A Two-connected graph with Gallai's property

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Abstract:

In this note two graphs are provided with the property that everyone vertex is missed by some longest cycle.

Keywords: Hypohamiltonian; hypotraceable; Hamiltonian; Gallai's property; Zamfirescu criterion;

Introduction

A cycle that passes through each of the vertices once and only once and ends on the same vertex in graph G is called Hamiltonian cycle (or Hamiltonian circuit). A path that also visit through every vertex one time with no recurrences, however does not have to start and end at the similar vertex in graph G is said to be Hamiltonian path. A graph is said to be traceable if it has a Hamiltonian path and a graph is said to be hamiltonian if it has a Hamiltonian cycle. A graph G is hypo-hamiltonian, if it is not hamiltonian however capable by omitting at all one vertex from G is hamiltonian, a well-known counterexample of existence hypo-hamiltonian is Petersen graph.

Motivated by way of the presence of hypo-hamiltonian graphs and earlier the modernization of the analogously clear hypotraceable graphs, In 1966, Tibor Gallai [1] raised the question about the existence of a finite graph with the property that everyone vertex is missed by some largest path, a situation feebler than hypotraceability. Just later, in 1969, Gallai's question was first replied through H. Walther [2], who set up a graph on 25 vertices satisfying Gallai's criterion and was a planar. Far ahead H. Walther and H. Voss [3] introduced with 12 nodes also self-sufficiently, by way of Tudor Zamfirescu [4]. For planar graphs, the lowest illustration as far as this, with 17 vertices, was set up by W. Schmitz [5]. The first two-connected planar graph created through Tudor Zamfirescu [6] with 82 nodes. The lowest illustration famous nowadays has 26 nodes [7], conversely the lowest planar example up to now has order 32 [6].

In 1972, Tudor Zamfirescu [4] questioned a number of nontrivial questions related to the Gallai's property. let $\mathrm{P}_i^j = \infty$ ($\overline{\mathrm{P}}_i^j = \infty$) if there is not any i −connected graph (planar graph) such that individually set of j points remains disjoint from some longest path condition $\mathrm{P}_i^j \neq \infty$ ($\overline{\mathrm{P}}_i^j \neq \infty$), let P_i^j($\overline{\mathrm{P}}_i^j$) indicate the smallest quantity of vertices of an i −connected graph (planar graph) such that individually set of j vertices be there disjoint from some largest path. Analogously are clear C_i^j and $\bar{C}_i^j$ for longest circuits as a replacement for longest path. As a number of moral instances replying Tudor Zamfirescu's demands were circulated in the succeeding years by means of W. Schmitz [5], H. Walther [8] provided examples of $C_2^2 \leq 220$ and $\bar{C}_2^1 \leq 105$ [2], B. Grunbaum, [9], W. Hatzel [10], Tudor. Zamfirescu [6], see also [11], [12].

The purpose of this note is to show graphs, the one non-planar graph with 12 vertices and the other created on Mobius strip is also with 12 vertices.

Theorem 1. There be existing a non-planar graph G with 12 vertices and is a 2- connected enjoying Gallai's property.

Proof: One has only to plane that, for individually vertex v in graph G, exist langest cycle missing v. We use Fig. 1 and 2.

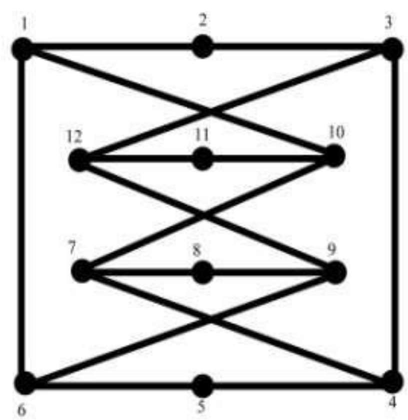


Figure 1.

