Graphs with non-concurrent Longest Paths

ABDUL NAEEM KALHORO [a] **,** ***ALI DINO JUMANI*** [b]

[a, b] *Department of Mathematics, Shah Abdul Latif University Khairpur 66020, Pakistan*

[a] *anksalu@gmail.com* , [b] *alidino.jumani@salu.edu.pk*

Abstract

In this two graphs are presented with the property that each vertex is missed by some longest Path.

Keywords: Hamiltonian path; longest path; hypotraceable; Gallai's property;

Introduction

A path that passes every vertex once time with no repetitions, however does not have to start and end at the similar vertex in graph G is said to be Hamiltonian path. A graph is said to be traceable if it has a Hamiltonian path. A graph G is a hypotraceable if G has no Hamiltonian path but $G - v$ has a Hamiltonian path for every $v \in V$. A cycle which contains all the vertices of G is called Hamiltonian cycle of the graph and the graph is called the Hamiltonian graph. A graph which is non-Hamiltonian but $G - v$ is Hamiltonian for all vertices v is called a hypo-Hamiltonian. The most famous example of hypohamiltonian graph is Petersen graph.

The existence of hypotraceable graphs, on the contrary, was exposed much later. Motivated by way of the presence of hypo-hamiltonian graphs and earlier the modernization of the analogously clear hypotraceable graphs, in 1966 T. Gallai [1] asked whether there exist connected graphs in which the with the property that every vertex is missed by some longest path. Just later, in 1969, Gallai's question was first responded through H. Walther [2], who constructed a graph on 25 vertices satisfying Gallai's statement and was a planar. Later H. Walther and H. Voss [3] introduced such kind of graph with 12 nodes also autonomously, by Tudor Zamfirescu [4]. For planar graphs, the lowest illustration as far as this, with 17 nodes, was constructed by W. Schmitz [5]. A smallest non-planar hpotraceable graph of order 34 introduced by Thomassen [14].The first 2-connected planar graph generated through Tudor Zamfirescu [6] with 82 nodes. The lowest illustration famous nowadays has 26 nodes [7], conversely the lowest planar example up to now has order 32 [6].

In 1972, Tudor Zamfirescu [4] questioned a number of nontrivial questions related to the Gallai's property. let $\mathrm{P}_{\mathrm{i}}^{\mathrm{j}} = \infty$ $(\overline{\mathrm{P}}_{\mathrm{i}}^{\mathrm{j}} = \infty)$ if there is not any i −connected graph (planar graph) such that individually set of j

points remains disjoint from some longest path condition $P_i^j \neq \infty$ ($\overline{P}_i^j \neq \infty$), let P_i^j ($\overline{P}_i^j$) indicate the smallest quantity of vertices of an i −connected graph (planar graph) such that individually set of j chosen vertices be there disjoint from some largest path. Analogously are clear C_i^j and $\bar{C}_i^j$ for longest circuits as a replacement for longest path. As a number of moral instances responding Tudor Zamfirescu's demands were circulated in the succeeding years by means of W. Schmitz [5], Tudor. Zamfirescu [6] , [11] , [12]. H. Walther [8] provided an example of $C_2^2 \leq 220$ as well $\bar{C}_2^1 \leq 105$ [2]. B. Grunbaum, [9], W. Hatzel [10].

The purpose of this note is to show, that an example of a $1-$ connected graph G, of order 22 and other is 18 vertices satisfying by Gallai's criterion.

Theorem 1: $P_i^j \leq 22$

Proof: Consider the graph G of figure 1, With 22 vertices, let W be a longest path in G , the longest paths of G joining two of its end points have length $p(G) = 18$ avoiding v with empty intersection of all its longest Path. Figure 2, shows longest paths and all vertices are missed by each of them, underlined vertices are as of 1 to 22, where each vertex is avoided by some longest paths. Also to confirmation that all vertices avoided by individually of the longest Path, in table 1.

Lemma: The graph G has no hamililtonian path.

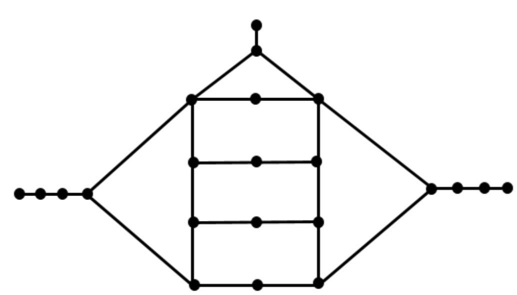
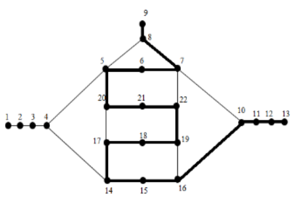
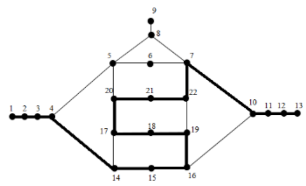


Figure 1. A Planar graph with 22 vertices



(I)



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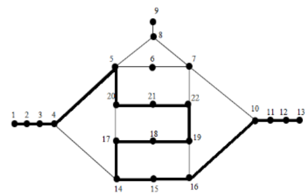

