

Question 403: Newton fractal for $f(z) = \frac{(1-z^5)^2}{1+z^{10}} - z$.

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abstract

This note presents some fractals related with the function:

$$f(z) = \frac{(1-z^5)^2}{1+z^{10}} - z, z \in \mathbb{C}$$

1. Introduction

$$\int_0^1 \left(\frac{(1-x^5)^2}{1+x^{10}} - x \right) dx = \frac{1}{2} + \frac{\sqrt{5}}{20} \ln(7+3\sqrt{5}) - \frac{\pi}{10} \sqrt{2 - \frac{2}{\sqrt{5}}} - \frac{\sqrt{5} \ln 2}{20} - \frac{\ln 2}{5} \quad (1)$$

❖ Remark: $\pi = 4 \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} = 3.141592\dots$

2. Newton fractals related with the function: $f(z) = \frac{(1-z^5)^2}{1+z^{10}} - z, z \in \mathbb{C}$

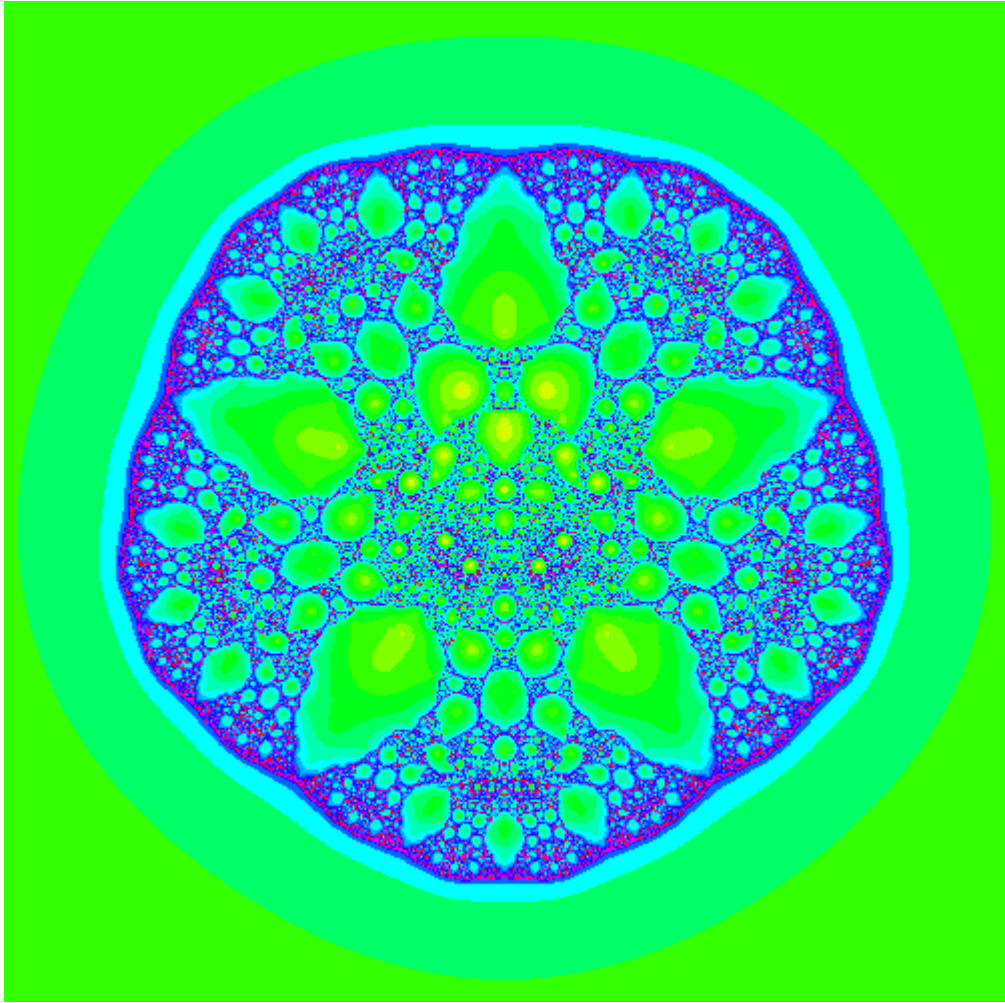


Figure 1.

