On Achieving Superluminal Communication

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Abstract

What is proposed here is a simple modification in the quantum protocol [1] for achieving *instantaneous* teleportation of any arbitrary quantum state from Alice to Bob even when Bob is several light years away. This modified quantum protocol is constructed by adding a step the in the celebrated quantum teleportation protocol [1]. It consists of the action of a unitary operator to be performed by Alice on the qubits in her possession before she does the Bell basis measurement. It is important to note that the existing quantum teleportation protocol [1] requires certain classical communication between the participents, Alice and Bob, and we are going to eliminate this classical communication through our modification. In the existing protocol [1] Alice requires to send the classical bits generated during her Bell basis measurement over a classical channel to Bob for Bob to use these classical bits to determine the exact recovery operation to be performed on the qubit(s) in his possession for creating the exactly identical copy of unknown quantum state that was with Alice and got destroyed during her Bell basis measurement. Alice cannot send the classical bits she obtained to Bob with the speed faster than that of light since it is the well known experimentally verified universal upper limit on the speed. We show that by incorporating suitable unitary operation to be performed by Alice on her qubits before the Bell basis measurement the requirement of the transmission of classical bits by Alice to Bob for the creation of the unknown quantum state by Bob at his place can be completely eliminated. Our modification in the teleportation protocols [1], [2] thus clearly demonstrates the enormous advantage of remaining in the quantum regime and avoiding the requirement of any classical communication for the teleportation of the quantum states.

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1. Introduction:

We present a modified quantum teleportation protocol to *instantaneously* teleport an arbitrary unknown quantum state from Alice to Bob even when Alice and Bob are spacelike separated from each other. This is not possible to achieve for the existing well known quantum teleportation protocol developed by C. Bennett et al [1] because in this protocol Alice requires to send certain classical information over a classical channel to Bob to complete the protocol. This classical information is required by Bob to determine the exact recovery operation that he needs to carry out on the qubit(s) in his possession for successfully creating the same unknown quantum state at his place which was originally present with Alice and got destroyed during her Bell basis measurement. This classical information in her possession, generated during the Bell basis measurement, in terms of certain classical bits cannot be sent to Bob with the speed faster than that of light, c, which is the well known experimentally verified universal upper limit on the speed for the transmission of signals over a classical channel. In this paper we suggest an additional unitary operation to be done by Alice on her qubits to circumvent this hurdle for superluminal communication present in terms of the upper limit on speed that exists for any classical communication. This additional unitary operation can cause the elimination of the requirement of transmitting the classical information from Alice to Bob. We circumvent this apparantly unsurmountable difficulty by enforcing the appearance of the classical bits 00 during Alices Bell basis measurement. We show that by carrying out one additional unitary operation in the teleportation protocol [1] before carrying out the Bell basis measurement by Alice we can enforce as the outcome of this measurement the appearance of the classical bits 00 which automatically implies that Bob needs to carry out no operation at all on the qubits in his possession as the desired quantum state has already appeared at his location.

Due to the quantum teleportation protocol developed by C. Bennett et al [1] one can make use of entangled states and non-local influences to teleport an arbitrary unknown quantum state between two locations. This technique transfers the quantum state of the particle to be teleported to another remote particle without the original particle having to traverse the intervening distance. However, in this process, the quantum state of the original particle is necessarily destroyed and the quantum state of the receiving particle becomes a perfect reincarnation of the quantum state of the original particle. But the successful completion of this teleportation protocol requires classical communication between the parties. For the quantum state associated with Bob's particle to become a perfect reincarnation of the quantum state associated with Alice's particle Bob needs to receive two classical bits from Alice received over a classical channel, with speed limited by the velocity of light, to decide about the correct operator to be operated by Bob on the quantum state of his particle to identify it with the quantum state of Alice's particle. Our proposed modification of the existing protocols [1], [2] consists of an additional unitary operation to be performed by Alice before she will carry out the Bell basis measurement offers a way out to achieve total freedom from any classical communication.

