# Quantum mechanics expressed in terms of the approach "Emission \& Regeneration" UFT. 

Osvaldo Domann<br>odomann@yahoo.com<br>First publication 2003 and last revision November 2020<br>(This paper is an extract of [11] listed in section Bibliography.)<br>Copyright. All rights reserved.


#### Abstract

Quantum mechanics differential equations are based on the de Broglie postulate. This paper presents the repercussions on quantum mechanics differential equations when the de Broglie wavelength is replaced by a relation between the radius and the energy of a particle. This relation results from the theoretical work [11] about the interaction of charged particles, where the particles are modelled as focal points of rays of fundamental particles with longitudinal and transversal angular momentum. Interaction of subatomic particles is described as the interaction of the angular momenta of their fundamental particles. The relationship between the solution of the differential equation for a radial Coulomb field and the Correspondence Principle is presented. All four known forces are the result of electromagnetic interactions, so that only QED is required to describe them. The potential well of an atomic nucleus is shown with the regions that are responsible for the four type of interactions defined in quantum mechanics. Also the compatibility of the gravitation model derived in 11 with quantum mechanics is shown, model where gravitation is the result of the reintegration of migrated electrons and positrons to their nuclei.


## Contents

## 1 Introduction.

1.1 General considerations. . . . . . . . . . . . . . . . . . . . . . . . . . . . 4
1.2 The wave package. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
2 Differential equations. ..... 6
2.1 Unrestricted differential equations. ..... 6
2.1.1 The wave equation ..... 7
2.1.2 The time independent differential equation. ..... 7
2.1.3 The space independent differential equation. ..... 8
2.2 Non relativistic differential equations ..... 8
2.2.1 General non relativistic differential equation. ..... 8
2.2.2 The time independent non relativistic differential equation. ..... 9
2.2.3 Space independent non relativistic differential equation. ..... 9
2.3 Uncertainty principle ..... 10
2.4 Operators. ..... 10
2.4.1 Relativistic operator for the linear momentum. ..... 10
2.4.2 Relativistic operators for the energy. ..... 11
2.4.3 Non-relativistic operator for the kinetic energy. ..... 11
2.4.4 Non-relativistic Hamilton operator. ..... 12
2.4.5 Non-relativistic operator for the orbital-angular-momentum. ..... 12
2.5 The proposed theory and the Correspondence Principle ..... 13
2.6 The mass conservation equation. ..... 14
2.7 The wave equation for relativistic speeds. ..... 15
3 Applications of the non-relativistic differential equation ..... 16
3.1 Potential pot ..... 16
3.2 Harmonic oscillator ..... 17
3.3 Hydrogen atom ..... 18
3.4 Stern-Gerlach experiment ..... 24
4 Wave equations for free moving particles. ..... 27
4.1 The relativistic wave equation for the free moving particle. ..... 27
4.2 The slightly relativistic wave equation for the free moving particle. ..... 28
4.3 The non-relativistic wave equation for the free moving particle ..... 29
5 Stable and unstable particles. ..... 30
5.1 The potentials of the four interactions. ..... 31
6 Compatibility of gravitation with Quantum mechanics. ..... 34
6.1 Quantized gravitation. ..... 34
6.2 Relation between energy levels and space. ..... 35
6.3 Superposition of gravitation and Coulomb forces. ..... 36
7 Table comparing the SM and the 'E \& R' model. ..... 37
8 Summery of main characteristics of the proposed model. ..... 39
9 Bibliography ..... 42

## 1 Introduction.

Quantum mechanics differential equations are based on the de Broglie postulate. In the theoretical work [3] about the interaction of charged particles, where particles are represented by a non local model emitting and absorbing continuously fundamental particles, a relation between the radius $r_{o}$ and the energy of a particle is derived.

$$
\begin{equation*}
r_{o}=\frac{\hbar c}{E} \quad \text { with } \quad E=\sqrt{E_{o}^{2}+E_{p}^{2}} \quad \text { the relativistic energy. } \tag{1}
\end{equation*}
$$

This relation is used instead of the de Broglie wavelength, to build wave packages with a Gauss distribution, and to derive the corresponding probability differential equations of quantum mechanics.

The effects on the uncertainty relations and the most important quantum mechanics operators are presented.

Note: When deriving the wave-package with the radius-energy relation, the mass of a particle is considered as concentrated in a sphere with a diameter equal approximately to two times the radius $r_{o}$ given by the radius energy-relation. This is not according to the approach that represents particles as Focal Points which led to the radius-energy relation where the mass (energy) of a particle is distributed from $r_{o}$ to infinity, outside the sphere with radius $r_{o}$.

### 1.1 General considerations.

To make use of the of Fourier-Transformation, the movement of a particle is first described as a sequence of particles represented by a sinus wave, having a wavelength $\lambda$ equal to $2 \pi r_{o}$. Then the Fourier-Transformation of a wave package of sinus waves with a Gauss shaped amplitude is build.

We have that

$$
\begin{equation*}
\lambda=2 \pi r_{o}=2 \pi \frac{\hbar c}{E_{\text {rel }}} \quad \text { with } \quad E_{\text {rel }}=\sqrt{E_{o}^{2}+E_{p}^{2}} \tag{2}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{o}=m_{o} c^{2} \quad E_{p}=p c \quad p=\frac{m_{o} v}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{3}
\end{equation*}
$$

The sinus wave on the x -axis is

$$
\begin{equation*}
\xi_{x}=A e^{i\left(k_{x} x-\omega_{x} t\right)} \quad \text { with } \quad k_{x}=\frac{2 \pi}{\lambda_{x}} \quad \text { and } \quad \omega_{x}=2 \pi \frac{v_{x}}{\lambda_{x}} \tag{4}
\end{equation*}
$$

If we now introduce in the expression that $\lambda_{x}=2 \pi r_{o_{x}}=2 \pi \hbar c / E_{\text {rel }}$ we get

$$
\begin{equation*}
\xi_{x}=A \exp \left[i \frac{c}{\hbar}\left(\frac{E_{r e l_{x}}}{c^{2}} x-\frac{v_{x}}{c^{2}} E_{r e l_{x}} t\right)\right] \tag{5}
\end{equation*}
$$

or

$$
\begin{equation*}
\xi_{x}=A \exp \left[i \frac{c}{\hbar}\left(\frac{E_{r e l_{x}}}{c^{2}} x-p_{x} t\right)\right] \tag{6}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{r e l_{x}}=m_{o} c^{2}\left(1-\frac{v_{x}^{2}}{c^{2}}\right)^{-1 / 2} \quad \text { and } \quad p_{x}=\frac{v_{x}}{c^{2}} E_{r e l_{x}} \tag{7}
\end{equation*}
$$

with $E_{\text {rel }}^{x}$ the relativistic energy of the particle on the x-axis.

Note: The wave-length used by Schroedinger is based exclusively on the kinetic energy $E_{k i n_{x}}$ for the non-relativistic case as follows.

$$
\begin{equation*}
\lambda=2 \pi r_{o}=2 \pi \frac{\hbar c}{E_{\text {rel }}} \text { with } \quad E_{o}=0 \quad \text { and } \quad E_{p}=p c \quad \text { where } \quad p=m v \tag{8}
\end{equation*}
$$

The proposed approach includes for the calculation of the wave-length the total energy with the rest energy of a particle. For the relativistic cases we get

$$
\begin{equation*}
\lambda=2 \pi r_{o}=2 \pi \frac{\hbar c}{E_{\text {rel }}}=2 \pi \frac{\hbar}{m c \gamma} \quad \text { with } \quad \gamma=\frac{1}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \tag{9}
\end{equation*}
$$

For $v \rightarrow c$ we get that $\lambda \rightarrow 0$.

### 1.2 The wave package.

We define the Fourier-Transformation of a wave package [1,2]; on the x -axis as

$$
\begin{equation*}
\phi_{x}(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \kappa_{x}\left(p_{x}\right) \exp \left\{i \frac{c}{\hbar}\left[m_{\text {relx }}\left(p_{x}\right) x-p_{x} t\right]\right\} d p_{x} \tag{10}
\end{equation*}
$$

with a Gauss distribution $\kappa_{x}\left(p_{x}\right)$ on the $p_{x}$-axis

$$
\begin{equation*}
\kappa_{x}\left(p_{x}\right)=B \exp \left\{-\frac{\left(p_{x}-p_{x_{o}}\right)^{2}}{4\left(\Delta p_{x}\right)^{2}}\right\} \tag{11}
\end{equation*}
$$

and the dispersion $m_{r e l_{x}}=m_{\text {rel }}^{x}\left(p_{x}\right)$ with

$$
\begin{equation*}
m_{r e l_{x}}=\frac{E_{r e l_{x}}}{c^{2}} \quad m_{r e l_{x}}=m_{r e l_{x}}\left(p_{x}\right)=\frac{1}{c^{2}} \sqrt{E_{o}^{2}+p_{x}^{2} c^{2}} \quad \text { and } \quad E_{o}=m_{o} c^{2} \tag{12}
\end{equation*}
$$

Because of symmetry reasons we can write also a wave package

$$
\begin{equation*}
\psi_{x}(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \chi_{x}\left(m_{r e l_{x}}\right) \exp \left\{i \frac{c}{\hbar}\left[m_{r e l_{x}} x-p_{x}\left(m_{r e l_{x}}\right) t\right]\right\} d m_{r e l_{x}} \tag{13}
\end{equation*}
$$

with the Gauss distribution on the $m_{r e l_{x}}$-axis

$$
\begin{equation*}
\chi_{x}\left(m_{r e l_{x}}\right)=A \exp \left\{-\frac{\left(m_{r e l_{x}}-m_{r e l_{x}}\right)^{2}}{4\left(\Delta m_{r e l_{x}}\right)^{2}}\right\} \tag{14}
\end{equation*}
$$

and the dispersion

$$
\begin{equation*}
p_{x}\left(m_{r e l_{x}}\right)=c \sqrt{m_{r e l_{x}}^{2}-m_{o}^{2}} \quad \text { and } \quad m_{o}=\frac{E_{o}}{c^{2}} \tag{15}
\end{equation*}
$$

## 2 Differential equations.

### 2.1 Unrestricted differential equations.

In this and the following section the probability differential equations are derived. The differential equations are classified into unrestricted and non-relativistic. Then they are subclassified in groups of general, time or space independent.

The unrestricted differential equations are valid for the whole range of speed $0 \leq$ $v \leq c$.

We start with the wave package

$$
\begin{equation*}
\psi_{x}(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \chi_{x}\left(m_{r e l_{x}}\right) \exp \left\{i \frac{c}{\hbar}\left[m_{r e l_{x}} x-p_{x}\left(m_{r e l_{x}}\right) t\right]\right\} d m_{\text {rel }}^{x} \tag{16}
\end{equation*}
$$

with

$$
\begin{equation*}
m_{r e l_{x}}=\frac{E_{r e l_{x}}}{c^{2}} \quad \text { and } \quad p_{x}\left(m_{r e l_{x}}\right)=c \sqrt{m_{r e l_{x}}^{2}-m_{o}^{2}} \tag{17}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{r e l_{x}}=E_{o}+E_{k i n_{x}}=\sqrt{E_{o}^{2}+E_{p_{x}}^{2}} \quad E_{o}=m_{o} c^{2} \quad E_{p_{x}}=p_{x} c \tag{18}
\end{equation*}
$$

For the unrestricted range of velocities $0 \leq v \leq c$ we have that

$$
\begin{equation*}
p_{x}=\frac{v_{x}}{c^{2}} E_{\text {rel }}^{x} \tag{19}
\end{equation*}
$$

and $E_{k i n_{x}}$ represents the kinetic energy for the whole range of speed.

