On Achieving Superluminal Communication

Dhananjay P. Mehendale*

Department of Physics, Savitribai Phule University of Pune,Pune,India-411007.

(Dated: October 23, 2017)

Abstract

We develop a simple yet impossible looking quantum protocol for achieving *instantaneous* teleportation of any arbitrary quantum state from Alice to Bob even when Bob is several light years away. We construct this quantum protocol by approriately combining two celebrated results: the existing quantum teleportation protocol [1] and the quantum algorithm for searching an unknown target [2]. The existing quantum teleportation protocol [1] requires certain classical communication between the participents, Alice and Bob. Alice has to send certain classical information in terms of classical bits generated during her Bell basis measurement over a classical channel to Bob using which Bob determines the exact recovery operation to be performed on the qubit(s) in his possession for the creation of the same unknown quantum state at his place and thus to complete the protocol. This classical information in Alice's possession in terms of certain classical bits cannot be sent to Bob with the speed faster than that of light which is the well known experimentally varified universal upper limit on the speed for the transmission of signals over a classical channel. We show that by appropritely using Grover's algorithm [2] at the appropriate place in the teleportation protocol [1] and its extension for teleporting multiqubit state [6] we can eliminate the requirement of the transmission of the classical bits by Alice over a classical channel to Bob for the creation of the unknown quantum state at his place and thus provide an eloquent way out to free ourselves from the universal upper limit on speed that is preventing us from the superluminal information transfer. Thus our new modified teleportation protocol clearly demonstrates the enormous advantage of remaining in the quantum regime and avoiding the requirement of any classical communication.

* dhananjay.p.mehendale@gmail.com

1. Introduction:

We present a modified quantum teleportation protocol to *instantaneously* teleport an arbitrary unknown quantum state from Alice to Bob even when Alice and Bob are spacelike separated from each other. This is not possible to achieve for the existing well known quantum teleportation protocol developed by C. Bennett et al [1] because in this protocol Alice requires to send certain classical information over a classical channel to Bob to complete the protocol. This classical information is required by Bob to determine the exact recovery operation that he needs to carry out on the qubit(s) in his possession for successfully creating the same unknown quantum state at his place which was originally present with Alice and got destroyed during her Bell basis measurement. This classical information in her possession, generated during the Bell basis measurement, in terms of certain classical bits required by Bob to complete the quantum teleportation protocol cannot be sent to Bob with the speed faster than that of light, c, which is the well known experimentally varified "universal upper limit on the speed" for the transmission of signals over a classical channel. This is justified according to special theory of relativity (STR) due to A. Einstein [3]. According to STR the energy, E, of a body with positive rest mass m is γmc^2, where c stands for velocity of light and $\gamma = \frac{1}{\sqrt{(1-\frac{v^2}{c^2})}}$, so, clearly $E \rightarrow \infty$ when velocity of the body, $v \rightarrow c$. This implies that infinite energy is required to take the velocity, v, of a body with positive rest mass beyond c, i.e. c is the upper limit for velocity. In this paper we suggest a clever trick to circumvent this hurdle for superluminal communication in terms of the requirement of classical communication for the successful completion of the existing quantum teleportation protocol [1] which is holding us back from achieving the superluminal communication. The trick we suggest here is extermely simple. We circumvent this apparantly unsurmountable difficulty by just eliminating the requirement of transfering this classical information to Bob and still successfully construct the unknown quantum state at Bob's location which could be any number of light years away from Alice and we gurrentedly manage this with hundred percent assurance. We show that by appropritely using Grover's algorithm [2] at the appropriate place in the teleportation protocol [1] and amplifying the amplitude of the appropriately defined target state before carrying out the Bell basis measurement by Alice we can enforce as the outcome of this measurement the appearance of the classical bits 00 in this Bell basis measurement which will further imply that Bob requires to operate only identity operator on the state in his possession, i.e. Bob needs to carry out no operation at all and thus this appropriately appending of Grover's algorithm before the step of the Bell basis measurement, as is done in [1], enables us to eliminate the requirement of the transmission of the classical bits by Alice over a classical channel to Bob and thus provides us an eloquent way out to free ourselves from the universal upper limit on speed that was preventing us from the superluminal information transfer. Thus our new modified teleportation protocol clearly demonstrates the enormous advantage of remaining in the quantum regime and managing the attainment of the goal of superluminal information transfer without any kind of classical communication.

