Nuclear Quantum Gravity – A First Review

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Abstract: To have a unified model of nuclear quantum gravity, it seems quite reasonable to consider a large nuclear gravitational constant, $G_s \cong (3.3 \pm 0.03) \times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$. In this context, we show practical applications pertaining to micro physics as well as macro physics. We would like to suggest that: (1) G_s plays a crucial role in understanding quantum theory of light, photoelectric work functions, superconductivity, nuclear binding energy, nuclear root mean square charge radii, root mean square radius of proton, neutron life time, neutron-proton mass difference, nuclear stability, nuclear magnetic dipole moments, weak coupling angle, Fermi's weak coupling constant, proton melting point and total energy of electron in Hydrogen atom etc.; (2) Nuclear binding energy can be understood with a single energy coefficient of magnitude 10 MeV. (3) Newtonian gravitation al constant G_N and the proposed G_s play a joint role in understanding neutron star mass generation as well as proton mass generation; and (4) Considering G_s as a characteristic feature of magnetism, celestial bodies 'mass dependent' magnetic dipole moments can be estimated. (5) Magnitude of G_N can certainly be estimated from microscopic elementary physical constants.

Keywords: Final unification, nuclear gravitational constant, Newtonian gravitational constant, nuclear quantum gravity.

1. Introduction

In this paper, we review our recently published views on nuclear quantum gravity [1] for better presentation on nuclear stability and binding energy [2-9], root mean square radius of proton, neutron life time, issues connected with coupling constants and other minor changes that may help in developing this new subject. The most desirable cases of any unified description [10-13] are:

1. To implement gravity in microscopic physics.
2. To develop a model of quantum gravity.
3. To simplify the complicated issues of known physics.
4. To predict new effects, arising from a combination of the fields inherent in the unified description.

In this context, for a better understanding, we would like to review our recent publications [14-25] in the following way: "By replacing $\left(\hbar c / m_p^2\right)$ with a large gravitational constant of magnitude, $G_s \cong 3.32956 \times 10^{28}$ assumed to be associated with nuclear structure, a basic model of 'nuclear quantum gravity' can be developed". Qualitatively, our assumption is not new and is having a long standing history [26-34]. For more information, readers are strongly encouraged to see Abdus Salam's 'Strong gravity' concept [32]. Recently, O. F. Akinto and Farida Tahir elaborated their work on 'modified strong gravity concepts' pertaining to QCD and general relativity in arXiv preprint [33]. In 2013, Roberto Onofrio [34] proposed a very interesting concept: Weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi coupling constant is related to the Newtonian constant of gravitation. In his opinion, at atto-meter scale, Newtonian gravitational constant seems to reach a magnitude of $8.205 \times 10^{22}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

2. **To unite nuclear and sub nuclear interactions**

The modern theory of strong interaction is Quantum chromodynamics (QCD) [35-38]. It explores baryons and mesons in broad view with 6 quarks and 8 gluons. According to QCD, the four important properties of strong interaction are: 1) color charge;2) confinement; 3) asymptotic freedom ;4) short-range nature($<10^{-15}$ m). Color charge is assumed to be responsible for the strong force to act on quarks via the force carrying agent, gluon. Experimentally it is well established that, strength of strong force depends on the energy of the interaction or the distance between particles. At lower energies or longer distances: a) color charge strength increases; b) strong force becomes 'stronger'; c) nucleons can be considered as fundamental nuclear particles and quarks seem to be strongly bound within the nucleons leading to 'Quark confinement'. At high energies or short distances: a) color charge strength decreases; b) strong force gets 'weaker'; 3) colliding protons generate 'scattered free quarks' leading to 'Quark Asymptotic freedom'. Based on these points, low energy nuclear scientists assume 'strong interaction' as a strange nuclear interaction associated with binding of nucleons and its implications were not considered. High energy nuclear scientists consider nucleons as composite states of quarks and try to understand the nature and strength of strong interaction (α_s) at sub nuclear level. According to QCD, (α_s) decreases with increasing interaction energy. By definition, at low energy scales, $\alpha_s \cong 1$ and by experiments and observations, at 80 to 90 GeV energy scales, $\alpha_s \cong 0.1186$.