### 2.1.1 The wave equation.

The wave differential equation we obtain by derivation of $\psi_{x}$ two times versus $t$ and two times versus $x$. The results are then connected through

$$
\begin{equation*}
p_{x}=\frac{v_{x}}{c^{2}} E_{r e l_{x}} \tag{20}
\end{equation*}
$$

We get

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=\frac{1}{v_{x}^{2}} \frac{\partial^{2}}{\partial t^{2}} \psi_{x} \tag{21}
\end{equation*}
$$

For $v_{x} \rightarrow c$ we have

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}(x, t)=\frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \psi_{x}(x, t) \tag{22}
\end{equation*}
$$

the well known wave equation

### 2.1.2 The time independent differential equation.

Time independent differential equations are deduced derivating one time and two times the wave function $\psi_{x}$.
a) We derivate the wave function $\psi_{x}$ one time versus $x$ and get the following time independent differential equation on the $x$ coordinate

$$
\begin{equation*}
\frac{\partial}{\partial x} \psi_{x}=\frac{i}{\hbar c} E_{r e l_{x}} \psi_{x}=\frac{i}{\hbar c}\left(E_{o}+E_{k i n_{x}}\right) \psi_{x} \tag{23}
\end{equation*}
$$

$E_{k i n_{x}}$ represents the kinetic energy for the whole range of speed, relativistic and non-relativistic.
b) We derivate the wave function $\psi_{x}$ two times versus $x$ and get the following time independent differential equation on the $x$ coordinate

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=-\frac{c^{2}}{\hbar^{2}} m_{r e l_{x}}^{2} \psi_{x} \tag{24}
\end{equation*}
$$

With

$$
\begin{equation*}
m_{r e l_{x}}=\frac{1}{c^{2}} \sqrt{E_{o}^{2}+E_{p_{x}}^{2}} \quad E_{o}=m_{o} c^{2} \quad \text { and } \quad E_{p_{x}}=p_{x} c \tag{25}
\end{equation*}
$$

we get

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=-\frac{1}{\hbar^{2} c^{2}}\left(E_{o}^{2}+E_{p_{x}}^{2}\right) \psi_{x} \tag{26}
\end{equation*}
$$

### 2.1.3 The space independent differential equation.

We derivate the wave function $\psi_{x}$ two times versus $t$

$$
\begin{equation*}
\frac{\partial^{2}}{\partial t^{2}} \psi_{x}=-\frac{c^{2}}{\hbar^{2}} p_{x}^{2} \psi_{x} \tag{27}
\end{equation*}
$$

and with

$$
\begin{equation*}
E_{p_{x}}=p_{x} c \quad \text { and } \quad E_{p}^{2}=E_{p_{x}}^{2}+E_{p_{y}}^{2}+E_{p_{z}}^{2} \tag{28}
\end{equation*}
$$

we get

$$
\begin{equation*}
-\hbar^{2} \frac{\partial^{2}}{\partial t^{2}} \psi_{x}=E_{p_{x}}^{2} \psi_{x} \tag{29}
\end{equation*}
$$

and for the space

$$
\begin{equation*}
-\hbar^{2} \Delta_{\mathbf{t}} \psi=E_{p}^{2} \psi \tag{30}
\end{equation*}
$$

with the operator $\Delta_{\mathrm{t}}$ defined in sec. 2.4.

### 2.2 Non relativistic differential equations

For non relativistic speeds we have that $v \ll c$ and that $E_{k i n_{x}} \approx p^{2} /\left(2 m_{o}\right)$.

### 2.2.1 General non relativistic differential equation.

The general non relativistic differential equation we obtain by deriving $\psi_{x}$ two times versus $t$ and one time versus $x$. The results are then connected through $E_{\text {rel }}^{x}-E_{o}=$ $E_{k i n_{x}} \approx p^{2} /\left(2 m_{o}\right)$. We get

$$
\begin{equation*}
-i \hbar c \frac{\partial}{\partial x} \psi_{x}(x, t)-E_{o} \psi_{x}(x, t) \approx-\frac{\hbar^{2}}{2 m_{o} c^{2}} \frac{\partial^{2}}{\partial t^{2}} \psi_{x}(x, t) \text { with } \quad E_{o}=m_{o} c^{2} \tag{31}
\end{equation*}
$$

The differential equation with the constant energy $E_{o}$ describes the movement of a non-accelerated particle in a cero potential energy field.

With $E_{\text {tot }}$ the total energy, $E_{k i n}$ the kinetic energy, $E_{p o t}$ the potential energy and $E_{\text {rel }}$ the relativistic energy, the above equation is equivalent to $E_{\text {rel }}-E_{o}=E_{\text {kin }}$. If we add at to the kinetic energy $E_{k i n}$ the potential energy $E_{p o t}=U_{x}(x, t)$ we get the total energy $E_{t o t}$ for an accelerated movement. The result is

$$
\begin{equation*}
-i \hbar c \frac{\partial}{\partial x} \psi_{x}(x, t)-E_{o} \psi_{x}(x, t)+U_{x}(x, t) \psi_{x}(x, t)=E_{t o t} \psi_{x}(x, t) \tag{32}
\end{equation*}
$$

$$
\begin{equation*}
-\frac{\hbar^{2}}{2 m_{o} c^{2}} \frac{\partial^{2}}{\partial t^{2}} \psi_{x}(x, t)+U_{x}(x, t) \psi_{x}(x, t)=E_{t o t} \psi_{x}(x, t) \tag{33}
\end{equation*}
$$

In a conservative system the total energy is time independent with $E_{\text {tot }}=$ constant .
Comparing equation (31) with the General Schrödinger differential equation, the main difference is that equation (31) derives one time versus space and two times versus time, in other words, time and space are interchanged.

### 2.2.2 The time independent non relativistic differential equation.

Differential equations are deduced in derivating one time or two times the wave function $\psi_{x}$.
a) We derivate the wave function $\psi_{x}$ one time versus $x$

$$
\begin{equation*}
\frac{\partial}{\partial x} \psi_{x}=\frac{i}{\hbar c} E_{r e l_{x}} \psi_{x}=\frac{i}{\hbar c}\left(E_{o}+E_{k i n_{x}}\right) \psi_{x} \tag{34}
\end{equation*}
$$

For a conservative field $U_{x}=q_{e} V_{x}$ with a total energy $E_{\text {tot }_{x}}$ we have

$$
\begin{equation*}
E_{\text {tot }_{x}}=E_{k i n_{x}}+U_{x} \quad \text { and with } \quad E_{k i n_{x}} \approx \frac{1}{2 m_{o}} p_{x}^{2} \tag{35}
\end{equation*}
$$

we get

$$
\begin{equation*}
\left\{-i \hbar c \frac{\partial}{\partial x}+U(x)\right\} \psi(x) \approx E_{x} \psi(x) \tag{36}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{x}=E_{t o t_{x}}+E_{o} \tag{37}
\end{equation*}
$$

the Eigenvalue.
b) For the time independent differential equation deduced derivating the wave function $\psi_{x}$ two times versus $x$ see sec. 2.5.

### 2.2.3 Space independent non relativistic differential equation.

We take two times the derivate of the wave function $\psi_{x}$ versus $t$

$$
\begin{equation*}
\frac{\partial^{2}}{\partial t^{2}} \psi_{x}=-\frac{c^{2}}{\hbar^{2}} p_{x}^{2} \psi_{x} \tag{38}
\end{equation*}
$$

and with eq. (30)

$$
\begin{equation*}
-\hbar^{2} \Delta_{\mathbf{t}} \psi=E_{p}^{2} \psi \tag{39}
\end{equation*}
$$

and $v \ll c$ and a conservative potential $U$

$$
\begin{equation*}
E_{k i n} \approx \frac{1}{2 m_{o}} p^{2}=\frac{E_{p}^{2}}{2 E_{o}} \quad \text { and } \quad E_{t o t}=E_{k i n}+U \tag{40}
\end{equation*}
$$

we obtain the space independent non relativistic differential equation

$$
\begin{equation*}
\left\{-\frac{\hbar^{2}}{2 E_{o}} \Delta_{\mathbf{t}}+U\right\} \psi \approx E_{t o t} \psi \tag{41}
\end{equation*}
$$

which is equivalent to the time inependent equation from Schroedinger.

### 2.3 Uncertainty principle.

In the proposed model the pairs of canonical conjugated variables lead to the following uncertainty relations

$$
\begin{equation*}
(\Delta E) \cdot(\Delta x) \geq \frac{1}{2} \hbar c \tag{42}
\end{equation*}
$$

and

$$
\begin{equation*}
(\Delta p) \cdot(\Delta t) \geq \frac{1}{2} \frac{\hbar}{c} \tag{43}
\end{equation*}
$$

Noticeable at this point is the relation

$$
\begin{equation*}
E r_{o}=h c \tag{44}
\end{equation*}
$$

for a particle, that connects the radius $r_{o}$ and the relativistic energy $E$ through $h c$.

### 2.4 Operators.

### 2.4.1 Relativistic operator for the linear momentum.

The relativistic operator for the linear momentum of a particle is

$$
\begin{equation*}
\hat{p}=i \frac{\hbar}{c} \frac{\partial}{\partial t} \tag{45}
\end{equation*}
$$

The linear momentum we get with

$$
\begin{equation*}
\bar{p} \chi=i \frac{\hbar}{c} \nabla_{t} \chi \tag{46}
\end{equation*}
$$

where $\chi$ is the total mass-probability function

$$
\begin{equation*}
\chi=\psi_{x} \psi_{y} \psi_{z} \tag{47}
\end{equation*}
$$

and $\nabla_{\mathrm{t}}$

$$
\begin{equation*}
\nabla_{\mathbf{t}}=\left.\frac{\partial}{\partial t}\right|_{x} \mathbf{e}_{x}+\left.\frac{\partial}{\partial t}\right|_{y} \mathbf{e}_{y}+\left.\frac{\partial}{\partial t}\right|_{z} \mathbf{e}_{z} \tag{48}
\end{equation*}
$$

### 2.4.2 Relativistic operators for the energy.

For the relativistic energy of a non-accelerated particle we obtain the operator

$$
\begin{equation*}
\hat{E}_{r e l_{x}}=-i \hbar c \frac{\partial}{\partial x} \tag{49}
\end{equation*}
$$

## Application example.

If we apply the relativistic operators to the relativistic energy of a particle

$$
\begin{equation*}
E_{x}^{2}=m_{o}^{2} c^{4}+p_{x}^{2} c^{2} \tag{50}
\end{equation*}
$$

we get

$$
\begin{equation*}
-\hbar^{2} c^{2} \frac{\partial^{2}}{\partial x^{2}} \psi_{x}=m_{o}^{2} c^{4} \psi_{x}-\hbar^{2} \frac{\partial^{2}}{\partial t^{2}} \psi_{x} \tag{51}
\end{equation*}
$$

the Klein-Gordon equation.
With $m_{o}=0$ we have

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=\frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \psi_{x} \tag{52}
\end{equation*}
$$

### 2.4.3 Non-relativistic operator for the kinetic energy.

The non-relativistic operator for the kinetic energy on the $x$ coordinate is

$$
\begin{equation*}
\hat{E}_{k i n_{x}}=-\left.\frac{\hbar^{2}}{2 m_{o} c^{2}} \frac{\partial^{2}}{\partial t^{2}}\right|_{x} \tag{53}
\end{equation*}
$$

and the total kinetic energy $E_{\text {kin }}$ in the three dimensional space

$$
\begin{equation*}
E_{k i n}=E_{k i n_{x}}+E_{k i n_{y}}+E_{k i n_{z}}=-\frac{\hbar^{2}}{2 m_{o} c^{2}} \Delta_{\mathbf{t}} \chi \tag{54}
\end{equation*}
$$

with

$$
\begin{equation*}
\Delta_{\mathbf{t}}=\left.\frac{\partial^{2}}{\partial t^{2}}\right|_{x}+\left.\frac{\partial^{2}}{\partial t^{2}}\right|_{y}+\left.\frac{\partial^{2}}{\partial t^{2}}\right|_{z} \tag{55}
\end{equation*}
$$

### 2.4.4 Non-relativistic Hamilton operator.

The operator for the non-relativistic total energy on the $x$ coordinate has the form

$$
\begin{equation*}
\hat{E}_{x}=\frac{1}{2 m_{o}}\left(\left.i \frac{\hbar}{c} \frac{\partial}{\partial t}\right|_{x}\right)^{2}+\hat{U}_{x} \tag{56}
\end{equation*}
$$

or

$$
\begin{equation*}
\hat{E}_{x}=\frac{\hat{p}_{x}^{2}}{2 m_{o}}+\hat{U}_{x} \tag{57}
\end{equation*}
$$

which is equal to the Hamilton operator $\hat{H}_{x}$.
The general non-relativistic differential equation thus takes the form

$$
\begin{equation*}
i \hbar c \frac{\partial}{\partial x} \psi_{x}(x, t)=\hat{H}_{x} \psi_{x}(x, t) \tag{58}
\end{equation*}
$$

with

$$
\begin{equation*}
\hat{H}_{x}=\frac{\hat{p}_{x}^{2}}{2 m_{o}}+\hat{U}_{x} \tag{59}
\end{equation*}
$$

the non-relativistic Hamilton operator.

