, we would have that: $x^n + y^n - z^n = 0$, $x^{n-1} + y^{n-1} - z^{n-1} > 0$, $x^{n-2} + y^{n-2} - z^{n-2} > 0$, ..., $x^2 + y^2 - z^2 > 0$, $x + y - z > 0$, where the succession of inequalities represents an increasing deviation from zero, which would imply that

$$x^{n-1} + y^{n-1} - z^{n-1} < x^{n-2} + y^{n-2} - z^{n-2} \tag{3}$$

for $n > 2$. Now, let $a = x^{n-2}$, $b = y^{n-2}$ and $c = z^{n-2}$, and as $x < y < z$, then $x^{n-2} < y^{n-2} < z^{n-2}$ and $a < b < c$, and from (1) it would be

$$ax^2 + by^2 = cz^2 \tag{4}$$

$$a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}y = c^{\frac{n-1}{n-2}}z \tag{5}$$

As $x < y < z$, let $y = x + d$ and $z = x + e$, where d and e are non-zero positive integers numbers, with $d < e$ because $y < z$. Substituting these values into (4) and (5):
$ax^2 + b(x + d)^2 = c(x + e)^2$ and $a^{\frac{n-1}{n-2}}x + b^{\frac{n-1}{n-2}}(x+d) = c^{\frac{n-1}{n-2}}(x+e)$,

$(a + b - c)x^2 - 2(ce - bd)x - (ce^2 - bd^2) = 0$ and $\left(a^{\frac{n-1}{n-2}} + b^{\frac{n-1}{n-2}} - c^{\frac{n-1}{n-2}}\right)x = c^{\frac{n-1}{n-2}}e - b^{\frac{n-1}{n-2}}d$, then

$$x = \frac{(ce-bd) + \sqrt{(ce-bd)^2 + (a+b-c)(ce^2 - bd^2)}}{x^{n-2} + y^{n-2} - z^{n-2}} \quad (6a)$$

$$x = \frac{(ce-bd) - \sqrt{(ce-bd)^2 + (a+b-c)(ce^2 - bd^2)}}{x^{n-2} + y^{n-2} - z^{n-2}} \quad (6b)$$

since $a + b - c = x^{n-2} + y^{n-2} - z^{n-2} > 0$ (that is, ≥ 1, because it is an integer number), and also note that: $ce - bd > 1$ and $ce^2 - bd^2 > 1$, since $b < c$ and $d < e$, and

$$x = \frac{c^{\frac{n-1}{n-2}}e - b^{\frac{n-1}{n-2}}d}{a^{\frac{n-1}{n-2}} + b^{\frac{n-1}{n-2}} - c^{\frac{n-1}{n-2}}} = \frac{z^{n-1}e - y^{n-1}d}{x^{n-1} + y^{n-1} - z^{n-1}} = \frac{zce - ybd}{x^{n-1} + y^{n-1} - z^{n-1}} \quad (7)$$

From (6a): $x = \frac{f + \sqrt{f^2 + g}}{x^{n-2} + y^{n-2} - z^{n-2}}$, where $f = ce - bd > 1$ and $g = (a + b - c)(ce^2 - bd^2) > 1$, then $x < \frac{fg + \sqrt{f^2 g^2}}{x^{n-2} + y^{n-2} - z^{n-2}} = \frac{2g(ce-bd)}{x^{n-2} + y^{n-2} - z^{n-2}}$, because $f > 1$, $g > 1$ (that is, ≥ 2, because it is an integer number) and $f^2 + g < f^2g^2$, since $g < f^2g^2 - f^2 = f^2(g^2 - 1) = f^2(g - 1)(g + 1)$, and from (7): $\frac{zce - ybd}{x^{n-1} + y^{n-1} - z^{n-1}} < \frac{2g(ce-bd)}{x^{n-2} + y^{n-2} - z^{n-2}}$, then, from (3), $\frac{zce - ybd}{2g(ce-bd)} < \frac{x^{n-1} + y^{n-1} - z^{n-1}}{x^{n-2} + y^{n-2} - z^{n-2}} < 1$, and $zce - ybd < 2gce - 2gbd$, $zce - 2gce < ybd - 2gbd$, $(z - 2g)ce < (y - 2g)bd$, $\frac{z-2g}{y-2g} < \frac{bd}{ce} < 1$, $z - 2g < y - 2g$ and $z < y$, which is impossible because $z > y$. And (6b) is also impossible because it would be $x < 0$. Therefore, (1) is false for $n > 2$.

Note: in this proof, I have followed the proving method used in the reference cited in a previous article below. This cited reference was submitted to viXra in 2017-09-07 (yyyy-mm-dd), but was withdrawn from viXra in 2017-09-25.

José Francisco García Juliá, A Minor Theorem Related with the Fermat Conjecture, viXra: 1709.0227 [Number Theory].
http://vixra.org/abs/1709.0227