2. The teleportation of an arbitrary unknown single qubit quantum state:

In this section we will describe our modified quantum teleportation protocol to *instantaneously* teleport an arbitrary single qubit quantum state from Alice to Bob. We describe here how Alice can *instantaneously* teleport the arbitrary unknown quantum state in her possession to Bob even when Bob could be several light years away from Alice. The modified quantum teleportation protocol involves as before two parties, namely Alice and Bob. Alice starts this protocol with a single qubit state $|\psi\rangle = a|0\rangle + b|1\rangle$, in her possession which is unknown to her except that $|a|^2 + |b|^2 = 1$. Alice and Bob also start out by sharing between them a Bell state, $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ such that the first qubit is in the possession of Alice and the second qubit is with Bob and suppose that Alice wants to teleport the single qubit state $|\psi\rangle$ in her possession to Bob.

The joint state of three qubits, say $|\Phi\rangle$, is

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle].$$

Rewriting the above equation we have

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[|00\rangle a|0\rangle + |01\rangle a|1\rangle + |10\rangle b|0\rangle + |11\rangle b|1\rangle].$$

By expressing the computational basis states made up of first two qubits $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ in the above equation in terms of the standard Bell basis states $|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle$ where $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ as above, $|\beta_{01}\rangle = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$, $|\beta_{10}\rangle = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]$, and $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}[|01\rangle - |10\rangle]$ we get

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(a|0\rangle + b|1\rangle) + |\beta_{01}\rangle(a|1\rangle + b|0\rangle) + |\beta_{10}\rangle(a|0\rangle - b|1\rangle) + |\beta_{11}\rangle(a|1\rangle - b|0\rangle)].$$

It is easy to check further that the above equation can also be expressed as follows:

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(I|\psi\rangle) + |\beta_{01}\rangle(X|\psi\rangle) + |\beta_{10}\rangle(Z|\psi\rangle) + |\beta_{11}\rangle(X.Z|\psi\rangle)],$$

where I, X, Z are standard Pauli operators.

Now, if Alice will perform the partial measurement on the first two qubits in her possession, i.e. if she will perform Bell basis measurement on the two qubits in her possession then any one out of the four Bell basis states $|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle$ will be the outcome with equal probability equal to $\frac{1}{4}$. The Bell basis measurement by Alice yields any one out of the four Bell basis elements with equal probability and this outcome decides what the posteriory state (third qubit) will be with Bob. So, the next step of the well known protocol [1] is to convey the result of the Bell basis measurement done by Alice to Bob in terms of two classical bits over a classical channel so that Bob can perform the appropriate recovery operation for yielding the exactly identical copy of $|\psi\rangle$ as his qubit (third qubit).

It is a well known result that given any two normalized quantum states, $|\mu\rangle$, $|\nu\rangle$ there always exists a unitary operator, U, such that $U|\mu\rangle = |\nu\rangle$. To prove this result one takes $|\mu\rangle$ as the first basis vector, i.e. $|\mu_1\rangle = |\mu\rangle$ and using Gram-Schmidt procedure one constructs other basis vectors, $|\mu_2\rangle$, $|\mu_3\rangle$, $\cdots$ $|\mu_k\rangle$ such that all these vectors together form the orthonormal basis for the vector space. Similarly, for $|\nu\rangle$. One takes $|\nu\rangle$ as the first basis vector, i.e. $|\nu_1\rangle = |\nu\rangle$ and using Gram-Schmidt procedure one constructs other basis vectors, $|\nu_2\rangle$, $|\nu_3\rangle$, $\cdots$ $|\nu_k\rangle$ such that all these vectors together form the orthonormal basis for that vector space. The unitary operator, U, is defined as $U = \sum_j |\nu_j\rangle\langle\mu_j|$. One can easily check that U is indeed unitary and it further satisfies $U|\mu\rangle = U|\mu_1\rangle = |\nu_1\rangle = |\nu\rangle$.