At this juncture, one important question to be answered and reviewed at basic level is: **How to understand nuclear interactions in terms of sub nuclear interactions?** Unfortunately, 1) At 1.2 fm scale, there is no practical evidence or applications for the basic definition of $\alpha_s \cong 1$. 2) With current concepts of QCD, one cannot explain the observed nuclear binding energy scheme. 3) Famous nuclear models like, Liquid drop model and Fermi gas model [39-41] are lagging in answering this question. To find a way, we would like to suggest that, by implementing the 'strong coupling constant' of magnitude 0.1186 in low energy nuclear physics, nuclear charge radius, Fermi's weak coupling constant and strong coupling constant can be studied in a unified picture. Proceeding further, close to beta stability line, nuclear binding energy can be addressed with a single energy coefficient of (8.9 to 10.0) MeV [3-6].

5. Microscopic and macroscopic Applications of the large nuclear gravitational constant

With the proposed assumption, quantum theory of light [42], photo electric work functions [43], super conductivity [44] nuclear physics, sub nuclear (particle) physics, electroweak theory, physics pertaining to planetary dipole magnetic moments, nuclear astrophysics and Bohr's theory of Hydrogen atom can be studied in a unified approach.

1) Quantum theory of light can be understood with, $hc \cong \sqrt{\left(\frac{m_p}{m_e}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)\left(G_s m_p^2\right)}.$
2) Magnetic flux quantum in super conductivity can be understood with, $\Phi_0 \cong \frac{h}{2e} \cong \frac{1}{2}\sqrt{\left(\frac{m_p}{m_e}\right)\left(\frac{\mu_0}{4\pi}\right)\left(G_s m_p^2\right)}$
3) G_F being the Fermi's weak coupling constant, $G_s m_p m_e \cong \left(\frac{\pi^3 \varepsilon_0 G_F^2 c^8}{e^2 G_s^2}\right)^{\frac{1}{3}}.$
4) Photoelectric work functions can be understood with, $W_Z \approx -Z^{\frac{1}{3}}\left(\frac{G_s m_p m_e}{2r_Z}\right)$ to $-A^{\frac{1}{3}}\left(\frac{G_s m_p m_e}{2r_Z}\right)$ where r_Z is the radius of atom [45], Z is the atomic number and A is the mass number.
5) Near to beta stability line, nuclear binding energy can be understood with, $\frac{1}{2}\left(e^2 G_s m_p^3 / 4\pi\varepsilon_0 \hbar^2\right) \cong 10.09$ MeV.
6) Nucleon mass difference, nuclear stability and neutron life time can be understood with $\left[e^2 G_s m_p^3 / 4\pi\varepsilon_0 \left(\hbar/2\right)^2\right] \cong 80.7$ MeV.
7) Nuclear charge radius can be addressed with, $R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.239$ fm Or $\left(\ln\sqrt{\frac{G_s}{G_N}}\right)\sqrt{\frac{e^2 G_s}{4\pi\varepsilon_0 c^4}} \cong 1.374$ fm.
8) For medium, heavy and super heavy atomic nuclei, root mean square charge radii can be fitted with $R_{(Z,A)} \cong \left\{Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right) \cong \left\{Z^{1/3} + \left[Z(A-Z)\right]^{1/6}\right\}\left(\frac{G_s m_p}{c^2}\right).$
9) Hadronic melting points [46] can be understood with, $T_{hadron} \cong \frac{\hbar c^3}{8\pi k_B G_s m_{hadron}}.$
10) Strong coupling constant can be understood with, $\frac{1}{\alpha_s} \cong \left(\frac{\hbar c}{G_s m_p^2}\right)^2 \cong 0.1152$ or $\frac{1}{\alpha_s} \cong \exp\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e}} \Rightarrow \alpha_s \cong 0.11855.$
11) Weak coupling angle can be understood with, $\sin^2\theta_W \cong \frac{4\pi\varepsilon_0 G_s m_p m_e}{e^2} \cong 0.2198.$
12) Up and down quark mass ratio can be understood with, $\left(\frac{m_u}{m_d}\right) \cong \sqrt{\frac{4\pi\varepsilon_0 G_s m_p m_e}{e^2}} \cong \sin\theta_W \cong 0.469$
13) Proton's magnetic dipole moment can be understood with, $\mu_{proton} \approx \left(\frac{m_u}{m_d}\right)\left(\frac{eG_s m_p}{c}\right) \approx 1.396\times10^{-26}$ J/Tesla
14) Neutron's magnetic dipole moment can be understood with, $\mu_{neutron} \approx \left(\frac{m_u}{m_d}\right)^{\frac{3}{2}}\left(\frac{eG_s m_n}{c}\right) \approx 9.573\times10^{-27}$ J/Tesla
15) Ratio of proton to neutron magnetic diploe moment can be addressed with $\frac{\mu_{proton}}{\mu_{neutron}} \cong \sqrt{\frac{m_u}{m_d}} \cong \sqrt{\sin\theta_W} \cong 0.685$
16) Ratio of proton-electron magnetic dipole moments can be understood with, $\left(\frac{\mu_{proton}}{\mu_{electron}}\right) \approx \frac{G_s m_p m_e}{\hbar c}.$
17) Nuclear Planck mass can be defined as, $m_{npl} \cong \sqrt{\hbar c/G_s} \cong 546.6\text{ MeV}/c^2$. Based on this new mass unit, a quantized model mechanism can be developed for understanding the hadronic mass spectrum. In our recent publication [14], by considering $546.6\text{ MeV}/c^2$ as a characteristic neutral hadronic fermion, we have developed a toy model for understanding the hadronic mass spectrum.
18) $m_{npl} \cong \sqrt{\hbar c/G_s} \cong 546.6\text{ MeV}/c^2$ can be considered as a characteristic dark matter candidate [33].
19) Total energy of electron in Hydrogen atom can be understood with,