The true teleportation was considered impossible till recently because every one felt that it would require some sort of scanning, or measurement, operation in order to extract a precise description of all the particles of which the object to be teleported is made up of. At the very least one requires the simultaneous knowledge of positions and momenta of all those particles composing the object and this is forbidden by the well known "uncertainty principle" [4]. The true teleportation was also considered impossible because every one felt that it would at least require making a copy of the object to be teleported and making such a copy is forbidden by the well known "no cloning theorem" [5]. This situation changed in 1993 due to a paper by C. Bennett et al [1] as they showed that one can exploit entangled states and non-local influences to circumvent the limitations of uncertainty principle and no cloning theorem and can teleport an arbitrary unknown quantum state between two locations in such a manner that the state did not traverse the intervenibg distance. This technique transfers the quantum state of the particle to be teleported to another remote particle without the original particle having to traverse the intervening distance. however, in this process, the quantum state of the original particle is necessarily destroyed and the quantum state of the receiving particle becomes a perfect reincarnation of the original. But the successful completion of this teleportation protocol requires classical communication between the parties. For the quantum state associated with Bob's particle to become a perfect reincarnation of the quantum state associated with Alice's particle Bob needs to receive two classical bits from Alice received over a classical channel, with speed limited by the velocity of light, to decide about the correct operator to be operated on the quantum state of his particle to identify it with the quantum state of Alice's particle. Thus, the existing quantum teleportation protocol [1] and its extension for teleporting multiqubit states [6] has shown us a way for teleportation but this teleportation does not take us in the realm of superluminal communication. Our quantum teleportation protocol on the other hand is a strikingly simple and also very effective modification of the existing protocol [1] and its extension for teleporting multiqubit states [6] carried out with the help of Grover's algorithm [2] which offers us a total freedom from any classical communication which creats a hindrance to attain the aim of achieving superluminal communication.

2. The teleportation of an arbitrary unknown single qubit quantum state:

In this section we will describe our modified quantum teleportation protocol to *instantaneously* teleport an arbitrary single qubit quantum state from Alice to Bob. We describe here how Alice can *instantaneously* teleport the arbitrary unknown quantum state in her possession to Bob even when Bob could be several light years away from Alice. The modified quantum teleportation protocol involves as previous two parties, namely Alice and Bob. Alice starts this protocol with a single qubit state $|\psi\rangle = a|0\rangle + b|1\rangle$, in her possession which is unknown to her such that $|a|^2 + |b|^2 = 1$. Alice and Bob also start out by sharing between them a Bell state, $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ such that the first qubit is in the possession of Alice and the second qubit is with Bob who has now gone several light years away from her and suppose that Alice wants to teleport the single qubit state $|\psi\rangle$ in her possession to Bob.

Now, the joint state of three qubits is

$$|\psi\rangle|\beta_{00}\rangle = a|000\rangle + a|011\rangle + b|100\rangle + b|111\rangle.$$

Rewriting the above equation we have

$$|\psi\rangle|\beta_{00}\rangle = |00\rangle a|0\rangle + |01\rangle a|1\rangle + |10\rangle b|0\rangle + |11\rangle b|1\rangle.$$