In our modified protocol Alice will first devise a unitary operator, O, which will relate as described in the above paragraph, the two normalized states, $|\Phi\rangle$ and $|\Omega\rangle$, where as given above

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(I|\psi\rangle) + |\beta_{01}\rangle(X|\psi\rangle) + |\beta_{10}\rangle(Z|\psi\rangle) + |\beta_{11}\rangle(X.Z|\psi\rangle)],$$

and where I, X, Z are standard Pauli operators i.e. $|\Phi\rangle$ is the same quantum state given above and let $|\Omega\rangle$ be the quantum state defined as $|\Omega\rangle = |\beta_{00}\rangle|\psi\rangle$ such that the following relation is satisfied, namely, $O|\Phi\rangle = |\Omega\rangle$. As described in the above paragraph Alice will form the orthonormal bases, e.g. Alice constructs the orthonormal basis $|\Omega\rangle = |\Omega_1\rangle = |\beta_{00}\rangle|\psi\rangle$, $|\Omega_2\rangle = |\beta_{01}\rangle|\psi\rangle$, $|\Omega_3\rangle = |\beta_{10}\rangle|\psi\rangle$, and $|\Omega_4\rangle = |\beta_{11}\rangle|\psi\rangle$. It is easy to check that $\langle\Omega_i|\Omega_j\rangle = \delta_{ij}$, where δ_{ij} stands for Kronecker delta function.

Similarly, Alice constructs the orthonormal basis $|\Phi\rangle = |\Phi_1\rangle$, where $|\Phi\rangle$ is as given above and also she further builds $|\Phi_2\rangle$, $|\Phi_3\rangle$, $|\Phi_4\rangle$, where

$$|\Phi_2\rangle = |\psi\rangle|\beta_{01}\rangle = \frac{1}{2}[|\beta_{00}\rangle(X|\psi\rangle) + |\beta_{01}\rangle(I|\psi\rangle) + |\beta_{10}\rangle(X.Z|\psi\rangle) + |\beta_{11}\rangle(Z|\psi\rangle)].$$

$$|\Phi_3\rangle = |\psi\rangle|\beta_{10}\rangle = \frac{1}{2}[|\beta_{00}\rangle(Z|\psi\rangle) - |\beta_{01}\rangle(X.Z|\psi\rangle) + |\beta_{10}\rangle(I|\psi\rangle) - |\beta_{11}\rangle(X|\psi\rangle)].$$

$$|\Phi_4\rangle = |\psi\rangle|\beta_{11}\rangle = \frac{1}{2}[|\beta_{00}\rangle(X.Z|\psi\rangle) - |\beta_{01}\rangle(Z|\psi\rangle) + |\beta_{10}\rangle(X|\psi\rangle) - |\beta_{11}\rangle(I|\psi\rangle)].$$

As before I, X, Z are standard Pauli operators. It is easy to verify that $\langle\Phi_i|\Phi_j\rangle = \delta_{ij}$, as required. Note that as before δ_{ij} stands for Kronecker delta function. We now create the unitary operator $O = \sum_j |\Omega_j\rangle\langle\Phi_j|$. It is straightforward to check that this operator, O, is unitary and it further satisfies the relation

$$O|\Phi\rangle = O|\Phi_1\rangle = |\Omega_1\rangle = |\Omega\rangle = |\beta_{00}\rangle|\psi\rangle.$$

Thus, in our modified protocol Alice first operates the above described unitary operator O, on the first two qubits in her possession and leaves the third qubit in possession of Bob as it is in the state $|\Phi\rangle$ which takes the state $|\Phi\rangle$ to state $|\Omega\rangle = |\beta_{00}\rangle|\psi\rangle$. After this unitary operation Alice then proceeds with the Bell basis measurement as done before in [1]. This action automatically enforces the desired outcome, namely, the classical bits 00, through the Bell bssis measurement by Alice.

The result of this Bell basis measurement by Alice is now as good as determined and certain. It will be $|\beta_{00}\rangle$ with hundred percent guarantee. As a result the classical bits that Alice needs to convey to Bob will (always) be equal to 00 and so need not be conveyed to Bob. Further as a result of this fixed outcome of Alice's measurement Bob requires to operate Identity operator, I, on his qubit i.e. in other words he does not require to perform any recovery operation to get the desired exactly identical quantum state, $|\psi\rangle$, as his qubit since he has already got it.