$$-\frac{1}{2}\left(\frac{G_s m_p m_e}{\hbar c}\right)^2 \frac{\sqrt{m_p m_e}c^2}{2n^2} \cong -\frac{14.0}{n^2} \text{ eV, where}$$

$\left(\frac{1}{2n^2}\right)$ can be considered as the probability of finding electron in its orbits labeled as $n = 1, 2, 3, ..$

20) Neutron star mass [47] or radius can be understood with, $\sqrt{\frac{G_s}{G_N}}$.

21) Mass dependent planetary magnetic dipole moments [48] can be understood with, $\left(\frac{G_s m_p m_e}{\hbar c}\right)\left(\frac{e G_s M_{planet}}{2c}\right)$.

22) Root mean square radius of proton can be fitted with:

$$R_p \cong \ln\left(\frac{e^2}{4\pi\varepsilon_0 G_N m_p^2}\right)\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s m_p^2}}\left(\frac{\hbar}{m_p c}\right) \le 0.87 \text{ fm}$$

or $2\pi R_p \cong 2\ln\left(\frac{G_s}{G_N}\right)\sqrt{\frac{e^2 G_s}{4\pi\varepsilon_0 c^4}} \cong 0.8746$ fm.

23) Neutron life time can be understood with,

$$t_n \cong \left(\frac{G_s}{G_N}\right)^{\frac{2}{3}} \sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{\hbar}{m_n c^2}\right) \cong 885.45 \text{ sec}$$

24) Proton-electron mass ratio can be understood with,

$$\begin{cases} \left(\frac{m_p}{m_e}\right) \cong \left(\left(\frac{G_s}{G_N}\right)\left(\frac{G_s m_e^2}{\hbar c}\right)\right)^{\frac{1}{10}} \\ \cong \left(\frac{G_s m_e^2}{G_N m_{npl}^2}\right)^{\frac{1}{10}} \cong 1836.3 \end{cases}$$

where $m_{npl} \cong \sqrt{\hbar c / G_s} \cong 546.6 \text{ MeV/}c^2$.

6. To understand proton's melting point

With reference to Hawking black hole temperature formula [46], melting point of proton [36] can be understood with:

$$T_{proton} \cong \frac{\hbar c^3}{8\pi k_B G_s m_p} \cong 0.15\times 10^{12} \text{ K} \tag{1}$$

Based on this relation and with reference to up quark, quark melting points can be expressed with the following kind of relation.

$$T_{quark} \cong \left(\frac{m_q}{m_{up}}\right)\frac{\hbar c^3}{8\pi k_B G_s m_{up}} \tag{2}$$

where can be $\left(\frac{m_q}{m_{up}}\right)$ represents the ratio of mass of any quark to mass of up quark. Based on this relation, for up quark of rest energy 2 MeV, its corresponding $T_{up} \cong 69$ *Tera K* and $8\pi k_B T_{up} \cong 236$ MeV. This energy can be compared with currently believed QCD energy scale of 270 MeV.