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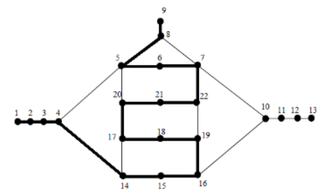

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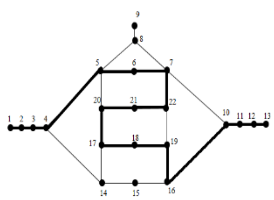

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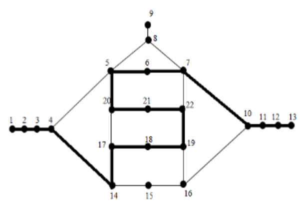
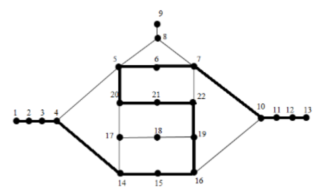
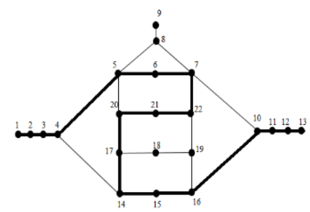


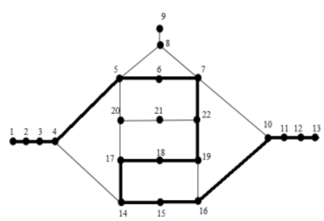
(VI)



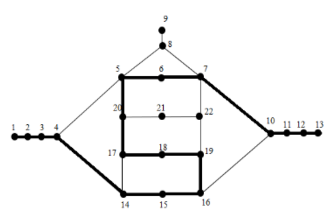
(VII)



(VIII)



(IX)



(X)

Table 1.

Fig.	Longest paths	Missed vertices
I	9, 8, 7, 6, 5, 20, 21, 22, 19, 18, 17, 14, 15, 16, 10, 11, 12, 13	1, 2, 3, 4
II	1, 2, 3, 4, 14, 15, 16, 19, 18, 17, 20, 21, 22, 7, 10, 11, 12, 13	5, 6, 8, 9
III	1, 2, 3, 4, 5, 20, 21, 22, 19, 18, 17, 14, 15, 16, 10, 11, 12, 13	6, 7, 8, 9
IV	9, 8, 5, 6, 7, 22, 21, 20, 17, 18, 19, 16, 15, 14, 4, 3, 2, 1	10, 11, 12, 13
V	1, 2, 3, 4, 5, 6, 7, 20, 21, 22, 17, 18, 19, 16, 10, 11, 12, 13	14, 15, 8, 9
VI	1 ,2, 3, 4, 14, 17, 18, 19, 22, 21, 20, 5, 6, 7, 10, 11, 12, 13	15, 16, 9, 8
VII	1, 2, 3, 4, 14, 15, 16, 19, 22, 21, 20, 5, 6, 7, 10, 11, 12, 13	17, 18, 9, 8
VII	1, 2, 3, 4, 5, 6, 7, 22, 21, 20, 17, 14, 15, 16, 10, 11, 12, 13	18, 19, 9, 8

IX	1, 2, 3, 4, 5, 6, 7, 22, 19, 18, 17, 14, 15, 16, 10, 11, 12, 13	20, 21, 9, 8
X	1, 2, 3, 4, 14, 15, 16, 19, 18, 17, 20, 5, 6, 7, 10, 11, 12, 13	21, 22, 9, 8

Theorem 2: $\bar{P}_i^j \leq 18$

Proof: Consider the graph G of figure 3. With 18 vertices, let W be a longest path in G, the longest paths of G joining two of its end points have length $p(G) = 13$ avoiding v with $W \cap V = \emptyset$ of all its longest Paths. Figure 4, shows 24 longest paths and all vertices are missed by each of them. The paths shown below underlined vertices are as of 1 to 18, where each vertex is avoided by some longest paths.

Lemma: The graph G has no hamililtonian path.