Now expressing computational basis states made up of first two qubits $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ in the above equation in terms of the standard Bell basis states $|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle$ where $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ as above, $|\beta_{01}\rangle = \frac{1}{\sqrt{2}}[|01\rangle + |10\rangle]$, $|\beta_{10}\rangle = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle]$, and $|\beta_{11}\rangle = \frac{1}{\sqrt{2}}[|01\rangle - |10\rangle]$ we get

$$|\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(a|0\rangle + b|1\rangle) + |\beta_{01}\rangle(a|1\rangle + b|0\rangle) + |\beta_{10}\rangle(a|0\rangle - b|1\rangle) + |\beta_{11}\rangle(a|1\rangle - b|0\rangle)].$$

It is easy to check further that the above equation can also be expressed as follows:

$$|\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(I|\psi\rangle) + |\beta_{01}\rangle(X|\psi\rangle) + |\beta_{10}\rangle(Z|\psi\rangle) + |\beta_{11}\rangle(X.Z|\psi\rangle)],$$

where I, X, Z are standard Pauli operators.

Now, if Alice will perform the partial measurement on the first two qubits in her possession, i.e. if she will perform Bell basis measurement on the two qubits in her possession then any one out of the four Bell basis states $|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle$ will be the outcome with equal probability which will be equal to $\frac{1}{4}$. After Bell basis measurement by Alice yielding one out of the four Bell basis elements what the posteriory state (third qubit) Bob will have will depend on the outcome of Alice's Bell basis measurement. So, the next step of the well known protocol [1] is to convey the result of the Bell basis measurement done by Alice to Bob in terms of two classical bits over a classical channel so that Bob can perform the appropriate recovery operation for yielding the exactly identical copy of $|\psi\rangle$ as his qubit (third qubit). Our modified protocol $differs$ at this step.

Before carrying out the Bell basis measurement we perform following standard steps carried out for searching an unknown target in the Grover's algorithm [2] as follows:

Step 1: We take the starting state (representing equally weighted superposition in standard Grover's algorithm) as follows:

$$|\Phi\rangle = |\psi\rangle|\beta_{00}\rangle = \frac{1}{2}[|\beta_{00}\rangle(I|\psi\rangle) + |\beta_{01}\rangle(X|\psi\rangle) + |\beta_{10}\rangle(Z|\psi\rangle) + |\beta_{11}\rangle(X.Z|\psi\rangle)],$$

where I, X, Z are standard Pauli operators.

Step 2: We take the target state, $|t\rangle$, as follows:

$$|t\rangle = |\beta_{00}\rangle(I|\psi\rangle).$$

Step 3: We define the so called phase inversion operator, O, as follows:

$$O = I - 2|t\rangle\langle t|.$$

Step 4: We define the operator causing the so called inversion about the mean, W, as follows:

$$W = 2|\Phi\rangle\langle\Phi| - I.$$

Step 5: We finally operate the operators in succession, first O and then W on $|\Phi\rangle$ yielding

$$WO|\Phi\rangle = |\beta_{00}\rangle(I|\psi\rangle) = |\beta_{00}\rangle(|\psi\rangle).$$

We now ask Alice to carry out Bell basis measurement on her qubits (first two qubits) as is done in the usual protocol [1]. The result of this Bell basis measurement is as good as determined and certain now. It will be $|\beta_{00}\rangle$ with hundred percent gurrentee. As a result the classical bits that Alice needs to convey to Bob will (always) be equal to 00 and so need not be conveyed to Bob. Further as a result of this fixed outcome of Alice's measurement Bob requires to operate Identity operator, I, on his qubit i.e. in other words he does not require to perform any recovery operation to get the desired exactly identical quantum state, $|\psi\rangle$, as his qubit since he has already got it.

3. The teleportation of an arbitrary unknown 2-qubit quantum state:

In this section we will show that it is possible to extend our modified quantum teleportation protocol for *instantaneous* teleportation of an arbitrary two qubit quantum state from Alice to Bob. For this case suppose that the arbitrary quantum state which is unknown to Alice and which Alice wants to teleport to Bob is now a two qubit state

$$|\chi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

such that $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. So, this time Alice starts the protocol with quantum state, $|\chi\rangle$ and also Alice and Bob start out with a shared four qubit Generalized Bell basis state or simply G-state. Following G. Rigolin, [6], we take this G-state as

$$|g_1\rangle = \frac{1}{2}[|0000\rangle + |0101\rangle + |1010\rangle + |1111\rangle]$$

such that the first two qubits of this G-state are in possession of Alice and the last two qubits are in possession of Bob. Let us suppose that (after shring this state Bob has moved now several light years away from Alice.