7. To fit and understand Fermi's weak coupling constant

Fermi's weak coupling constant [35] can be fitted with a relation of the kind,

$$\begin{aligned} G_F &\cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \cong G_s m_e^2 \left(2\sqrt{\frac{G_s \hbar}{c^3}}\right)^2 \\ &\cong \frac{4 G_s^2 m_e^2 \hbar}{c^3} \cong 1.44021\times 10^{-62} \text{ J.m}^3 \end{aligned} \tag{3}$$

where $\sqrt{\frac{G_s \hbar}{c^3}} \cong 0.36$ fm can be considered as the characteristic Nuclear Planck length and $2\sqrt{\frac{G_s\hbar}{c^3}} \cong \frac{2G_s m_{npl}}{c^2}$.

8. To fit and understand strong coupling constant

Based on the proposed G_s, strong coupling constant can be fitted with the following kind of relation.

$$\alpha_s \cong \left[\frac{\hbar c}{G_s m_p^2}\right]^2 \cong 0.115543 \tag{4}$$

Based on this relation,

$$R_0 \cong \left(\frac{1}{\sqrt{\alpha_s}}\right)\frac{2\hbar}{m_p c} \cong 1.239 \text{ fm} \tag{5}$$

$$\alpha_s F_W \cong \frac{4\hbar^3 m_e^2}{m_p^4 c} \tag{6}$$

$$\alpha_s \cong \frac{4\hbar^3 m_e^2}{m_p^4 c F_W} \cong \left(\frac{m_e}{m_p}\right)^2\left(\frac{4\hbar^3 m_e^2}{m_p^2 c F_W}\right) \tag{7}$$

Based on this relation and considering relativistic energy of proton, it is possible to show that, $\alpha_s \propto \left\{\left[1-(v/c)^2\right]/m_p^2\right\}$. Qualitatively, this kind of observation seems to be in-line with modern QCD concepts.

9. Two characteristic energy units

Based on the proposed G_s, one can construct two (characteristic and practical) energy units in the following way.

$$\begin{aligned} E_X &\cong \left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)\left(\frac{G_s m_p^2}{\hbar c}\right)\left(m_p c^2\right) \\ &\cong \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2}\right) \cong 20.174 \text{ MeV} \end{aligned}$$

(8)

By considering 'Uncertainty relation' and replacing $(\hbar)$ with $(\hbar/2)$, it is possible to construct another energy unit.

$$E_Y \cong \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0 (\hbar/2)^2}\right) \cong \left(\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2}\right) \cong 4E_X \cong 80.696 \text{ MeV} \quad (9)$$

It may be noted that, with reference to Fermi gas model of the nucleus, E_X seems to represent the nucleon's mean kinetic energy per nucleon and $2E_X \cong (E_Y/2) \cong \sqrt{E_X E_Y} \cong 40.35$ MeV seems to represent the depth of nuclear potential energy.

10. Fitting neutron-proton mass difference

Neutron-proton mass difference can be understood with the following relation:

$$\left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \cong \ln\sqrt{\frac{E_Y}{(m_e c^2)}} \cong \ln\sqrt{\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2 m_e c^2}} \cong \ln\sqrt{\frac{80.696 \text{ MeV}}{0.511 \text{ MeV}}} \cong \ln(4\pi) \cong 2.531 \quad (10)$$

11. Fitting neutron life time

Neutron life time t_n can be understood with the following relation:

$$t_n \cong \exp\left\{\frac{E_Y}{(m_n - m_p)c^2}\right\} \times \left(\frac{\hbar}{m_n c^2}\right) \cong \exp\left(\frac{80.696 \text{ MeV}}{1.2933 \text{ MeV}}\right) \times \left(\frac{\hbar}{m_n c^2}\right) \cong 877.9 \sec \quad (11)$$

This fitted value can be compared with material bottle and cold beam experimental results: (878.5 ± 0.8) sec and (887.7 ± 2.2) sec [49]. See section 22 for correlating bottle and cold beam experimental results.