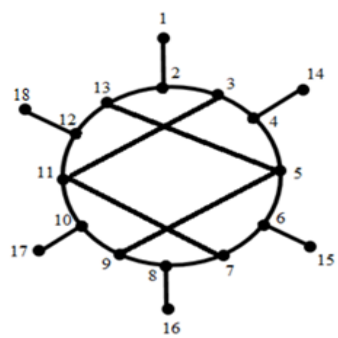


Fig.3 A non-planar graph with 18 vertices

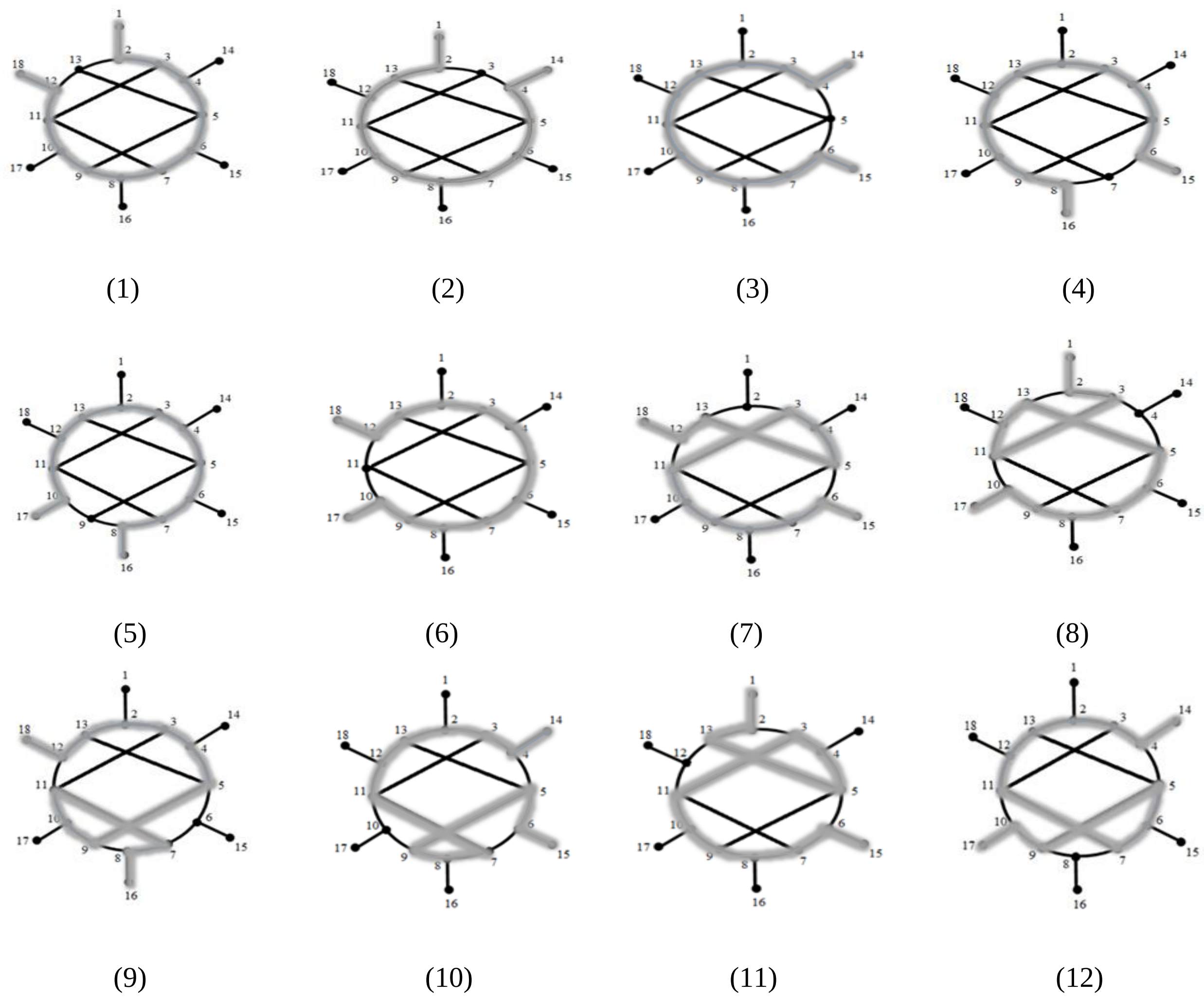


Fig.4 Longest Paths

Table 2.

Paths	Missed vertices	Longest paths
1	13, 14, 15, 16, 17	1, 2, 3, 4 ,5, 6, 7, 8, 9, 10, 11, 12, 18
2	3, 15, 16, 17, 18	14, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 2, 1
3	1, 5, 16, 17, 18	15,6, 7, 8, 9, 10, 11, 12, 13, 2, 3, 4, 14
4	1, 7, 14, 17, 18	16, 8, 9, 10, 11, 12, 13, 2, 3, 4, 5, 6, 15
5	1, 9, 14, 15, 18	17, 10, 11, 12, 13, 2, 3, 4, 5, 6,7, 8, 16

6	1,11,14, 15, 16	18, 12, 13, 2, 3, 4 ,5, 6, 7, 8, 9, 10, 17
7	1, 2, 14, 16, 17	18, 12, 13, 5, 4,3, 11, 10, 9, 8, 7, 6, 15
8	4, 14, 15, 16, 18	1, 2, 3, 11, 12, 13, 5, 6, 7, 8, 9, 10, 17
9	1, 6, 14, 15, 17	18, 12, 13, 2, 3, 4, 5, 9, 10, 11, 7, 8, 16
10	1, 10, 16, 17, 18	14, 4, 3, 2, 13, 12, 11, 7, 8, 9, 5, 6, 15
11	12, 14, 16, 17, 18	1, 2, 13, 5, 4, 3, 11, 10, 9, 8, 7, 6, 1
12	1, 8, 15, 16, 18	14, 4, 3, 2, 13, 12, 11, 7, 6, 5, 9, 10, 17

To confirmation that all vertices avoided by individually of the longest Path, in table 1 and 2.

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