The joint state, $|\Theta\rangle$, this time is made up of six qubits, $|\Theta\rangle = |\chi\rangle|g_1\rangle$, can be expressed as follows:

$$
\begin{aligned}
|\Theta\rangle = & \frac{a}{2}(|000000\rangle + |000101\rangle|001010\rangle + |001111\rangle) \\
& + \frac{b}{2}(|010000\rangle + |010101\rangle + |011010\rangle + |011111\rangle) \\
& + \frac{c}{2}(|100000\rangle + |100101\rangle + |101010\rangle + |101111\rangle) \\
& + \frac{d}{2}(|110000\rangle + |110101\rangle + |111010\rangle + |111111\rangle).
\end{aligned}
$$

In this joint state the first four qubits belong to Alice and the last two qubits belong to Bob. Rewriting the above state we have

$$
\begin{aligned}
|\Theta\rangle = & |0000\rangle\frac{a}{2}|00\rangle + |0001\rangle\frac{a}{2}|01\rangle + |0010\rangle\frac{a}{2}|10\rangle + |0011\rangle\frac{a}{2}|11\rangle \\
& + |0100\rangle\frac{b}{2}|00\rangle + |0101\rangle\frac{b}{2}|01\rangle + |0110\rangle\frac{b}{2}|10\rangle + |0111\rangle\frac{b}{2}|11\rangle \\
& + |1000\rangle\frac{c}{2}|00\rangle + |1001\rangle\frac{c}{2}|01\rangle + |1010\rangle\frac{c}{2}|10\rangle + |1011\rangle\frac{c}{2}|11\rangle \\
& + |1100\rangle\frac{d}{2}|00\rangle + |1101\rangle|\frac{d}{2}01\rangle + |1110\rangle\frac{d}{2}|10\rangle + |1111\rangle\frac{d}{2}|11\rangle.
\end{aligned}
$$

Now, expressing all the sixteen computational basis states, $|0000\rangle, |0001\rangle, \cdots, |1111\rangle$, in the above equation which are now made up of first four qubits, in terms of Generalized Bell basis states or simply G-states following [6], namely, in terms of $|g_1\rangle, |g_2\rangle, \cdots, |g_{16}\rangle$, as given in [6] we can rewrite above equation as:

$$|\Theta\rangle = \frac{1}{4}\sum_{j=1}^{16} |g_j\rangle|\phi_j\rangle$$

where $|\phi_j\rangle = O_j|\chi\rangle$ and O_j for all j are certain unitary operators composed of standard Pauli operators which can be easily determined and further it can be easily checked that $O_1 = I$, the Identity operator. Therefore, the above equation can be further written as

$$|\Theta\rangle = \frac{1}{4}|g_1\rangle(I|\chi\rangle) + \frac{1}{4}\sum_{j=2}^{16} |g_j\rangle|(O_j|\chi\rangle).$$

If Alice now will make the Generalized Bell basis measurement (this is actually a partial measurement on her four qubits) as is done in usual teleportation method [1] then she will obtain with equal probabilities one of the 16 Generalized Bell basis states or simply G-states and the value of these equal probabilities will be $\frac{1}{16}$. In this case the posteriory state with Bob will be dependent upon the outcome of Generalized Bell basis measurement by her and she will be required to convey this outcome in terms of four classical bits to Bob on a classical channel for Bob to decide the correct recovery operation to be carried out on the qubits (the quantum state) in his possession to recover the exactly identical copy of Alice's quantum state, $|\chi\rangle$ (i.e. for Bob to determine the appropriate inverse operator, $(O_j)^{-1}$, to be operated on the qubits in his possession to recover the exactly identical copy of $|\chi\rangle$ as his qubits).