12. Understanding nuclear stability

Stable mass number corresponding to Z can be estimated with the following relation [2]:

$$(A_s - 2Z) \approx \left\{\frac{m_e c^2}{E_Y}\right\} Z^2 \approx \left(\frac{4\pi\varepsilon_0 \hbar^2 m_e c^2}{4e^2 G_s m_p^3}\right) Z^2 \approx \left(\frac{0.511 \text{ MeV}}{80.696 \text{ MeV}}\right) Z^2 \approx 0.00633 (Z)^2 \approx kZ^2 \quad (12)$$

where $k \cong \left\{\frac{m_e c^2}{E_Y}\right\} \cong \left(\frac{4\pi\varepsilon_0 \hbar^2 m_e c^2}{4e^2 G_s m_p^3}\right) \cong 0.00633$. Using this new number, nuclear binding can be estimated. With further study, it is also possible to show that,

$$k \cong \left\{\frac{m_e c^2}{E_Y}\right\} \approx \frac{0.71 \text{ MeV}}{8.89 \text{ MeV}}. \quad (13)$$

where 0.71 MeV represents the coulombic energy coefficient and 8.89 MeV can be considered as the maximum binding energy per nucleon.

13. Understanding nuclear binding energy

Based on the new integrated model proposed by N. Ghahramany et al [3,4] and with reference to relation (12), it is possible to show that, $Z \cong (40 \text{ to } 83)$, close to the beta stability line,

$$(B)_{A_s} \cong \left[A_s - \left(\frac{N_s^2 - Z^2}{3Z}\right)\right] \times 9.5 \text{ MeV} \cong \left[A_s - \left(\frac{kZA_s}{3}\right)\right] \times 9.5 \text{ MeV} \quad (14)$$

where, $\left[\frac{N_s^2 - Z^2}{Z}\right] \cong kZA_s$. Based on this strange and simple relation and with reference to our recent publications [5,6] and first four terms of the semi empirical mass formula (SEMF), close to the beta stability line, for $(Z = 2 \text{ to } 100)$, it is possible to show that,

$$(B)_{A_s} \cong \left[A_s - A_s^{1/3} - \frac{kA_s\sqrt{N_s Z}}{3.42} - 1\right] \times \left\{\frac{E_X}{2}\right\} \cong \left[A_s - A_s^{1/3} - \frac{kA_s\sqrt{N_s Z}}{3.35} - 1\right] \times 10.09 \text{ MeV} \quad (15)$$

where, $\left(\frac{k}{3.35}\right) \approx \alpha_s \left(\frac{a_c}{2a_a}\right)$. See table-1.

For $Z = 50$ and $A = 100$ to 136, estimated binding energy range is (857 to 1140) MeV and can be compared with reference binding energy [40] range of (806 to 1105) MeV. It is for further study. With reference to SEMF, close to the beta stability line, it is also possible to show that,

$$\frac{(A_s - 2Z)^2}{A_s} \cong \left(k^2 A_s N_s \sqrt{Z}\right) \quad (16)$$

Let,

$$\left\{\begin{array}{l} a_v \cong a_s \cong a_a \approx 14.8 \text{ MeV} \approx (3/2) \times 10.0 \text{ MeV} \\ \text{and } a_c \cong 0.71 \text{ MeV} \end{array}\right\}.$$

If so,

$$(B)_{A_s} \approx \begin{bmatrix} A_s - A_s^{2/3} \\ -0.0473\left[\frac{Z(Z-1)}{A_s^{1/3}}\right] - \left(k^2 A_s N_s \sqrt{Z}\right) \end{bmatrix} \times 14.8 \text{ MeV} \quad (17)$$

In comparison with SEMF, by replacing A_s with A in relation (17) and by considering a multiplication factor of the kind $(A_s/A)^{1-(Z/A)}$ associated with each term, binding energy of A can be estimated approximately. For relations (15) and (17), see figure 1 (red and violet curves respectively) for the estimated binding energy per nucleon close to beta stability line of Z= 2 to 100 compared with first four terms of the semi empirical mass formula (Green curve) where : $a_v \cong 15.77$ MeV, $a_s \cong 18.34$ MeV, $a_a \cong 23.2$ MeV and $a_c \cong 0.71$ MeV.