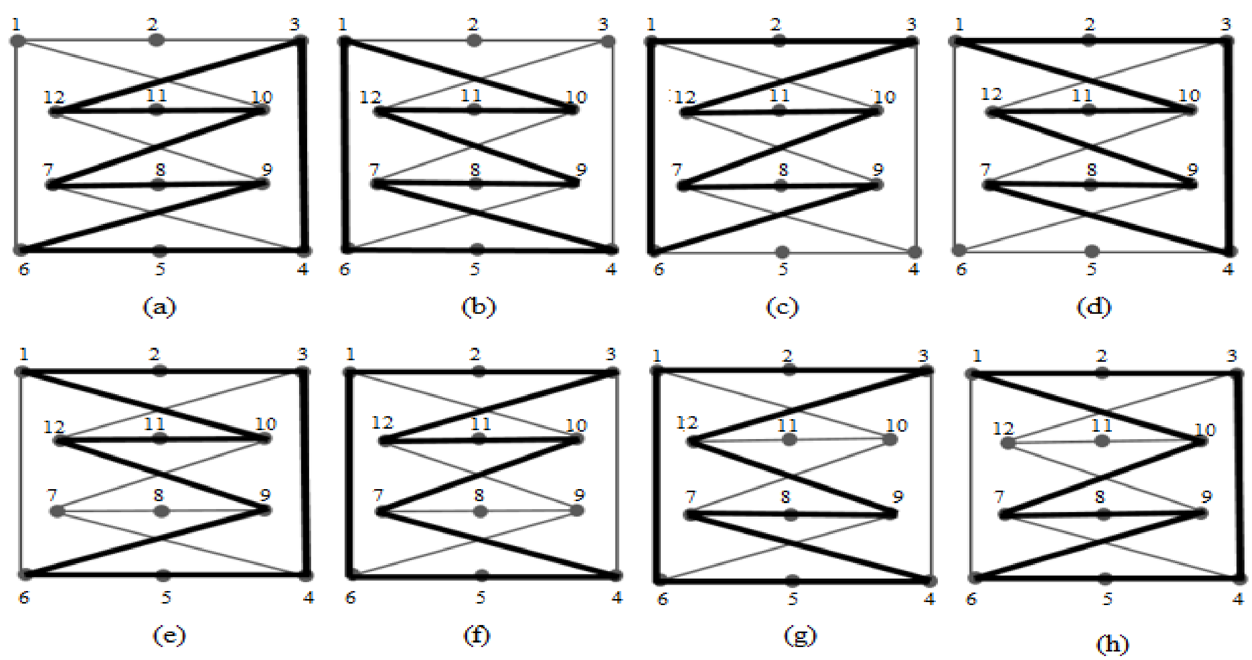


Figure 2. Longest Cycles

Consider the graph G of figure 1, With 12 vertices, let W be a longest cycle in G , the longest cycle of G have length $C(G) = 10$ avoiding v with $W \cap V = \emptyset$ empty intersection of all its longest Cycles. Figure 2, shows longest Cycles and all vertices are missed by each of them, underlined vertices are as of 1 to 12, where each vertex is avoided by some longest Cycle. Also to confirmation that all vertices avoided by individually of the longest cycles, in table 1.

Theorem 2. The graph created on Moebius band (or Mobius strip) satisfying the corresponding Gallai's criterion

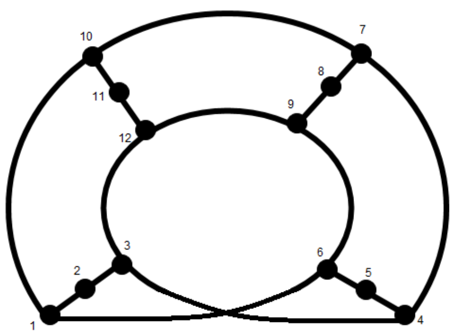
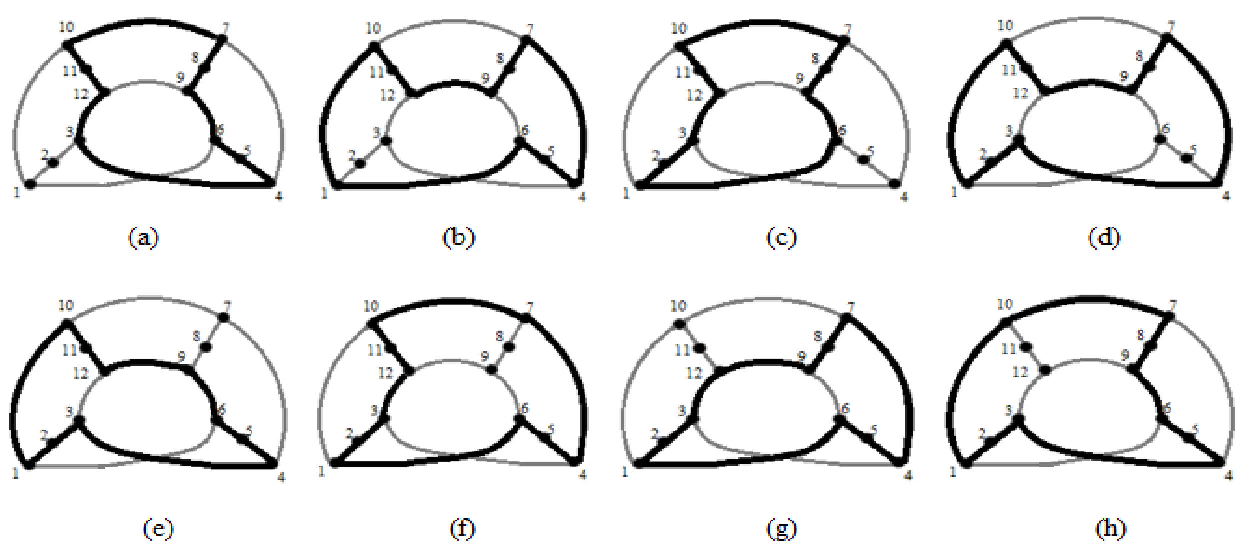


Figure 3. Moebius band



To show that completely vertices avoided as each of the largest cycle, in table 1

Table 1

Fig.	Largest Cycle	Vertices missed	Fig.	Largest Cycle	Vertices missed
(a)	12, 11, 10,7,8, 9, 6, 5, 4, 3, 12	1, 2	(e)	2, 1, 10, 11, 12, 9, 6, 5, 4, 3, 2	7, 8
(b)	10, 11, 12, 9, 8, 7, 4,5, 6, 1, 10	2, 3	(f)	2, 3, 12, 11, 10, 7, 4, 5, 6, 1, 2	8, 9
(c)	2, 3, 12, 11, 10, 7, 8, 9, 6, 1, 2	4, 5	(g)	2, 3, 12, 9, 8, 7, 4, 5, 6, 1, 2	10, 11
(d)	7, 8, 9, 12, 11, 10, 1, 2, 3, 4, 7	5, 6	(h)	2, 1, 10, 7, 8, 9, 6, 5, 4, 3, 2	11, 12

Respectively appearances the point out vertices are missed starting 1 to 12, as well appearances the highlighted, vertices are avoided consequently for each vertex is missed through some largest cycle.

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