### 2.4.5 Non-relativistic operator for the orbital-angular-momentum.

The wave function for the three dimentional space is

$$
\begin{equation*}
\psi_{x}(\mathbf{r}, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \chi\left(m_{\text {rel }}\right) \exp \left\{i \frac{c}{\hbar}\left[m_{\text {rel }} \mathbf{r}-\mathbf{p}\left(m_{\text {rel }}\right) t\right]\right\} d m_{\text {rel }} \tag{60}
\end{equation*}
$$

with

$$
\begin{equation*}
\mathbf{r}=x \mathbf{e}_{x}+y \mathbf{e}_{y}+z \mathbf{e}_{z} \quad \text { and } \quad \mathbf{p}=p_{x} \mathbf{e}_{x}+p_{y} \mathbf{e}_{y}+p \mathbf{e}_{z} \tag{61}
\end{equation*}
$$

We define the linear momentum operator for the different coordinates as:

$$
\begin{equation*}
\hat{p}_{k}=\left.i \frac{\hbar}{c} \frac{\partial}{\partial t}\right|_{k} \tag{62}
\end{equation*}
$$

The orbital-angular-momentum-operator can be expressed as

$$
\begin{equation*}
\mathbf{M}\left(\mathbf{r}, i \frac{\hbar}{c} \nabla_{\mathrm{t}}\right)=\left(\mathbf{r} \times i \frac{\hbar}{c} \nabla_{\mathrm{t}}\right) \tag{63}
\end{equation*}
$$

with

$$
\begin{equation*}
\nabla_{\mathbf{t}}=\left.\frac{\partial}{\partial t}\right|_{x} \mathbf{e}_{x}+\left.\frac{\partial}{\partial t}\right|_{y} \mathbf{e}_{y}+\left.\frac{\partial}{\partial t}\right|_{z} \mathbf{e}_{z} \tag{64}
\end{equation*}
$$

The operators for the vectorcomponents are:

$$
\begin{equation*}
\hat{M}_{x}=\hat{y} \hat{p}_{z}-\hat{z} \hat{p}_{y} \quad \hat{M}_{y}=\hat{z} \hat{p}_{x}-\hat{x} \hat{p}_{z} \quad \hat{M}_{z}=\hat{x} \hat{p}_{y}-\hat{y} \hat{p}_{z} \tag{65}
\end{equation*}
$$

The conmutations are as known

$$
\begin{equation*}
\left[\hat{M}_{k}, \hat{M}_{k+1}\right] \neq 0 \quad\left[\hat{M}_{k}, \hat{Q}\right]=0 \quad \text { with } \quad \hat{Q}=\hat{M}_{x}^{2}+\hat{M}_{y}^{2}+\hat{M}_{z}^{2} \tag{66}
\end{equation*}
$$

### 2.5 The proposed theory and the Correspondence Principle.

The present theory is based on the radius-energy relation that substitutes the de Broglie wavelength.

The accordance of the proposed theory with the correspondence principle of quantum mechanics is ensured, in that the time independent differential equation from Schroedinger, deduced from the wave package constructed with the de Broglie wavelength, can be derived from the wave package constructed with the radius-energy relation presented in this work.

We start derivating the wave function $\psi_{x}$ two times versus space, to get the time independent differential equation

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=-\frac{c^{2}}{\hbar^{2}} m_{r e l_{x}}^{2} \psi_{x} \tag{67}
\end{equation*}
$$

With

$$
\begin{equation*}
m_{r e l_{x}}=\frac{1}{c^{2}} \sqrt{E_{o}^{2}+E_{p_{x}}^{2}} \quad E_{o}=m_{o} c^{2} \quad \text { and } \quad E_{p_{x}}=p_{x} c \tag{68}
\end{equation*}
$$

we get

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=-\frac{1}{\hbar^{2} c^{2}}\left(E_{o}^{2}+E_{p_{x}}^{2}\right) \psi_{x} \tag{69}
\end{equation*}
$$

For non-relativistic velocities $v \ll c$ we have that

$$
\begin{equation*}
E_{k i n_{x}}=\frac{p_{x}^{2}}{2 m_{o}} \quad \text { and } \quad E_{p_{x}}^{2}=p_{x}^{2} c^{2}=2 m_{o} c^{2} E_{k i n_{x}} \tag{70}
\end{equation*}
$$

and we get

$$
\begin{equation*}
\frac{\partial^{2}}{\partial x^{2}} \psi_{x}=-\frac{2 m_{o}}{\hbar^{2}}\left[\frac{1}{2} E_{o}+E_{k i i_{x}}\right] \psi_{x} \tag{71}
\end{equation*}
$$

With a conservative potential $E_{t o t_{x}}=U_{x}+E_{k i n_{x}}$ we get finally

$$
\begin{equation*}
\left[-\frac{\hbar^{2}}{2 m_{o}} \frac{\partial^{2}}{\partial x^{2}}+U_{x}\right] \psi_{x}=E_{x} \psi_{x} \quad \text { with } \quad E_{x}=\frac{1}{2}\left[E_{o}+2 E_{\text {tot }_{x}}\right] \tag{72}
\end{equation*}
$$

For the three dimensional space we have

$$
\begin{equation*}
\left[-\frac{\hbar^{2}}{2 m_{o}} \Delta_{\mathbf{r}}+U\right] \chi=E \chi \tag{73}
\end{equation*}
$$

with $\Delta_{\mathbf{r}}$ the Laplace operator and

$$
\begin{equation*}
E=\frac{1}{2}\left[E_{o}+2 E_{t o t}\right] \tag{74}
\end{equation*}
$$

If we make $E_{o}=0$ we get

$$
\begin{equation*}
\left[-\frac{\hbar^{2}}{2 m_{o}} \Delta_{\mathbf{r}}+U\right] \chi=E_{t o t} \chi \tag{75}
\end{equation*}
$$

Eq. (75) is exactly the time independent differential equation constructed by Schroedinger with $E_{t o t}$ the Eigenvalue.

### 2.6 The mass conservation equation.

The mass conservation differential equation we obtain by derivating $\psi_{x}$ one time versus $t$ and one time versus $x$. The results are then connected through

$$
\begin{equation*}
p_{x}=\frac{v_{x}}{c^{2}} E_{r e l_{x}} \tag{76}
\end{equation*}
$$

We get

$$
\begin{equation*}
\frac{\partial}{\partial t} \psi_{x}(x, t)=-v_{x} \frac{\partial}{\partial x} \psi_{x}(x, t) \tag{77}
\end{equation*}
$$

We define the mass probability density as

$$
\begin{equation*}
\rho_{x}(x, t)=\psi_{x}^{*}(x, t) \psi_{x}(x, t) \quad \text { or } \quad \rho(\mathbf{r}, t)=\psi^{*}(\mathbf{r}, t) \psi(\mathbf{r}, t) \tag{78}
\end{equation*}
$$

We derive the mass probability density versus time

$$
\begin{equation*}
\frac{\partial}{\partial t} \rho_{x}(x, t)=\frac{\partial}{\partial t}\left[\psi_{x}^{*}(x, t) \psi_{x}(x, t)\right]=\frac{\partial}{\partial t} \psi_{x}^{*}(x, t) \psi_{x}(x, t)+\psi_{x}^{*}(x, t) \frac{\partial}{\partial t} \psi_{x}(x, t) \tag{79}
\end{equation*}
$$

With eq. (77) we get

$$
\begin{equation*}
\frac{\partial}{\partial t} \rho_{x}(x, t)=-v_{x}\left[\frac{\partial}{\partial x} \psi_{x}^{*}(x, t) \psi_{x}(x, t)+\psi_{x}^{*}(x, t) \frac{\partial}{\partial x} \psi_{x}(x, t)\right] \tag{80}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\partial}{\partial t} \rho_{x}(x, t)=-v_{x} \frac{\partial}{\partial x}\left[\psi_{x}^{*}(x, t) \psi_{x}(x, t)\right]=-\frac{\partial}{\partial x}\left[v_{x} \rho_{x}(x, t)\right]=-\frac{\partial}{\partial x} j(x, t) \tag{81}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\partial}{\partial t} \rho(\mathbf{r}, t)=-\nabla_{\mathbf{r}} \mathbf{j}(\mathbf{r}, t) \quad \text { with } \quad \mathbf{j}(\mathbf{r}, t)=\mathbf{v} \psi^{*}(\mathbf{r}, t) \psi(\mathbf{r}, t) \tag{82}
\end{equation*}
$$

where $\mathbf{j}(\mathbf{r}, t)$ is the mass-current probability density.

### 2.7 The wave equation for relativistic speeds.

We start with the wave eq. (13) from sec. 1.2

$$
\begin{equation*}
\psi_{x}(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \chi_{x}\left(m_{r e l_{x}}\right) \exp \left[i \frac{c}{\hbar}\left(m_{r e l_{x}} x-p_{x}\left(m_{\text {rel }}^{x}\right) ~ t\right)\right] d m_{r e l_{x}} \tag{83}
\end{equation*}
$$

and analyze the equation for relativistic speeds where $\Delta v=c-v \ll c$. We get

$$
\begin{equation*}
E_{\text {rel }}=E_{p}=p c=\frac{m v}{\beta} c \quad \beta=\sqrt{1-\frac{v^{2}}{c^{2}}} \quad \lambda=\frac{h}{p} \tag{84}
\end{equation*}
$$

The resulting wave equation is

$$
\begin{equation*}
\psi_{x}(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \chi_{x}\left(m_{r e l_{x}}\right) \exp \left[\frac{i}{\hbar}\left(p x-E_{p v} t\right)\right] d m_{r e l_{x}} \tag{85}
\end{equation*}
$$

where

$$
\begin{equation*}
E_{p v}=p v=\frac{m v}{\beta} v \tag{86}
\end{equation*}
$$

With $E_{\text {rel }}=p c^{2} / v$ and $E_{o}^{2} \ll E_{p}^{2}$ we get

$$
\begin{equation*}
E_{p v}=p v=\frac{p^{2} c^{2}}{E_{r e l}}=\frac{p^{2} c^{2}}{\sqrt{E_{o}^{2}+E_{p}^{2}}} \approx p c=E_{p} \tag{87}
\end{equation*}
$$

We now derive the wave equation one time versus space and one time versus time and connect the results with $E_{p v}=p c$. We get

$$
\begin{equation*}
\frac{\partial}{\partial t} \psi_{x}=-c \frac{\partial}{\partial x} \psi_{x} \tag{88}
\end{equation*}
$$

## 3 Applications of the non-relativistic differential equation

The solutions of the time independent non-relativistic differential equation (32) for a potential pot, the harmonic oscillator and the hydrogen atom are derived.