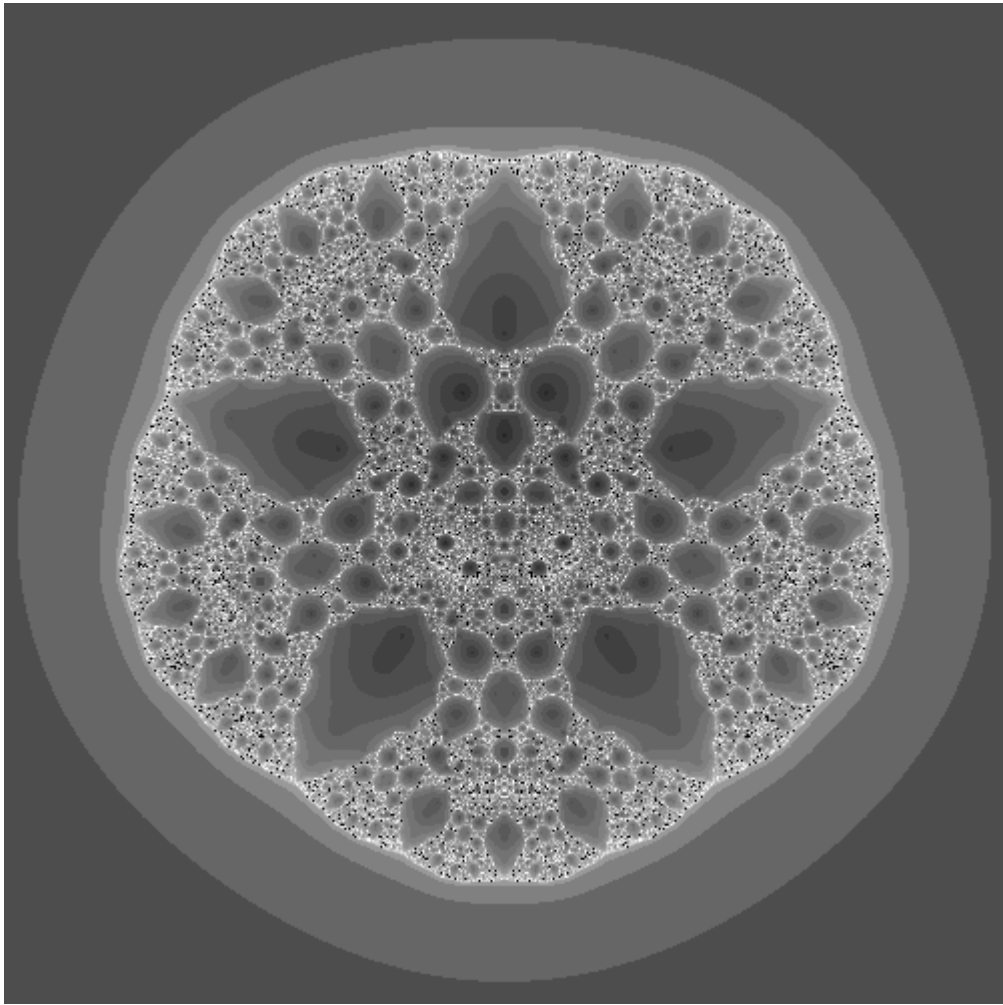


Figure 2.