14. Fitting medium, heavy and super heavy nuclear charge radii

For medium, heavy and super heavy atomic nuclei, nuclear charge radii [50-54] can be fitted with the following simple relation.

$$\begin{aligned} R_{(Z,A)} &\cong \left\{Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3}\right\}\left(\frac{G_s m_p}{c^2}\right) \\ &\cong \left\{Z^{1/3} + \left[Z(A-Z)\right]^{1/6}\right\}\left(\frac{G_s m_p}{c^2}\right) \end{aligned} \quad (18)$$

where $Z = (2 \text{ to } 100)$ and $\left(G_s m_p/c^2\right) \cong 0.62$ fm

See table-2. It may be noted that, this relation is free from arbitrary numbers and can be compared with the following relation available in recent literature [52].

$$R_{(Z,N)} \cong \left\{1 - 0.349\left(\frac{N-Z}{N}\right)\right\} N^{\frac{1}{3}} \; 1.262 \text{ fm} \quad (19)$$

15. To understand neutron star mass and radius

A) If (M_{NS}, m_n) represent the masses of neutron star [33] and neutron, then,

$$\frac{G_N M_{NS} m_n}{\hbar c} \approx \sqrt{\frac{G_s}{G_N}} \to M_{NS} \approx 3.175 M_\odot \quad (20)$$

B) If R_{NS} represents the neutron star radius, then,

$$\frac{R_{NS}}{\left(\sqrt{G_s \hbar/c^3}\right)} \approx \sqrt{\frac{G_s}{G_N}} \to R_{NS} \approx 8.06 \text{ km} \quad (21)$$

16. To understand earth's magnetic dipole moment

Planet's earth's magnetic dipole moment can be understood with:

$$\mu_{earth} \cong \left(\frac{\mu_{proton}}{\mu_{electron}}\right) * \left(\frac{e G_s M_{earth}}{2c}\right) \cong 8.15 \times 10^{22} \text{ J.Tesla}^{-1} \quad (22)$$

It is very interesting to note that,

$\left(\frac{\mu_{proton}}{\mu_{electron}}\right) \approx \frac{G_s m_p m_e}{\hbar c}$. Based on this observation,

$$\begin{aligned} \mu_{earth} &\cong \left(\frac{G_s m_p m_e}{\hbar c}\right) * \left(\frac{e G_s M_{earth}}{2c}\right) \\ &\cong 8.566 \times 10^{22} \text{ J.Tesla}^{-1} \end{aligned} \quad (23)$$

Based on this relation, other solar planets, exo-planets and neutron star's "mass dependent" magnetic dipole moments can be estimated [48]. See table-3. It may be noted that, for 30 hot Jupiters, on an average, estimated value is roughly 0.2 times the reference value.

17. Fitting the Newtonian gravitational constant and proton-electron mass ratio

It is noticed that,

$$\frac{G_N}{G_s} \cong \left\{\left(\frac{m_e}{m_p}\right)^{10} \left(\frac{G_s m_p^2}{\hbar c}\right)\right\} \text{ and}$$

$$\begin{aligned} G_N &\cong \left\{\left(\frac{m_e}{m_p}\right)^{10} \left(\frac{G_s m_p^2}{\hbar c}\right)\right\} G_s \\ &\cong 6.67986 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \end{aligned} \quad (24)$$

By considering (G_s, G_N) as basic features of final unification and by considering $m_{npl} \cong \sqrt{\hbar c/G_s} \cong 546.6 \text{ MeV}/c^2$ as a characteristic hadronic fermion, it is possible to show that,

$$\left.\begin{aligned} \left(\frac{m_p}{m_e}\right) &\cong \left(\frac{G_s m_e^2}{G_N m_{npl}^2}\right)^{\frac{1}{10}} \cong 1836.3 \text{ and} \\ m_p &\cong \left(\frac{G_s m_e^2}{G_N m_{npl}^2}\right)^{\frac{1}{10}} m_e \cong 1836.3 \times m_e \end{aligned}\right\} \quad (25)$$

Here, interesting point to be noted is that, in RHS, m_e seems to be associated with G_s and m_{npl} seems to be associated with G_N.

18. Alternative expression for strong coupling constant

It is also noticed that,

$$\exp\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e}} \cong \frac{1}{\alpha_s} \cong 8.43562 \qquad (26)$$
$$\to \alpha_s \cong 0.11855$$

This can be compared with the recommended value of 0.1186 and can be given some consideration.