### 3.1 Potential pot

The non-relativistic time independent differential equation is

$$
\begin{equation*}
-i \hbar c \frac{\partial}{\partial x} \psi_{x}(x)+U_{x}(x) \psi_{x}(x)=\left[E_{t o t}+E_{o}\right] \psi_{x}(x)=E \psi_{x}(x) \tag{89}
\end{equation*}
$$

With $y=\psi_{x}(x)$ we can write

$$
\begin{equation*}
-i \hbar c \frac{d y}{y}=[E-U] d x \tag{90}
\end{equation*}
$$

After integration we get

$$
\begin{equation*}
-i \hbar c\left[\ln |y|+\ln C_{y}\right]=\int[E-U] d x \tag{91}
\end{equation*}
$$

resulting

$$
\begin{equation*}
|y|=\frac{1}{C_{y}} \exp \left\{\frac{i}{\hbar c} \int[E-U] d x\right\} \tag{92}
\end{equation*}
$$

Equation (92) is valid for all potential energies $U$ and gives real values for $y$ if

$$
\begin{equation*}
\left\{\frac{i}{\hbar c} \int[E-U] d x\right\}=k \pi \quad \text { and } \quad k=0, \pm 1, \pm 2, \pm 3, \cdots \tag{93}
\end{equation*}
$$

defining the quantization condition, which together with the normalization condition allows the calculation of the eigenfunctions.

The potential pot is defined as

$$
U= \begin{cases}\infty & \text { for } x \leq 0 \\ 0 & \text { for } 0<x<a \\ \infty & \text { for } x \geq a\end{cases}
$$

and we have for $U=0$

$$
\begin{equation*}
\frac{1}{\hbar c} E x=k \pi \quad \text { resulting with } x=a \quad E_{k}=\pi \frac{\hbar c}{a} k \tag{94}
\end{equation*}
$$

with $k=0, \pm 1, \pm 2, \pm 3, \cdots$.
The total energy is with $E_{k}=E_{t o t}+E_{o}$

$$
\begin{equation*}
E_{t o t}=E_{k}-E_{o}=\pi \frac{\hbar c}{a} k-E_{o} \tag{95}
\end{equation*}
$$

and for $E_{\text {tot }}=0$ we get

$$
\begin{equation*}
a_{o}=k \frac{\pi \hbar c}{E_{o}}=k \pi r_{o} \quad \text { with } \quad \frac{\hbar c}{E_{o}}=r_{o} \tag{96}
\end{equation*}
$$

the radius of of a rest electron or positron.
The eigenfunction is

$$
\begin{equation*}
y_{k}=\frac{1}{C_{y}} \exp \left\{\frac{i}{\hbar c} E_{k} x\right\} \tag{97}
\end{equation*}
$$

The integration constant $C_{y}$ we get with the normalization condition

$$
\begin{equation*}
\int_{-\infty}^{\infty} y_{k^{\prime}}^{*} y_{k} d x=\delta_{\left(k^{\prime}, k\right)} \tag{98}
\end{equation*}
$$

For $k^{\prime}=k$ we get

$$
\begin{equation*}
\frac{1}{C_{y}^{2}} \int_{0}^{a} \exp \left\{\frac{i}{\hbar c}\left[E_{k^{\prime}}-E_{k}\right] x\right\} d x=1 \tag{99}
\end{equation*}
$$

resulting

$$
\begin{equation*}
\frac{1}{C_{y}^{2}}=a \quad \text { or } \quad C_{y}=\sqrt{a} \tag{100}
\end{equation*}
$$

The normalized eigenfunction is

$$
\begin{equation*}
y_{k}=\frac{1}{\sqrt{a}} \exp \left\{\frac{i}{\hbar c} E_{k} x\right\} \tag{101}
\end{equation*}
$$

Conclusion: The main differences compared with the solution obtained with the Schroedinger equation is that the quantization of the energy is proportional to $k$ instead of $k^{2}$ and for defined values of $a$ the total energy $E_{\text {tot }}$ becomes zero.

### 3.2 Harmonic oscillator

The potential energy for the harmonic oscillator is

$$
\begin{equation*}
U(x)=\frac{K}{2} x^{2}=\frac{m \omega^{2}}{2} x^{2} \quad \text { with } \quad \omega^{2}=K / m \tag{102}
\end{equation*}
$$

With eq. (92) we get

$$
\begin{equation*}
|y|=\frac{1}{C_{y}} \exp \left\{\frac{i}{\hbar c} \int\left[E-\frac{K}{2} x^{2}\right] d x\right\} \tag{103}
\end{equation*}
$$

With the quantization condition we get

$$
\begin{equation*}
\frac{1}{\hbar c} \int_{0}^{a}\left[E-\frac{K}{2} x^{2}\right] d x=\frac{1}{\hbar c}\left[E a-\frac{K}{6} a^{3}\right]=k \pi \tag{104}
\end{equation*}
$$

resulting for the quantized energy with $E_{t o t}=E_{k}-E_{o}$

$$
\begin{equation*}
E_{t o t}=\pi \frac{\hbar c}{a}\left[k+\frac{1}{6} \frac{m \omega^{2}}{\pi \hbar c} a^{3}\right]-E_{o}=E_{k}-E_{o} \tag{105}
\end{equation*}
$$

The minimum quantum change between two adjacent energy levels is

$$
\begin{equation*}
\Delta E_{t o t}=\Delta E_{k}=\pi \frac{\hbar c}{a} \tag{106}
\end{equation*}
$$

For $E_{\text {tot }}=0$ we get

$$
\begin{equation*}
a\left[E_{o}-\frac{1}{6} m \omega^{2} a^{2}\right]=k \pi \hbar c \tag{107}
\end{equation*}
$$

which for $k=0$ gives

$$
\begin{equation*}
a_{1}=0 \quad \text { or } \quad a_{2,3}= \pm \sqrt{\frac{6 E_{o}}{m \omega^{2}}} \quad \text { for } \quad k=0 \tag{108}
\end{equation*}
$$

We get for the minimum quantum change between two adjacent energy levels

$$
\begin{equation*}
\Delta E_{t o t}= \pm \frac{\pi}{\sqrt{6}} \hbar \omega \tag{109}
\end{equation*}
$$

The minimum quantum energy difference $\Delta E_{\text {tot }}$ between two adjacent energy levels is proportional to $\hbar \omega$.

With the normalization condition given by equation (98) we have that

$$
\begin{equation*}
\int_{-\infty}^{\infty} y_{k^{\prime}}^{*} y_{k} d x=\frac{1}{C_{y}^{2}} \int_{-\infty}^{\infty} \exp \left\{\frac{i}{\hbar c}\left[E_{k^{\prime}}-E_{k}\right] x\right\} d x \tag{110}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{\hbar c}{C_{y}^{2}} \int_{-\infty}^{\infty} \exp \left\{i\left[E_{k^{\prime}}-E_{k}\right] \eta\right\} d \eta=\frac{\hbar c}{C_{y}^{2}} \delta_{\left(k^{\prime}, k\right)} \quad \text { with } \quad \eta=\frac{x}{\hbar c} \tag{111}
\end{equation*}
$$

With $k^{\prime}=k$ we get the integration constant $C_{y}=\sqrt{\hbar c}$ resulting the normalized eigenfunctions

$$
\begin{equation*}
y_{k}=\frac{1}{\sqrt{\hbar c}} \exp \left\{\frac{i}{\hbar c}\left[E_{k} x-\frac{K}{6} x^{3}\right]\right\} \tag{112}
\end{equation*}
$$

### 3.3 Hydrogen atom

We start with the deduction of the quantization conditions with eq. (32)

$$
\begin{equation*}
-i \hbar c \frac{\partial}{\partial x} \psi_{x}(x)+U_{x}(x) \psi_{x}(x)=\left[E_{o}+E_{t o t}\right] \psi_{x}(x)=E \psi_{x}(x) \tag{113}
\end{equation*}
$$

which is equivalent to

$$
\begin{equation*}
E_{r e l}+U=E_{o}+E_{k i n}+U=E \quad E_{t o t}=E_{k i n}+U \quad E_{\text {rel }}=E_{o}+E_{k i n} \tag{114}
\end{equation*}
$$

We define the operator

$$
\begin{align*}
& \vec{\nabla} \cdot \vec{E}=\nabla E=\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z} \text { with } \vec{E}=\vec{e}_{x}+\vec{e}_{y}+\vec{e}_{z}  \tag{115}\\
& \nabla E \psi(x, y, z)=\frac{\partial}{\partial x} \psi(x, y, z)+\frac{\partial}{\partial y} \psi(x, y, z)+\frac{\partial}{\partial z} \psi(x, y, z) \tag{116}
\end{align*}
$$

For polar coordinates we write

$$
\begin{equation*}
-i \hbar c \nabla \chi(r, \theta, \varphi)+U \chi(r, \theta, \varphi)=E \chi(r, \theta, \varphi) \tag{117}
\end{equation*}
$$

with the $\nabla$ operator expressed in polar coordinates

$$
\begin{equation*}
\nabla=\frac{\partial}{\partial r}+\frac{2}{r}+\frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi}+\frac{1}{r} \frac{\partial}{\partial \theta}+\frac{1}{r} \cot \theta \tag{118}
\end{equation*}
$$

The differential equation has now the form

$$
\begin{equation*}
\left[\nabla+\frac{i}{\hbar c} U\right] \chi=\frac{i}{\hbar c} E \chi \tag{119}
\end{equation*}
$$

We now assume that the wave function $\chi$ can be expressed as a product of a function exclusively of the distance $r$ and a function of the angular variables $\theta$ and $\varphi$.

$$
\begin{equation*}
\chi(r, \theta, \varphi)=R(r) Y(\theta, \varphi) \tag{120}
\end{equation*}
$$

We get

$$
\begin{equation*}
\left[\frac{d}{d r}+\frac{4}{r}\right] R \cdot Y+\frac{1}{r} \Lambda Y \cdot R+\frac{i}{\hbar c} U \cdot R \cdot Y=\frac{i}{\hbar c} E \cdot R \cdot Y \tag{121}
\end{equation*}
$$

with the operator $\Lambda$

$$
\begin{equation*}
\Lambda=\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi}+\frac{\partial}{\partial \theta}+2 \cot \theta \tag{122}
\end{equation*}
$$

We now assume that

$$
\begin{equation*}
\Lambda Y=-\lambda Y \tag{123}
\end{equation*}
$$

and get two separate differential equations for $R(r)$ and $Y(\theta, \varphi)$.

$$
\begin{equation*}
\frac{d}{d r} R-\frac{i}{\hbar c}[E-U] R+\frac{1}{r}[4-\lambda] R=0 \tag{124}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi}+\frac{\partial}{\partial \theta}+2 \cot \theta\right] Y=-\lambda Y \tag{125}
\end{equation*}
$$

After multiplying Eq. (124) with $d r / R$ and integrating we get

$$
\begin{equation*}
\ln R=\frac{i}{\hbar c} \int_{r_{u}}^{r}[E-U] d r-[4-\lambda] \ln \frac{r}{r_{u}} \tag{126}
\end{equation*}
$$