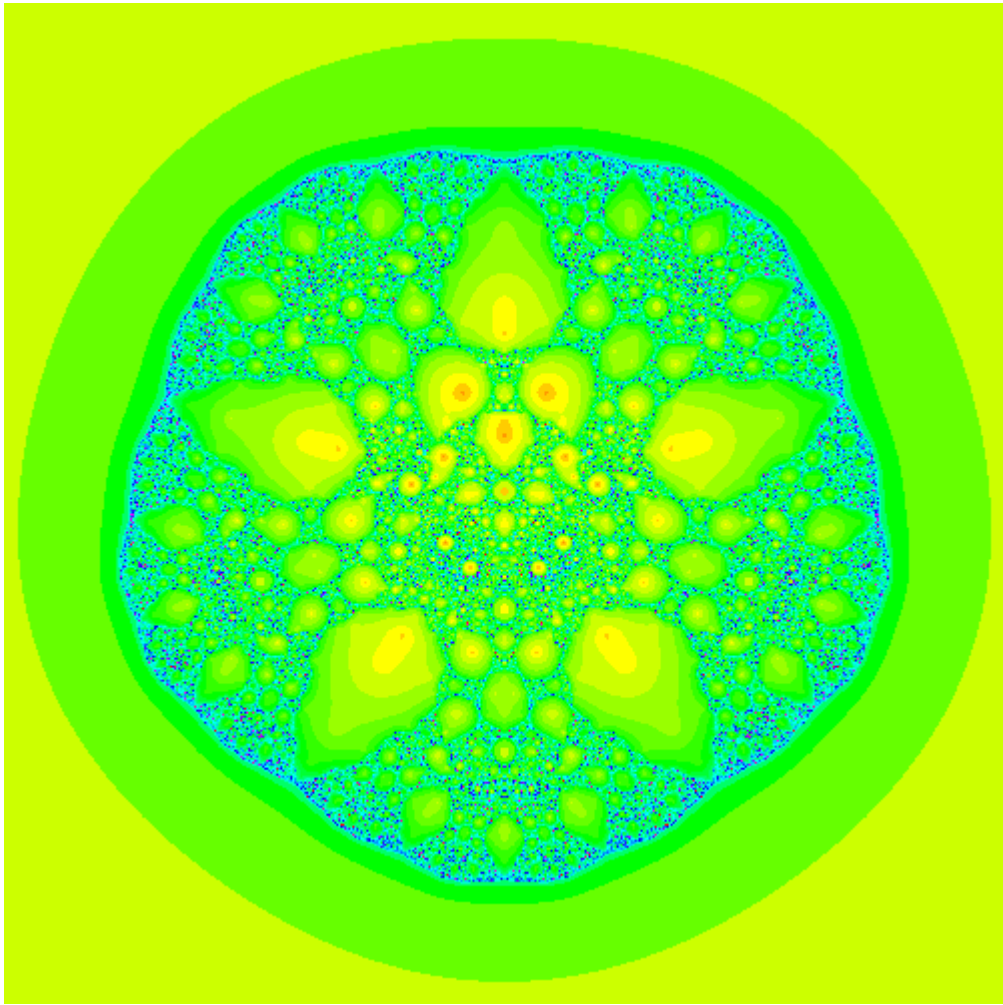


Figure 3.