3. The teleportation of an arbitrary unknown 2-qubit quantum state:

In this section we will show that it is possible to extend our modified quantum teleportation protocol for *instantaneous* teleportation of an arbitrary two qubit quantum state from Alice to Bob. For this case suppose that the arbitrary quantum state which is unknown to Alice and which Alice wants to teleport to Bob is now a two qubit state

$$|\chi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

such that $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. So, this time Alice starts the protocol with quantum state, $|\chi\rangle$ and also Alice and Bob start out with a shared four qubit Generalized Bell basis state or simply G-state. Following G. Rigolin, [2], we take this G-state as

$$|g_1\rangle = \frac{1}{2}[|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle]$$

such that the first two qubits of this G-state are in possession of Alice and the last two qubits are in possession of Bob.

The joint state, $|\Theta\rangle$, this time is made up of six qubits, $|\Theta\rangle = |\chi\rangle|g_1\rangle$, can be expressed as follows:

$$|\Theta\rangle = |\chi\rangle|g_1\rangle = \frac{a}{2}(|000000\rangle + |000101\rangle|001010\rangle + |001111\rangle)$$

$$+\frac{b}{2}(|010000\rangle + |010101\rangle + |011010\rangle + |011111\rangle)$$

$$+\frac{c}{2}(|100000\rangle + |100101\rangle + |101010\rangle + |101111\rangle)$$

$$+\frac{d}{2}(|110000\rangle + |110101\rangle + |111010\rangle + |111111\rangle).$$

In this joint state the first four qubits belong to Alice and the last two qubits belong to Bob. Rewriting the above state we have

$$|\Theta\rangle = |0000\rangle\frac{a}{2}|00\rangle + |0001\rangle\frac{a}{2}|01\rangle + |0010\rangle\frac{a}{2}|10\rangle + |0011\rangle\frac{a}{2}|11\rangle$$

$$+|0100\rangle\frac{b}{2}|00\rangle + |0101\rangle\frac{b}{2}|01\rangle + |0110\rangle\frac{b}{2}|10\rangle + |0111\rangle\frac{b}{2}|11\rangle$$

$$+|1000\rangle\frac{c}{2}|00\rangle + |1001\rangle\frac{c}{2}|01\rangle + |1010\rangle\frac{c}{2}|10\rangle + |1011\rangle\frac{c}{2}|11\rangle$$

$$+|1100\rangle\frac{d}{2}|00\rangle + |1101\rangle|\frac{d}{2}01\rangle + |1110\rangle\frac{d}{2}|10\rangle + |1111\rangle\frac{d}{2}|11\rangle.$$

Now, expressing all the sixteen computational basis states, $|0000\rangle, |0001\rangle, \cdots, |1111\rangle$, in the above equation which are now made up of first four qubits, in terms of G-states following [2], namely, in terms of $|g_1\rangle, |g_2\rangle, \cdots, |g_{16}\rangle$, as given in [2] we can rewrite above equation as:

$$|\Theta\rangle = \frac{1}{4}\sum_{j=1}^{16}|g_j\rangle|\phi_j\rangle$$

where $|\phi_j\rangle = O_j|\chi\rangle$ and O_j for all j are certain unitary operators composed of standard Pauli operators which can be easily determined and further it can be easily checked that $O_1 = I$, the Identity operator. Therefore, the above equation can be further written as

$$|\Theta\rangle = \frac{1}{4}|g_1\rangle(I|\chi\rangle) + \frac{1}{4}\sum_{j=2}^{16}|g_j\rangle|(O_j|\chi\rangle).$$

If Alice now will make the Generalized Bell basis measurement or simply G-basis measurement (this is actually a partial measurement on her four qubits) as is done in usual teleportation method [1] then she will obtain with equal probabilities one of the 16 number of G-states and the value of these equal probabilities will be $\frac{1}{16}$. In this case the posteriory state with Bob will be dependent upon the outcome of G-basis measurement by her and she will be required to convey this outcome in terms of four classical bits to Bob on a classical channel for Bob to decide the correct recovery operation to be carried out on the qubits (the quantum state) in his possession to recover the exactly identical copy of Alice's quantum state, $|\chi\rangle$ (i.e. for Bob to determine the appropriate inverse operator,

$(O_j)^{-1}$, to be operated on the qubits in his possession to recover the exactly identical copy of $|\chi\rangle$ as his qubits).