From the solution of eq. (125) results that $\lambda=i l$ with $l=0, \pm 1, \pm 2 ; \cdots$ as will be shown later on at eq. (141). We get

$$
\begin{equation*}
R=\exp \left\{-4 \ln \frac{r}{r_{u}}\right\} \exp \left\{\frac{i}{\hbar c}\left[\int_{r_{u}}^{r}(E-U) d r+l \hbar c \ln \frac{r}{r_{u}}\right]\right\} \tag{127}
\end{equation*}
$$

The quantization condition requires that

$$
\begin{equation*}
\frac{1}{\hbar c}\left[\int_{r_{u}}^{r}(E-U) d r+l \hbar c \ln \frac{r}{r_{u}}\right]=k \pi \quad \text { with } \quad k=0, \pm 1, \pm 2 ; \cdots \tag{128}
\end{equation*}
$$

Equation (128) is valid for all point symmetrical potentials $U$. We now introduce the potential of an atomic nucleus

$$
\begin{equation*}
U=-\frac{K_{u}}{r}=-Z \frac{K_{o}}{r} \quad \text { with } \quad K_{o}=\frac{e^{2}}{4 \pi \epsilon_{o}} \tag{129}
\end{equation*}
$$

where $Z$ is the atomic number, and get for the quantization condition for $E$

$$
\begin{equation*}
E=\left[k \pi \hbar c-\left(K_{u}+l \hbar c\right) \ln \frac{r}{r_{u}}\right] \frac{1}{r-r_{u}} \tag{130}
\end{equation*}
$$

In eq. (130) the terms represent $E=E_{o}+U+E_{\text {orb }}$ where

$$
\begin{equation*}
E_{o} \equiv \frac{k \pi \hbar c}{r-r_{u}} \quad U \equiv-\frac{K_{u}}{r-r_{u}} \ln \frac{r}{r_{u}} \quad E_{\text {orb }} \equiv-\frac{l \hbar c}{r-r_{u}} \ln \frac{r}{r_{u}} \tag{131}
\end{equation*}
$$

According to the " $\mathrm{E} \& \mathrm{R}$ " model, nucleons are composed of electrons and positrons that neither attract nor repel each other when the distance between them tend to zero. They can polarize the nucleons, so that an orbital electron can interact during a short time with more than one positron of the nucleon. In the case of the hydrogen, the
orbital electron can be attracted by one, two or more electrons of the proton. The concept is shown in Fig. 1.


Figure 1: Energy levels at the hydrogen atom.
This means that the potential $U$ becomes $n$ times stronger, where $n$ is the number of positrons of the nucleon that interact with the orbital electron.

$$
\begin{equation*}
U=-n \frac{K_{u}}{r-r_{u}} \ln \frac{r}{r_{u}}=-\frac{K_{u}}{r^{\prime}} \tag{132}
\end{equation*}
$$

we get

$$
\begin{equation*}
\ln \frac{r}{r_{u}}=\frac{1}{n} \frac{r-r_{u}}{r^{\prime}} \tag{133}
\end{equation*}
$$

which in eq. (130) with $k=0$ gives

$$
\begin{equation*}
E^{\prime}=-\left(K_{u}+l \hbar c\right) \frac{1}{n r^{\prime}} \tag{134}
\end{equation*}
$$

$E^{\prime}$ is a function of the product $n r^{\prime}=r_{n}$. We derive versus $r_{n}$ and get

$$
\begin{equation*}
\frac{d}{d r_{n}} E^{\prime}=\left(K_{u}+l \hbar c\right) \frac{1}{n^{2} r^{\prime 2}} \tag{135}
\end{equation*}
$$

As we want the energy that corresponds to a number $n$ of positrons of the nucleus, we maintaining $n$ fix and integrate only versus $r^{\prime}$. We get

$$
\begin{equation*}
E_{n}=\int_{\infty}^{r^{\prime}}\left(\frac{d}{d r_{n}} E^{\prime}\right) d r^{\prime}=-\left(K_{u}+l \hbar c\right) \frac{1}{r^{\prime}} \frac{1}{n^{2}} \tag{136}
\end{equation*}
$$

The difference between energy levels is

$$
\begin{equation*}
\Delta E_{n}=\left(K_{u}+l \hbar c\right) \frac{1}{r^{\prime}}\left[\frac{1}{n^{2}}-\frac{1}{(n+m)^{2}}\right] \tag{137}
\end{equation*}
$$

With $l=0$ the energy difference $\Delta E_{n}$ of eq. (137) is only due to the potential energy.

The $\Delta E$ of the hydrogen spectrum empirically deduced by Ballmer is only valid for the potential energy, namely

$$
\begin{equation*}
E=h c R_{H} \frac{1}{n^{2}} \quad \text { and } \quad \Delta E=h c R_{H}\left[\frac{1}{n^{2}}-\frac{1}{(n+1)^{2}}\right] \tag{138}
\end{equation*}
$$

with $R_{H}$ the Rydberg constant and $n=1,2, \ldots \ldots$
From the two equations (137) and (138) we get

$$
\begin{equation*}
\frac{K_{u}}{r_{H}^{\prime}}=h c R_{H} \quad r_{H}^{\prime}=\frac{K_{u}}{h c R_{H}}=1.05811 \cdot 10^{-10} \mathrm{~m} \tag{139}
\end{equation*}
$$

Now we deduce the condition $\lambda=i l$ introduced previously in eq. (125).

$$
\begin{equation*}
\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \varphi}+\frac{\partial}{\partial \theta}+2 \cot \theta\right] Y=-\lambda Y \tag{140}
\end{equation*}
$$

We assume that

$$
\begin{equation*}
Y(\theta, \phi)=\Theta(\theta) \Phi(\varphi) \quad \text { and } \quad \frac{d}{d \varphi} \Phi=m \Phi \tag{141}
\end{equation*}
$$

and with $\Phi(\varphi)=\Phi(\varphi+2 \pi)$ we get

$$
\begin{equation*}
\Phi=\exp \{m \varphi\} \quad \text { with } \quad m=i m_{l} \quad \text { and } \quad m_{l}= \pm 0, \pm 1, \pm 2 ; \cdots \tag{142}
\end{equation*}
$$

With eq. (141) we have that eq. (125) transforms to

$$
\begin{equation*}
\frac{m}{\sin \theta} \Theta+\frac{d}{d \theta} \Theta+2 \cot \theta \Theta=-\lambda \Theta \tag{143}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d \Theta}{\Theta}=-\left[\frac{m}{\sin \theta}+2 \cot \theta+\lambda\right] d \theta \tag{144}
\end{equation*}
$$

which gives the solution

$$
\begin{equation*}
\Theta=\frac{1}{C_{\Theta}} \exp \left\{-\int\left[\frac{i m_{l}}{\sin \theta}+2 \cot \theta+\lambda\right] d \theta\right\} \tag{145}
\end{equation*}
$$

With $\Theta(\theta)=\Theta(\theta+2 \pi)$ we conclude that

$$
\begin{equation*}
\Theta=\frac{1}{C_{\Theta}} \exp \{-2 \ln \sin \theta\} \exp \left\{-i\left[m_{l} \ln (\csc \theta-\cot \theta)+l \theta\right]\right\} \tag{146}
\end{equation*}
$$

with $\lambda=i l$ and $l= \pm 0, \pm 1, \pm 2 ; \cdots$ what we have anticipated for eq. (127).
Eq.( 143) we can now write as

$$
\begin{equation*}
\frac{d}{d \theta} \Theta+i \frac{m_{l}}{\sin \theta} \Theta=-2 \cot \theta \Theta-i l \Theta \tag{147}
\end{equation*}
$$

In this equation the real and the imaginary terms must be equal, and we get from the imaginary terms that

$$
\begin{equation*}
\frac{m_{l}}{l}=-\sin \theta \quad \text { with } \quad m_{l}= \pm 0, \pm 1, \pm 2 ; \cdots \quad \text { and } \quad l= \pm 0, \pm 1, \pm 2 ; \cdots \tag{148}
\end{equation*}
$$

We conclude, that the relation between the orbital quantum number $l$ and the magnetic quantum number $m_{l}$ is

$$
\begin{equation*}
\left|\frac{m_{l}}{l}\right| \leq 1 \quad \text { or } \quad\left|m_{l}\right| \leq|l| \tag{149}
\end{equation*}
$$

## The hydrogen atom and the energy stored in the orbital angular moment.

From eq. (136) we get with $l=-i \lambda$

$$
\begin{equation*}
E_{n}=-\left(K_{u}-i \lambda \hbar c\right) \frac{1}{r^{\prime}} \frac{1}{n^{2}} \quad r^{\prime}=r_{H}^{\prime}=1.05811 \cdot 10^{-10} \mathrm{~m} \tag{150}
\end{equation*}
$$

The energy level $E_{n}$ has a real and an imaginary component, namely

$$
\begin{equation*}
E_{n_{r}}=-K_{u} \frac{1}{r^{\prime}} \frac{1}{n^{2}} \quad E_{n_{i}}=\lambda \hbar c \frac{1}{r^{\prime}} \frac{1}{n^{2}} \tag{151}
\end{equation*}
$$

The real component corresponds to the potential energy of the orbital electron while the imaginary part to the kinetic energy stored in the orbital angular moment.

The energy variation due to the variation of the number $n$ of interactions with positrons of the nucleon is

$$
\begin{equation*}
\Delta E_{n_{r}}=K_{u} \frac{1}{r^{\prime}}\left[\frac{1}{n^{2}}-\frac{1}{(n+m)^{2}}\right] \tag{152}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta E_{n_{i}}=-i \lambda \hbar c \frac{1}{r^{\prime}}\left[\frac{1}{n^{2}}-\frac{1}{(n+m)^{2}}\right] \tag{153}
\end{equation*}
$$

$\Delta E_{n_{r}}$ is the energy irradiated and $\Delta E_{n_{i}}$ is the change of the energy stored in the orbital angular moment, which we have assumed to be zero in the previous deductions making $\lambda=0$.

Conclusions: The present approach is based on the "E \& R " model, where nucleons are composed of electrons and positrons that neither attract nor repel each other when the distance between them tends to zero. A nucleon can polarize, so that an orbital electron can interact during a short time with more than one positron of the nucleon. In the case of the hydrogen, the orbital electron can be attracted during a short time by two or more positrons of the proton defining the higher energy levels for the orbital electron.

As nucleons are composed of electrons and positrons, also quarks are composed of electrons and positrons. The fractional charges of quarks are simply the relation between the number of electrons or positrons that integrate the quark, to the total number of electrons and positrons that compose the quark. No fractional charges exist.

The electron shells of atoms is the result of the accommodation of the electrons and positrons of the atomic nuclei in the quarks. The combination principle used in spectroscopy becomes with the "E \& R" model a physical interpretation.

### 3.4 Stern-Gerlach experiment.

To explain the splitting of the atomic ray in the Stern-Gerlach experiment, electrons were assigned an intrinsic spin with a quantized magnetic field that takes two positions, up and down relative to an external magnetic field, although it is not possible to explain how the intrinsic spin and magnetic field are generated. Measurements with individual electrons to detect the magnetic spin are fruitless because of the strong Lorenz forze. The proposed approach with particles as focal points of rays of FPs has also no possibility to explain how such an intrinsic spin and magnetic field could exist on the electron.

An explanation is now given how the magnetic field at the orbit electron is generated.

The concept is shown in Fig: 2
The approach $E \& R$ UFT shows that electrons and positrons coexist in nucleons without repelling or attracting each other. They can be seen as swarms of electrons and positrons forming the nucleon. As nuclei are composed of nucleons they are also composed of electrons and positrons as shown in Fig. 2.