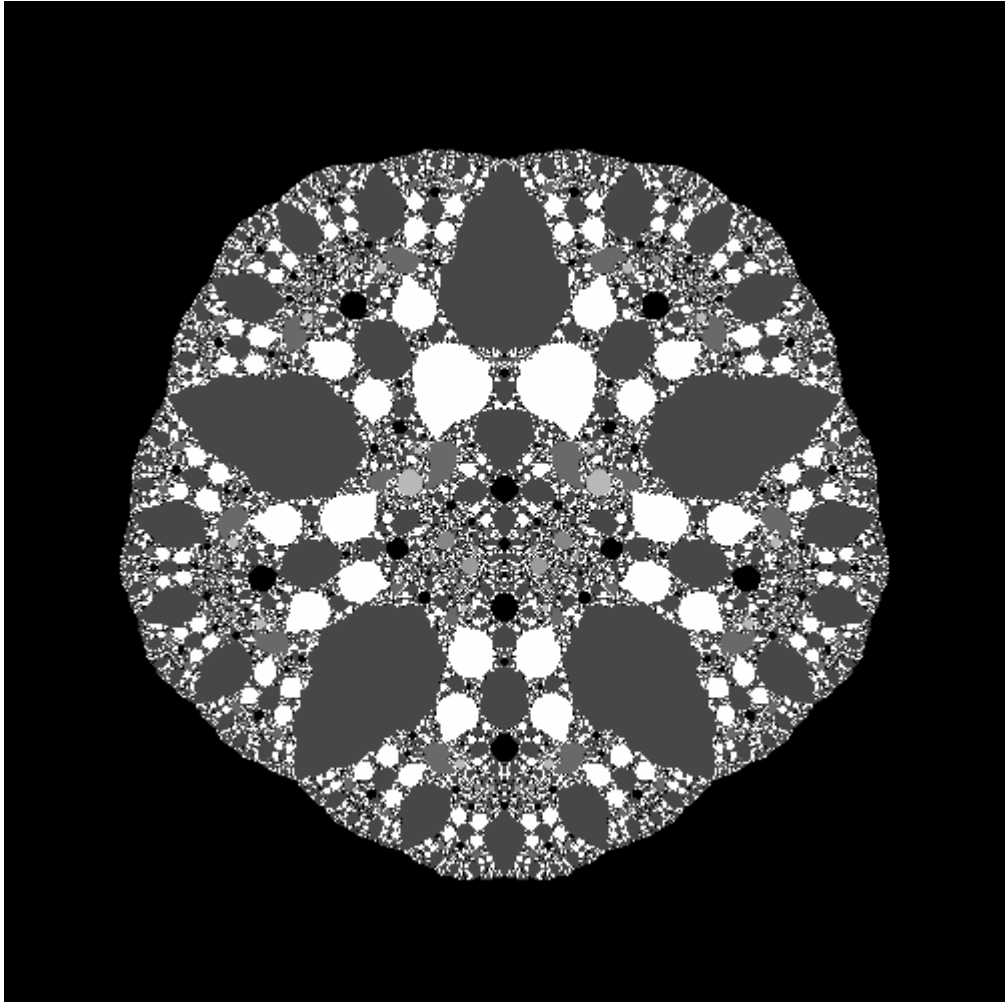


Figure 4.

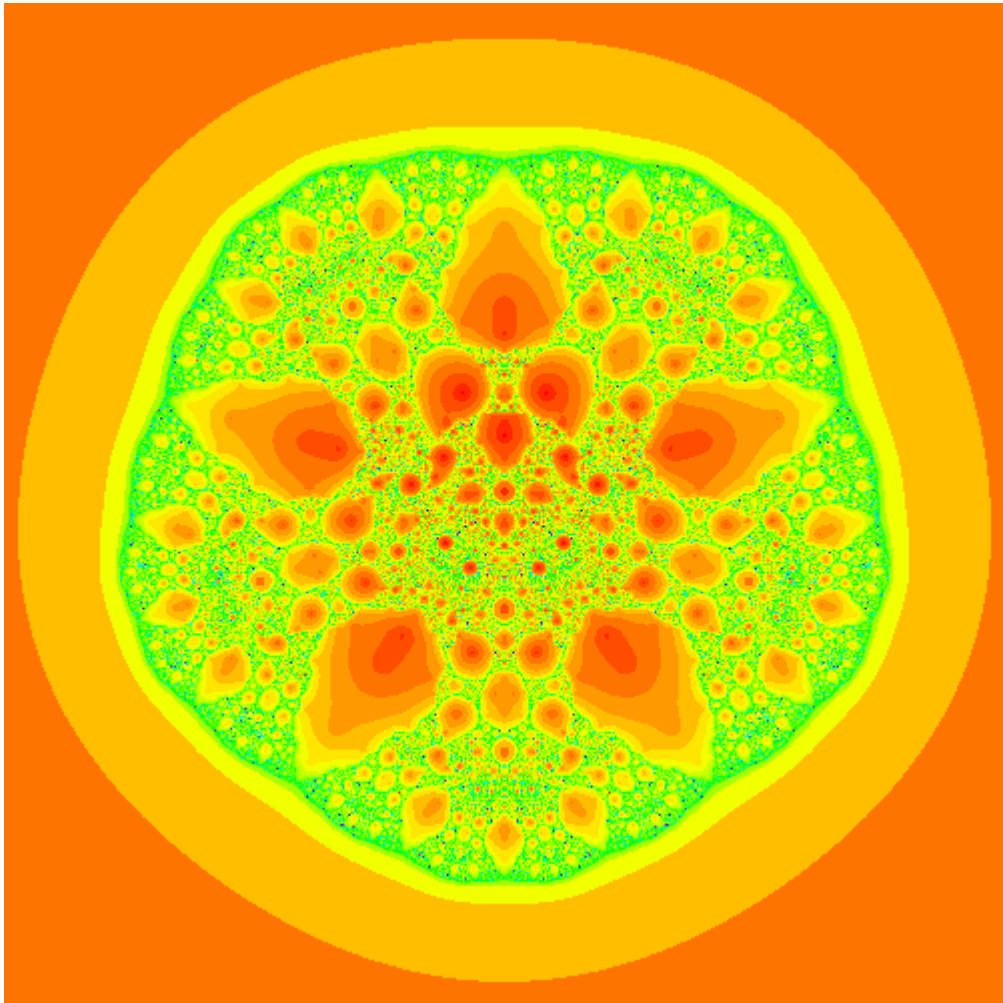


Figure 5.

References

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2. N. Steinmetz : Rational Iteration, Complex Analytic Dynamical Systems, Walter de Gruyter , Berlin , 1993.
3. P. Blanchard : Complex Analytic dynamics on the Riemann sphere, Bull. Amer. Math. Soc. (N.S.) 11 , 1984 , 85-141.