Our modified protocol differs at this step. Before carrying out the Bell basis measurement we perform following standard steps carried out for searching an unknown target in the Grover's algorithm [2] as follows:

Step 1: We take starting state (representing equally weighted superposition in standard Grover's algorithm) as follows:

$$|\Theta\rangle = \frac{1}{4}|g_1\rangle(I|\chi\rangle) + \frac{1}{4}\sum_{j=2}^{16}|g_j\rangle|(O_j|\chi\rangle).$$

where I, O_j for all j are standard Pauli operators which can be determined by following the steps as in [6].

Step 2: We take the target state, $|t\rangle$, as follows:

$$|t\rangle = |g_1\rangle(I|\chi\rangle).$$

Step 3: We define the so called phase inversion operator, O, as follows:

$$O = I - 2|t\rangle\langle t|.$$

Step 4: We define the operator causing the so called inversion about the mean, W, as follows:

$$W = 2|\Theta\rangle\langle\Theta| - I.$$

Step 5: We now finally operate the operators in succession, first O and then W on $|\Theta\rangle$, in the present case R times where $R = \frac{\pi}{4}\sqrt{2^4} = R*$, say (since in general n qubit case the number of required Grover's iterations are $R = \frac{\pi}{4}\sqrt{2^n}$, for reaching the target) yielding

$$(WO)^{R*}|\Theta\rangle = |g_1\rangle(I|\chi\rangle) = |g_1\rangle(|\chi\rangle).$$

Now we ask Alice to carry out Generalized Bell basis measurement on her qubits (first four qubits) as is done in the usual protocol [1] and its extension [6]. The result of this Bell basis measurement is as good as determined and certain now. It will be $|g_1\rangle$ with hundred percent gurrentee. As a result the classical bits that Alice needs to convey to Bob will be always equal to 0000 and so need not be conveyed to Bob and further as a result of this outcome of Alice's measurement Bob does not require to perform any recovery operation to get the desired exactly identical quantum state, $|\chi\rangle$, which originally was with Alice and got destroyed as a result of her Generalized Bell basis measurement, since he has already got the exactly identical copy of that desired 2-qubit quantum state, $|\chi\rangle$.

4. The teleportation of an arbitrary unknown n-qubit quantum state:

By proceeding on exactly similar lines of the above two sections it is possible to extend our modified quantum teleportation protocol to *instantaneous* teleportation of an arbitrary n-qubit quantum state unknown to Alice from Alice to Bob. As is done in previous two sections here Alice starts her protocol with an n-qubit state and also Alice and Bob start out with an $2n$-qubit shared state, namely, a suitably chosen Generalized bell basis state or simply G-state such that the first n qubits are with Alice and the last n qubits are with Bob. As is described in [6] we proceed on exactly same lines and finally execute our modification of exercising Grover's search on the joint state before proceeding with Generalized Bell basis measurement which will lead (as happened in previous two sections) to the desired possession by Bob of the exactly identical n-qubit quantum state with which Alice started her protocol and which got destroyed at the time of Alice's Generalized Bell basis measurement.

5. A Remark:

Achieving superluminal communication will now be possible and is no longer a misapprehension for our new modified quantum teleportation protocol.

Acknowledgement

I thank Dr. M. R. Modak, S. P. College, Pune-411030, India, for his encouragement.

---

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).

[2] L. K. Grover, Proceedings of the 28th Annual ACM Symposium on the Theory of Computing,ACM Press, New York, pp 212-219, (1996).

[3] A. Einstein, Annalen der Physik, 17, 890-921 (1905).

[4] W. Heisenberg, Zeitschrift fur Physik, 43(3-4), 172-198 (1927).

[5] W. Wootter, Zurek, Wojciech, Nature, 299, 802-803 (1982).

[6] G. Rigolin, Phys. Rev. A. 71(3), 032303, (2005).