The charge $Q$ of a nucleus is replaced by the expression $\Delta n=n^{+}-n^{-}$which gives


Figure 2: Magnetic spin $d H_{r}$ of an orbital electron.
the difference between the constituent numbers of electrons and positrons that form the nucleus. As the $n_{i}$ are integer numbers, the Charge of the nucleus is quantified.

As examples we have for the proton $n^{+}=919$ and $n^{-}=918$ with a binding Energy of $E_{B_{\text {prot }}}=-6.9489 \cdot 10^{-14} \mathrm{~J}=-0.43371 \mathrm{MeV}$, and for the neutron $n^{+}=919$ and $n^{-}=919$ with a binding Energy of $E_{B_{n e u t r}}=5.59743 \cdot 10^{-14} \mathrm{~J}=0.34936 \mathrm{MeV}$.

The tangential speed $v_{t}$ and the radial orthogonal speed $v_{r}$ of the orbital electron with its charge $q$ generate orthogonal transversal fields $d H_{n}$.

$$
\begin{equation*}
d H_{n_{t}} \propto \sqrt{m} v_{t} \quad \text { and } \quad d H_{n_{r}} \propto \sqrt{m} v_{r} \tag{154}
\end{equation*}
$$

The speed $v_{r}$ of the electron with mass $m_{1}$ together with the associated positron of the nucleus generate the field $d H_{n_{1}}$, which defines the state $\frac{1}{2}$ of its spin. The speed $v_{r}$ of the opposed orbital electron with mass $m_{2}$ together with the associated positron of the nucleus generate the opposed field $d H_{n_{2}}$, which defines the state $-\frac{1}{2}$ of its spin. Each electron that is added to the orbital electrons when building an atom, tends to occupy a position relative to the other electrons with the minimum interaction. In the case of the two electrons of Fig: 2 the electrons are placed at the opposed sides of the diameter of the atom.

The neutral atoms used in the Stern-Gerlach experiment have all complete shells plus the first electron of the next shell. The configuration of the $A g$ is $[K r] 4 d^{10} 5 s^{1}$. The $5 s^{1}$ electron has according to the Standard Model no orbital angular moment and also no magnetic moment associated with it . The only way to explain the Stern-Gerlach experiment in this case to assign the electron an intrinsic magnetic moment.

According to the $E \& R$ UFT approach the $5 s^{1}$ electron generates two perpendicular
magnetic fields, the orbital and the radial magnetic fields. The external inhomogeneous magnetic field can force the electron to move on an orbit where no net force is generated by the orbital field, but not simultaneously also for the radial field.

## 4 Wave equations for free moving particles.

### 4.1 The relativistic wave equation for the free moving particle.

We start with the dispersion equations for the relativistic mass $m_{\text {rel }}$ of sec. 1.2. In what follows we omit the sub-index $x$ and write $m_{r e l}$ instead of $m_{r e l_{x}}$.

$$
\begin{equation*}
m_{\text {rel }}=\frac{E_{\text {rel }}}{c^{2}} \quad m_{\text {rel }}=m_{\text {rel }}(p)=\frac{1}{c^{2}} \sqrt{E_{o}^{2}+p^{2} c^{2}} \quad \text { and } \quad E_{o}=m_{o} c^{2} \tag{155}
\end{equation*}
$$

which can be transformed to

$$
\begin{equation*}
m_{\text {rel }}=\frac{1}{c}\left[p^{2}+\frac{E_{o}^{2}}{c^{2}}\right]^{1 / 2}=\frac{1}{c}\left[p+p^{\prime}\right] \tag{156}
\end{equation*}
$$

with

$$
\begin{equation*}
p_{1,2}^{\prime}=-p \pm \sqrt{p^{2}+\frac{E_{o}^{2}}{c^{2}}} \tag{157}
\end{equation*}
$$

We also transform

$$
\begin{equation*}
p\left(m_{r e l}\right)=c \sqrt{m_{r e l}^{2}-m_{o}^{2}} \quad \text { and } \quad m_{o}=\frac{E_{o}}{c^{2}} \tag{158}
\end{equation*}
$$

to

$$
\begin{equation*}
p=\frac{1}{c}\left[E_{r e l}^{2}-m_{o}^{2} c^{4}\right]^{1 / 2} \quad \text { with } \quad E_{\text {rel }}=E_{o}+E_{k i n} \tag{159}
\end{equation*}
$$

and

$$
\begin{equation*}
p=\frac{1}{c}\left[E_{k i n}^{2}+2 E_{o} E_{k i n}\right]^{1 / 2}=\frac{1}{c}\left[E_{k i n}+E^{\prime}\right] \tag{160}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{1,2}^{\prime}=-E_{k i n} \pm \sqrt{E_{k i n}^{2}+2 E_{o} E_{k i n}} \tag{161}
\end{equation*}
$$

Note: In what follows we changed the symbol for the wave function from $\phi$ to $\Psi$ to follow the convention.

If we now introduce (156) and (160) in eq. ( 10 )

$$
\begin{equation*}
\Psi(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \kappa_{x}\left(p_{x}\right) \exp \left\{i \frac{c}{\hbar}\left[m_{\text {rel }}^{x}\left(p_{x}\right) x-p_{x} t\right]\right\} d p_{x} \tag{162}
\end{equation*}
$$

we get

$$
\begin{equation*}
\Psi(x, t) \propto \exp \left\{\frac{i}{\hbar}\left[\left[p+p^{\prime}\right] x-\left[E_{k i n}+E^{\prime}\right] t\right]\right\} \tag{163}
\end{equation*}
$$

what we can write in the form

$$
\begin{equation*}
\Psi(x, t) \propto \exp \left\{\frac{i}{\hbar}\left[p^{\prime} x-E^{\prime} t\right]\right\} \cdot \exp \left\{\frac{i}{\hbar}\left[p x-E_{k i n} t\right]\right\} \tag{164}
\end{equation*}
$$

We know that

$$
\begin{equation*}
E_{r e l}=E_{o}+E_{k i n}=E_{s}+E_{n} \tag{165}
\end{equation*}
$$

with

$$
\begin{equation*}
E_{s}=\frac{E_{o}^{2}}{\sqrt{E_{o}^{2}+E_{p}^{2}}} \quad E_{n}=\frac{E_{p}^{2}}{\sqrt{E_{o}^{2}+E_{p}^{2}}} \quad E_{p}=p c \tag{166}
\end{equation*}
$$

For relativistic speeds $v>0.95 c$ we have that

$$
\begin{equation*}
E_{s} \ll E_{n} \quad E_{\text {rel }} \approx E_{n} \quad E_{k i n} \approx E_{n}-E_{o} \tag{167}
\end{equation*}
$$

and

$$
\begin{equation*}
p_{1}^{\prime}=0 \quad p_{2}^{\prime}=-2 p \quad E_{1}^{\prime}=0 \quad E_{2}^{\prime}=-2 E_{k i n} \tag{168}
\end{equation*}
$$

and get

$$
\begin{equation*}
\Psi(x, t) \propto \exp \left\{ \pm \frac{i}{\hbar}\left[p x-E_{k i n} t\right]\right\}=\exp \left\{ \pm \frac{i}{\hbar}\left[p x-\left(E_{n}-E_{o}\right) t\right]\right\} \tag{169}
\end{equation*}
$$

### 4.2 The slightly relativistic wave equation for the free moving particle.

For $v \ll c$ we have that $p \approx m v$

$$
\begin{equation*}
E_{s} \approx E_{o} \quad \text { and } \quad E_{n} \approx E_{\text {rel }}-E_{o}=E_{k i n} \tag{170}
\end{equation*}
$$

Also for $v \rightarrow 0$ we get that

$$
\begin{equation*}
E_{k i n} \rightarrow 0 \quad \text { and } \quad E^{\prime} \rightarrow 0 \quad \text { for } \quad v \rightarrow 0 \tag{171}
\end{equation*}
$$

and

$$
\begin{equation*}
p \rightarrow 0 \quad \text { and } \quad p^{\prime} \rightarrow m c \quad \text { for } \quad v \rightarrow 0 \tag{172}
\end{equation*}
$$

From ( 164 ) we get

$$
\begin{equation*}
\Psi(x, t) \propto \exp \left\{\frac{i}{\hbar}[m c x]\right\} \cdot \exp \left\{\frac{i}{\hbar}\left[p x-E_{k i n} t\right]\right\} \tag{173}
\end{equation*}
$$

where we have that the first exponent is not a function of $p$ and $E_{k i n}$. As $p=m v$ from the second exponent is much smaller than $m c$ from the first exponent, the first exponent oscillates along the $x$-axis between plus and minus of its absolute value which is one. The frequency of the oscillation of the first factor is very high compared with the second, and the first factor can be made equal to one for all $x$.

$$
\begin{equation*}
\Psi(x, t) \propto \exp \left\{\frac{i}{\hbar}\left[p x-E_{k i n} t\right]\right\} \tag{174}
\end{equation*}
$$

With $p \approx m v$ we also can write

$$
\begin{equation*}
E_{k i n} \approx-\frac{c^{2}}{2 E_{o}} p^{2}+\frac{1 \cdot 3}{2 \cdot 4} \frac{c^{4}}{E_{o}^{3}} p^{4}-\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{c^{6}}{E_{o}^{5}} p^{6}+\cdots \tag{175}
\end{equation*}
$$

and arrive to the relativistic wave equation for a free moving particle

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \Psi=\left[\frac{\hbar^{2}}{2 m} \frac{\partial^{2}}{\partial x^{2}}+\frac{1 \cdot 3}{2 \cdot 4} \frac{\hbar^{4}}{m^{3} c^{2}} \frac{\partial^{4}}{\partial x^{4}} \cdots\right] \Psi \tag{176}
\end{equation*}
$$

If we take into consideration only the first two terms of $E_{k i n}$ and introduce an external potential $U(x)$, we get the following time independent wave equation for a slightly relativistic moving charged particle in an external potential.

$$
\begin{equation*}
\left[\frac{\hbar^{2}}{2 m} \frac{\partial^{2}}{\partial x^{2}}+\frac{1 \cdot 3}{2 \cdot 4} \frac{\hbar^{4}}{m^{3} c^{2}} \frac{\partial^{4}}{\partial x^{4}}+U(x)\right] \Psi=E \Psi \tag{177}
\end{equation*}
$$

To calculate the maximum velocity $v_{\max }$ for this case we make the third term of eq. (175) ten times smaller than the second term and get $v_{\max }=0.346 c$. It is not recommended to use more than two terms of eq. 175 because of the approximations made for the deduction.

Note: Eq. 177 allows to calculate the solutions for QM systems which are slightly relativistic instead of using the strong relativistic Dirac formulation.