In our modified protocol Alice will proceed on identical lines as in the case discussed in the above section for single qubit teleportation. Here also before Alice carrying out Bell basis measurement she prepares a unitary operator, P, to be operated on $|\Theta\rangle$ where

$$|\Theta\rangle = \frac{1}{4}|g_1\rangle(I|\chi\rangle) + \frac{1}{4}\sum_{j=2}^{16}|g_j\rangle|(O_j|\chi\rangle),$$

as given above such that this action produces the following result: $P|\Theta\rangle = |\Xi\rangle = |g_1\rangle|\chi\rangle$. Note that here also as done before, in order to build the unitary operator, P, for our modified protocol Alice will form the orthonormal bases, i.e. Alice will construct the orthonormal basis $|\Xi\rangle = |\Xi_1\rangle = |g_1\rangle|\chi\rangle$, $|\Xi_2\rangle = |g_2\rangle|\chi\rangle, \cdots, |\Xi_{16}\rangle = |g_{16}\rangle|\chi\rangle$. It is easy to check that $\langle\Xi_i|\Xi_j\rangle = \delta_{ij}$, where δ_{ij} stands for Kronecker delta function. Similarly, Alice constructs the orthonormal basis $|\Theta\rangle = |\Theta_1\rangle$, where $|\Theta\rangle$ is as given above and also she further proceed with constructing $|\Theta_2\rangle = |\chi\rangle|g_2\rangle$, $|\Theta_3\rangle = |\chi\rangle|g_3\rangle, \cdots, |\Theta_{16}\rangle = |\chi\rangle|g_{16}\rangle$, such that $\langle\Theta_i|\Theta_j\rangle = \delta_{ij}$, where as before δ_{ij} stands for Kronecker delta function. She then creates the above mentioned unitary operator $P = \sum_j |\Xi_j\rangle\langle\Theta_j|$. It is straightforward to check that this operator, P, is indeed a unitary operator and as required it satisfies the relation $P|\Theta\rangle = P|\Theta_1\rangle = |\Xi_1\rangle = |\Xi\rangle = |g_1\rangle|\chi\rangle$.

Thus, in our modified protocol Alice first operates the above described unitary operator P, on the first four qubits in her possession and leaves the next two qubits in possession of Bob as they are in the state $|\Theta\rangle$ which takes the state $|\Theta\rangle$ to state $|\Xi\rangle = |g_1\rangle|\chi\rangle$. After this unitary operation Alice then proceeds with the Bell basis measurement as done before in [1] on the first four qubits in her possession. This action automatically enforces the desired outcome, namely, the classical bits 0000, through the Bell basis measurement which automatically implies the appearance of the desired quantum state $|\chi\rangle$ as Bob's two qubits.

4. The teleportation of an arbitrary unknown n-qubit quantum state:

By proceeding on exactly similar lines of the above two sections it is possible to extend our modified quantum teleportation protocol to *instantaneous* teleportation of an arbitrary n-qubit quantum state unknown to Alice from Alice to Bob. As is done in previous two sections here Alice starts her protocol with an n-qubit state and also Alice and Bob start out with an $2n$-qubit shared state, namely, a suitably chosen G-state such that the first n qubits are with Alice and the last n qubits are with Bob. As is described in [2] we proceed on exactly same lines and finally execute our modification of operating by the suitably defined unitary operator on the qubits in possession of Alice and leaving qubits in Bob's possession as they are before proceeding with G-basis measurement which will lead (as happened in previous two cases discussed in the earlier sections) to the desired appearance at Bob's place the exactly identical n-qubit quantum state with which Alice started her protocol and which got destroyed at the time of Alice's G-basis measurement. The modified algorithm thus enforces the reincarnation of Alice's state at Bob's place.

5. A Remark:

Achieving superluminal communication will now be possible and is no longer a misapprehension for our new modified quantum teleportation protocol.

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