### 4.3 The non-relativistic wave equation for the free moving particle

If we make $E_{o}=0$ because we want an equation that describes only the kinetic energy we get $p^{\prime}=0$ and $E^{\prime}=0$, and if we reduce our observation to non-relativistic speeds
with $v \ll c$ we have from eq. (164)

$$
\begin{gather*}
\Psi(x, t) \propto \exp \left\{\frac{i}{\hbar}\left[p x-E_{k i n} t\right]\right\} \quad \text { with } \quad E_{k i n}=\frac{1}{2} \frac{p^{2}}{m}=E_{k i n}(p)  \tag{178}\\
\Psi(x, t)=\frac{1}{2 \pi} \int_{-\infty}^{+\infty} \kappa_{x}\left(p_{x}\right) \exp \left\{\frac{i}{\hbar}\left[p x-E_{k i n}(p) t\right]\right\} d p_{x} \tag{179}
\end{gather*}
$$

The wave function derived two times versus $x$ and one time versus $t$ gives the differential equation of the free moving particle of mass $m$. If we introduce an external potencial $U$ we have the Schrödinger equation for an accelerated particle.

$$
\begin{equation*}
i \hbar \frac{\partial}{\partial t} \Psi(x, t) \approx\left[-\frac{\hbar^{2}}{2 m_{o}} \frac{\partial^{2}}{\partial x^{2}}+U\right] \Psi(x, t) \tag{180}
\end{equation*}
$$

## 5 Stable and unstable particles.

Particles in the SM are classified as Gauge Bosons, Leptons, Quarks, Baryons and Mesons. The classification makes no difference between stable and unstable particles. Unstable particles with energies much grater than the energies of the stable electron ( $0.511 \mathrm{MeV} / \mathrm{c}^{2}$ ), positron or neutrino are defined as Basic Subatomic Particles (BSPs), violating the concept of basic particles which must be the constituents of all not basic particles. The result is the search for basic particles like the unstable Quarks with energies above $0.35 \mathrm{GeV} / \mathrm{c}^{2}$.

The approach "Emission and Regeneration" UFT

1. defines as BSPs the electron, positron and the neutrino which are stable particles, and defines all particles with higher energies, stable or unstable, as Composed Subatomic Particles (CSPs) which are integrated by BSPs.
2. defines electrons and positrons as focal points of rays of Fundamental Particles (FPs) which go from infinite to infinite and have longitudinal and transversal angular momenta. Interactions between electrons and positrons are the result of the interactions of the angular momenta of their FPs. No carrier bosons are required to describe interactions between subatomic particles.
3. defines neutrinos as pairs of FPs with opposed angular momenta which generate linear momenta, and photons as a sequence of pairs of FPs with opposed angular momenta that generate a sequence of opposed linear momenta.
4. shows that no strong forces are required to hold electrons and positrons together, which are the constituents of protons and neutrons. The forces between the
constituents electrons and positrons tend to zero for the distance between them tending to zero.
5. shows that weak forces which are responsible for the decay of atomic nuclei are electromagnetic forces.
6. shows that gravitation forces are also electromagnetic forces.

The conclusion is, that all interactions between subatomic particles are electromagnetic interactions and described by QED. Interactions as described by QCD and Gauge/Gravity Duality are simply the product of the deficiencies of the SM and not required.

### 5.1 The potentials of the four interactions.

Our SM differentiates between the following potentials to explain interactions between particles.

- Strong
- Weak
- Gravitation
- Electromagnetic

In $|11|$ the momentum curve between two static charged BSAs (electron/positron) was derived resulting Fig. 3 and the following regions were defined:

1. From $0 \ll \gamma \ll 0.1$ where $p_{\text {stat }}=0$
2. From $0.1 \ll \gamma \ll 1.8$ where $p_{s t a t} \propto d^{2}$
3. From $1.8 \ll \gamma \ll 2.1$ where $p_{\text {stat }} \approx$ constant
4. From $2.1 \ll \gamma \ll 518$ where $p_{\text {stat }} \propto \frac{1}{d}$
5. From $518 \ll \gamma \ll \infty$ where $p_{\text {stat }} \propto \frac{1}{d^{2}}$ (Coulomb)

The static momentum curve of Fig. 3 is part of the potential well of an atomic nucleus as shown in Fig. 4, which can be approximated by a piecewise constant potential for the analytical analysis in quantum mechanics.

The force on electrons or positrons that move in the defined regions of the potential well is given by the following equations derived in [11]:


Figure 3: Linear momentum $p_{s t a t}$ as function of $\gamma=d / r_{o}$ between two static BSPs with equal radii $r_{o_{1}}=r_{o_{2}}$

$$
\begin{gather*}
d \bar{F}_{i_{n}}=\frac{1}{8 \pi} \sqrt{m_{p}} r_{o_{p}} r o t \frac{d}{d t} \int_{r_{r}}^{\infty} d \bar{H}_{n} \quad \text { with }  \tag{181}\\
\frac{d}{d t} \int_{r_{r}}^{\infty} d \bar{H}_{n}=\frac{1}{2} \frac{d}{d t}\left[H_{n}\right] \frac{r_{o}}{r_{r}} \sin \varphi d \varphi \bar{s}_{\gamma}-H_{n} v \frac{r_{o}}{r_{r}^{2}} \sin \varphi \cos \varphi d \varphi \bar{s}_{\gamma}  \tag{182}\\
+\frac{1}{2} H_{n} \frac{1}{r_{r}} \sin \varphi d \varphi \frac{d r_{o}}{d t} \bar{s}_{\gamma}
\end{gather*}
$$

For the regions we have that:

- BSPs that are in region 1 don't attract nor repel each other. The static force is zero and no binding Gluons nor strong forces to hold them together are needed.
- BSPs that have migrated slowly from region 1 to region 2 where the potential groves approximately with $d^{2}$, are accelerated to or away from the potential wall by the static force according the charge of the particle and the charge of the remaining particles in region $\mathbf{1}$. We can differentiate between:
- BSPs that are accelerated away from the potential wall (region 3) induce on BSPs of other atoms the gravitation force. The accelerated BSPs transmit


Figure 4: Potential well of an atom.
their acquired momentum to BSPs of other atoms (induction) and stop their movement immediately according the conservation law of momentum. The force on accelerated BSPs is given with $\frac{d}{d t}\left[H_{n}\right]=\sqrt{m} \frac{d v}{d t}$.

- BSPs that are accelerated to the potential wall may tunnel the wall what results in the decay of the atom with the corresponding radiations. No special weak force is required.
- BSPs in the region 5 where the Coulomb force exists, orbit around the atom nucleus. This is called in the SM the electromagnetic force.

The "Emission \& Regeneration" UFT approach shows that all forces are derived from one Field, the $d H$ field. It also shows that all interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and that no other type of interactions are required. It shows that all particles are composed of electrons, positrons and neutrinos and that particles of very short lifetime are composed particles.

## 6 Compatibility of gravitation with Quantum mechanics.

The potential in which an orbital electron in an Hidrogen atom with $Z=1$ moves is

$$
\begin{equation*}
U(r)_{\text {Coul }}=-\left(\frac{Z e^{2}}{4 \pi \epsilon_{o}}\right) \frac{1}{r}=2.3072 \cdot 10^{-28} \frac{1}{r} J \quad \text { with } \quad Z=1 \tag{183}
\end{equation*}
$$

We know from $|5|$ page 178 that the discrete energy levels for the orbital electron of the H -atom is

$$
\begin{equation*}
E_{n_{\text {Coul }}}=-\frac{m}{2 \hbar^{2}}\left(\frac{Z e^{2}}{4 \pi \epsilon_{o}}\right)^{2} \frac{1}{n^{2}}=2.1819 \cdot 10^{-18} \frac{1}{n^{2}} J \tag{184}
\end{equation*}
$$

The difference between the energy levels is

$$
\begin{equation*}
\Delta E_{n_{\text {Coul }}}=2.1819 \cdot 10^{-18}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] J \tag{185}
\end{equation*}
$$

### 6.1 Quantized gravitation.

In the present approach of "Emission \& Regeneration" UFT gravitation is presented based on the reintegration of migrated electrons and positrons to their nuclei. According to that model the force on one electron/positron of a mass $M_{1}$ due to the reintegration of an electron/positron to an atomic nucleus of a mass $M_{2}$ is given by

$$
\begin{equation*}
F_{i}=\frac{d p}{\Delta t}=\frac{k c \sqrt{m} \sqrt{m_{p}}}{4 K d^{2}} \iint_{\text {Induction }} \quad \text { with } \quad \iint_{\text {Induction }}=2.4662 \tag{186}
\end{equation*}
$$

and the corresponding potential is

$$
\begin{equation*}
U(r)_{G r a v}=\left(2.4662 \frac{k c \sqrt{m} \sqrt{m_{p}}}{4 K}\right) \frac{1}{r}=2.3071 \cdot 10^{-28} \frac{1}{r} J \tag{187}
\end{equation*}
$$

If we write the Schroedinger equation with the gravitation potential instead of the Coulomb potential for the H -atom, we get discrete energy levels simply in replacing the expression in brackets of eq.(184) with the expression in brackets of eq. (187)

$$
\begin{equation*}
E_{n_{G r a v}}=-\frac{m}{2 \hbar^{2}}\left(2.4662 \frac{k c \sqrt{m} \sqrt{m_{p}}}{4 K}\right) \frac{1}{n^{2}}=2.1816 \cdot 10^{-18} \frac{1}{n^{2}} \mathrm{~J} \tag{188}
\end{equation*}
$$

In the same model of gravitation the number of reintegrating electrons/positrons for a mass $M$ is derived as $\Delta G=\gamma_{G} M$ with $\gamma_{G}=5.3779 \cdot 10^{8} \mathrm{~kg}^{-1}$. The resulting energy level due to all reintegrating electrons/positrons of $M_{1}$ and $M_{2}$ is

$$
\begin{equation*}
E_{n_{\text {Grav tot }}}=2.1816 \cdot 10^{-18} \Delta G_{1} \Delta G_{2} \frac{1}{n^{2}} J \tag{189}
\end{equation*}
$$

For the H -Atom $M_{2}$ is formed by one proton composed of 918 electrons and 919 positrons and $M_{1}$ is the mass of the electron. The mass of a proton is $M_{2}=m_{\text {prot }}=$ $1.6726 \cdot 10^{-27} \mathrm{~kg}$ and the mass of the electron $M_{1}=m_{\text {elec }}=9.1094 \cdot 10^{-31} \mathrm{~kg}$. We get $\Delta G_{2}=8.9951 \cdot 10^{-19}$ and $\Delta G_{1}=4.8989 \cdot 10^{-22}$. We get for the energy difference for orbital electrons at the H -Atom due to gravitation potential

$$
\begin{equation*}
\Delta E_{n_{\text {Proton }}}=9.6134 \cdot 10^{-58}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] J \tag{190}
\end{equation*}
$$

If we compare the factors of the brackets for the energy difference due to the Coulomb potential of eq. (185) and the gravitational potential of eq. (190), we see that even between very different energy levels $n_{1}$ and $n_{2}$ of the gravitational levels the energy differences of the gravitation are neglectible compared with the Coulomb.

For the energy difference between two levels $n_{1}$ and $n_{2}$ of an atom we can write:

$$
\begin{equation*}
\Delta E_{n_{\text {Coul }}} \pm \Delta E_{n_{\text {Grav }}}=h(\nu \pm \Delta \nu)=2.1819 \cdot 10^{-18}\left[1 \pm \Delta G_{1} \Delta G_{2}\right]\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] J \tag{191}
\end{equation*}
$$

with $\Delta G=\gamma_{G} M$ where $\gamma_{G}=5.3779 \cdot 10^{8} \mathrm{~kg}^{-1}$.
Now we make the same calculations for the difference between the energy levels due to the gravitation potential of the sun with $M_{2}=M_{\odot}=1.9891 \cdot 10^{30} \mathrm{~kg}$ and the earth with $M_{1}=M_{\dagger}=5.9736 \cdot 10^{24} \mathrm{~kg}$. We we get $\Delta_{G_{\odot}}=1.0697 \cdot 10^{39}$ and $\Delta_{G_{\dagger}}=3.2125 \cdot 10^{33}$ resulting

$$
\begin{equation*}
\Delta E_{n_{\odot, \uparrow}}=7.4968 \cdot 10^{54}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] J \tag{192}
\end{equation*}
$$

As the earth shows no quantization in its orbit around the sun, two adjacent levels $n_{1}$ and $n_{2}$ must be very large outer levels so that $\Delta E_{n_{\odot, t}} \approx 0$, similar to the large outer levels of the conducting electrons of conducting materials. Mathematically we can write with $n_{2}=n_{1}+1$

$$
\begin{equation*}
\lim _{n_{1} \rightarrow \infty} \Delta E_{n_{\odot, t}}=7.4968 \cdot 10^{54}\left[\frac{1}{n_{1}^{2}}-\frac{1}{\left(n_{1}+1\right)^{2}}\right]=0 \mathrm{~J} \tag{193}
\end{equation*}
$$

### 6.2 Relation between energy levels and space.

The compatibility of gravitation as the reintegration of migrated electrons/positrons to their nuclei is also shown by the following calculations. From eq. (189) we get the energy difference between two gravitation levels

$$
\begin{equation*}
\Delta E_{n_{G r a v}}=2.1816 \cdot 10^{-18} \Delta G_{1} \Delta G_{2}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] J \tag{194}
\end{equation*}
$$

and with the difference between two gravitation potentials at different distances

$$
\begin{equation*}
\Delta U_{\text {Grav }}=G M_{1} M_{2}\left[\frac{1}{r_{1}}-\frac{1}{r_{2}}\right] J \tag{195}
\end{equation*}
$$

we can write that $\Delta E_{n_{\text {Grav }}}=\Delta U_{\text {Grav }}$ what gives with $r_{1} r_{2} \approx r^{2}$

$$
\begin{equation*}
\frac{\Delta r}{r^{2}}=\frac{2.1816 \cdot 10^{-18} \gamma_{G}^{2}}{G}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] \tag{196}
\end{equation*}
$$

For the H -atom with $r \approx 10^{-13} m$ we get for the difference between the two first energy levels $n_{1}=1$ and $n_{2}=2$

$$
\begin{equation*}
\Delta r=\frac{2.1816 \cdot 10^{-18} \gamma_{G}^{2}}{G} r^{2}\left[\frac{3}{4}\right]=7.0926 \cdot 10^{-17} \mathrm{~m} \tag{197}
\end{equation*}
$$

what is a reasonable result because $\Delta r \ll r$.
Now we make the same calculations for the earth and the sun with $r_{\odot, \dagger} \approx 150.00 \cdot$ $10^{9} \mathrm{~m}$. We get

$$
\begin{equation*}
\Delta r_{\odot, \dagger}=2.1164 \cdot 10^{32}\left[\frac{1}{n_{1}^{2}}-\frac{1}{n_{2}^{2}}\right] \tag{198}
\end{equation*}
$$

As the earth shows no quantization in its orbit around the sun, two adjacent levels $n_{1}$ and $n_{2}$ must be very large outer levels so that $\Delta r_{\odot, \dagger} \approx 0$, similar to the large outer levels of the conducting electrons of conducting materials.

### 6.3 Superposition of gravitation and Coulomb forces.

The "Emission \& Regeneration" UFT shows that the Coulomb and the Ampere forces tend to zero for the distance between electrons/positrons tending to zero. The behaviour is explained with the cross product of the angular momenta of the regenerating rays of FPs that tends to zero.

The induction force is not a function of the cross product but simply the product between angular momenta of the regenerating rays of FPs. The result is that the induction force does not tend to zero with the distance between inducing particles tending to zero. As the gravitation was defined as the reintegration of migrated electrons/positrons to their nuclei and as a induction force, the gravitation force prevails over the Coulomb or Ampere forces for the distance tending to zero.

Fig. 5 shows qualitatively the resulting momentum due to Coulomb/Ampere and Gravitation momenta between an atomic nucleus of a target and a He nucleus.


Figure 5: Resulting linear momentum $p$ due to Coulomb/Ampere and Gravitation momenta.

Note: The gravitation model of "Emission \& Regeneration" UFT is based on a physical approach of reintegration of migrated electrons/positrons to their nuclei and compatible with quantum mechanics, while General Relativity, the gravitation model of the SM, based on a mathematical-geometric approach is not compatible with quantum mechanics.

## 7 Table comparing the SM and the 'E \& R' model.

| Model | Subdivision | Particle representation | Force Carriers | Fields | Interactions | Gauges | Comment |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SM <br> (Poly-particle) | Classic | Pointlike |  | Strong <br> Weak <br> Electromagnetic Gravitation | Strong <br> Weak <br> Electromagnetic Gravitation |  | Four fields one for each type of force |
|  | QM | Wave Packet |  |  |  | QCD <br> Electroweak QED <br> Gravity Duality |  |
| $\underset{\text { (Mono-particle) }}{E} \& \underset{R}{R}$ | Classic | Focal-point of rays of Fundamental Particles | Fundamental <br> Particle with <br> Longitudinal and <br> Transversal angular momenta | $d H$ field with Longitudinal and Transversal components | Electromagnetic <br> (Long x Long, <br> Trans x Trans, <br> Trans - Long) |  | One field for all forces |
|  | QM | Wave <br> Packet |  |  |  | QED |  |

Figure 6: Table comparing the SM and the 'E \& R' model.

Fig. 6 shows the SM and the 'E \& R' model subdivided in classical physics and QM. The classic part of the SM with its point-like representation of particles has four force-carriers, four fields and four interactions. QM based on the classical physics of the SM has correspondingly four gauge theories.

The classic part of the 'E \& R' model with its focal-point representation of particles has only one type of force-carrier, only one field and only one type of interaction. QM based on the classical physics of the 'E \& R' model has correspondingly only one type of gauge theory, namely QED.

The SM has four fields one for each type of force while the 'E \& R' model has only one field for all forces and is therfore a UFT.

The SM is a poly-particle model while the ' $\mathrm{E} \& \mathrm{R}$ ' model is a mono-particle model.

## 8 Summery of main characteristics of the proposed model.

The following abbreviations are used:

1. Basic Subatomic Particles (BSPs) are electrons, positrons and neutrinos.
2. Subatomic Particles (SPs)
3. Fundamental Particles (FPs)

## The main characteristics of the proposed model are:

- Subatomic particles (SPs) are represented as focal points of rays of Fundamental Particles (FPs) that go from infinite to infinite. FPs store the energy of the SPs as rotation defining longitudinal and transversal angular momenta.
- FPs are emitted at the focal point and regenerate the focal point. Regenerating FPs are the FPs that were emitted by other focal points in space.
- The charge of a SP is defined by the rotation sense of the longitudinal angular momenta of the emitted FPs.
- The interacting particles for all types of interactions (electromagnetic, strong, weak, gravitation) are the FPs with their longitudinal and transversal angular momenta.
- All known forces are derived from one vector field generated by the longitudinal and transversal angular momenta of fundamental particles.
- All the basic laws of physics (Coulomb, Ampere, Lorentz, Maxwell, Gravitation, bending of particles and interference of photons, Bragg, Schroedinger) are mathematically derived from the proposed model, making sure that the approach is in accordance with experimental data.
- Electrons and positrons neither attract nor repel each other for the distance between them tending to zero. Nucleons are interpreted as swarms of electrons and positrons.
- The coexistence of protons in the atomic nucleus does not require the definition of a special strong force nor additional mediating particles (gluons).
- Quarks are composed of electrons and positrons and the charge Q is the relation between the difference of positrons and electrons of the quark and the total number of electrons and positrons. Q is the relative charge of the quark.
- The emission of particles from a heavy atomic nucleus does not require the definition of a special weak force nor additional mediating particles.
- Gravitation has its origin in the linear momenta induced by the reintegration of migrated electrons and positrons to their nuclei. No special mediating particles are required (gravitons).
- The gravitation force is composed of an induced Newton component and an Ampere component due to parallel currents of reintegrating electrons and positrons. For galactic distances the induced component can be neglected. A positive Ampere component explains the flattening of galaxies' rotation curve (no dark matter is required) and a negative Ampere component explains the expansion of galaxies (no dark energy is required).
- The inertia of particles is explained with the time delay between the emission and the regeneration of FPs. No special mediating particles are required.
- Permanent magnets are explained with the synchronization along a closed path of reintegrating BSPs to their nuclei.
- The two possible states (spins) in one energy level of orbiting electrons are replaced by the two types of electrons defined in the present theory, namely the accelerating and decelerating electrons.
- The splitting of the atomic beam in the Stern-Gerlach experiment is explained with the magnetic field generated by the parallel currents composed of the orbital electron and the current induced in the atomic nucleus. The magnetic spin is not an intrinsic characteristic of the electron.
- Relativity deduced on speed variables instead of space-time variables gives the same equations as special relativity but without the fictitious concepts of time dilation and length contraction. Also the transversal Doppler effect, which was never experimentally detected, appears.
- The wave character of the photon is defined as a sequence of FPs with opposed transversal angular momenta which carry potential opposed transversal linear momenta.
- Light that moves trough a gravitation field can only lose energy, what explains the red shift of light from far galaxies (no expansion of the universe is required).
- Diffraction of particles such as the Bragg diffraction of electrons is now the result of the quantized interaction of parallel currents.
- As the model relies on BSPs permitting the transmission of linear momenta at infinite speed via FPs, it is possible to explain that entangled photons show no time delay when they change their state.
- The addition of a wave to a particle (de Broglie) is effectively replaced by a relation between the particles radius and its energy.
- The Schroedinger equation is replaced by an equation where the wave function is derived one time versus space and two times versus time in analogy to Newton's second law.
- The uncertainty relation of quantum mechanics derived with the new wave function form pairs of canonical conjugated variables between "energy and space" and "momentum and time".
- The time independent Schroedinger equation results deriving the new wave function two times versus space, the same as for the established wave function.
- The new quantum mechanics theory, based on wave functions derived from the radius-energy relation, is in accordance with the quantum mechanics based on the correspondence principle.
- All interactions are of electromagnetic type and described by QEDs (Quantum Electrodynamics) and no other type of interactions are required.
- The gravitation of the present approach "Emission \& Regeneration" UFT is compatible with quantum mechanics, what is not the case with General Relativity, which is the gravitation model of the SM.
- Finally the hypothesis is made that the apparent CMB radiation is a gravitational effect between the mass of the satellite and the signal evaluating part of the satellite, what would explaining the isotropy of the radiation.


## 9 Bibliography

Note: The present approach is based on the concept that fundamental particles are constantly emitted by electrons and positrons and constantly regenerate them. As the concept is not found in mainstream theory, no existing paper can be used as reference.

1. Guenter Lehner. Elektromagnetische Feldtheorie. 7. bearbeitete Auflage 2010. Springer Verlag.
2. Hering • Martin • Stohrer. Physik fuer Ingenieure. Vierte, verbesserte Auflage 1992. VDI Verlag.
3. Albrecht Lindner. Grundkurs Theoretische Physik. Teubner Verlag, Stuttgart 1994.
4. Georg Joos. Lehrbuch der Theoretischen Physik. 15., voellig neu bearbeitete Auflage 1989. AULA-Verlag Wiesbaden.
5. Max Schubert / Gerhard Weber. Quantentheorie, Grundlagen und Anwendungen. Spektrum, Akad. Verlag 1993.
6. Harald Klingbeil. Electromagnetische Feldtheorie. 2. ueberarbeitete Auflage 2011, Vieweg + Teubner Verlag.
7. Martin Ammon / Johanna Erdmenger. Gauge/Gravity Duality. Cambridge University Press 2015.
8. Benenson • Harris • Stocker • Lutz. Handbook of Physics. Springer Verlag 2001.
9. B.R. Martin \& G. Shaw. Particle Physics. John Wiley \& Sons 2003.
10. Stephen G. Lipson. Optik. Springer Verlag 1997.
11. Osvaldo Domann. "Emission \& Regeneration" Unified Field Theory. June 2003. www.odomann.com.