19. Understanding the total energy of electron in hydrogen atom

Let r_0 be the characteristic imaginary distance between proton and electron. In a quantum gravitational approach,

$$r_o \approx \left(\frac{\hbar c}{G_s m_p m_e}\right)\left(\frac{\hbar}{\sqrt{m_p m_e}c}\right) \approx \frac{\hbar^2}{G_s m_p^{3/2} m_e^{3/2}} \approx 5.6161\times10^{-12}\ \text{m} \qquad (27)$$

It may be noted that, this length is 9.42 times less than the Bohr radius a_o. It can be approximated by the following relation.

$$\frac{a_o}{r_o} \approx 2\left(\frac{e^2}{4\pi\varepsilon_0 G_s m_p m_e}\right) \approx 2\times4.5475 \approx 9.075 \qquad (28)$$

By considering $\left(\frac{1}{2n^2}\right)$ as the probability of finding electron in its orbits labeled as $n = 1,2,3,..,$ potential energy of electron can be understood with the following relation.

$$\left(E_{pot}\right)_n \approx -\left(\frac{1}{2n^2}\right)\frac{G_s m_p m_e}{r_0} \approx -\left(\frac{G_s m_p m_e}{\hbar c}\right)^2 \frac{\sqrt{m_p m_e}c^2}{2n^2} \approx -\frac{28.03}{n^2}\ \text{eV} \qquad (29)$$

This can be compared with $-\left(\frac{27.2}{n^2}\right)$ eV .With reference to Virial theorem, corresponding kinetic energy can be understood with the following relation.

$$\left(E_{kin}\right)_n \approx \frac{1}{2}\left|\left(\frac{1}{2n^2}\right)\frac{G_s m_p m_e}{r_0}\right| \approx \left[\left(\frac{G_s m_p m_e}{\hbar c}\right)^2 \frac{\sqrt{m_p m_e}c^2}{4n^2}\right] \approx \frac{14.014}{n^2}\ \text{eV} \qquad (30)$$

Thus, binding energy or total energy of electron can be understood with,

$$\left(E_{tot}\right)_n \approx \left(E_{pot}\right)_n - \left(E_{kin}\right)_n \approx -\left(\frac{1}{2n^2}\right)\frac{G_s m_p m_e}{2r_0} \approx -\left[\left(\frac{G_s m_p m_e}{\hbar c}\right)^2 \frac{\sqrt{m_p m_e}c^2}{4n^2}\right] \approx -\frac{14.014}{n^2}\ \text{eV} \qquad (31)$$

This can be compared with the experimental total energy of $-\left(\frac{13.6}{n^2}\right)$ eV . Based on this coincidence,

$$G_s \cong \left(\frac{\sqrt{2}e^2}{4\pi\varepsilon_0 m_p^{5/4} m_e^{3/4}}\right) \cong 3.27125\times10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \qquad (32)$$

20. Fitting and understanding the photoelectric work functions

Based on $\left(G_s m_p m_e\right)$, photoelectric work functions can be estimated with the following relation.

$$W_Z \approx -Z^{\frac{1}{3}}\left(\frac{G_s m_p m_e}{2r_Z}\right) \qquad (33)$$

where r_Z is the radius of atom and Z is the atomic number. With reference to light, medium and heavy atomic experimental data range, above relation can be slightly modified with $A^{\frac{1}{3}}$ where A is the atomic mass number.

$$W_Z \approx -A^{\frac{1}{3}}\left(\frac{G_s m_p m_e}{2r_Z}\right) \approx \left(2Z\right)^{\frac{1}{3}}\left(\frac{G_s m_p m_e}{2r_Z}\right) \qquad (34)$$

See table-4.

21. To estimate G_N with RMS radius of proton

Correlating elementary physical constants of different areas of physics is interesting and uncertain. With reference to the above semi empirical relations, in an optimistic approach, we tried to correlate the root mean square radius of proton and the Newtonian gravitational constant in the following way.

$$R_p \cong \ln\left(\frac{e^2}{4\pi\varepsilon_0 G_N m_p^2}\right)\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s m_p^2}}\left(\frac{\hbar}{m_p c}\right) \cong 0.87\ \text{fm} \qquad (35)$$

$$\frac{e^2}{4\pi\varepsilon_0 G_N m_p^2} \cong \exp\left[\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{m_p R_p c}{\hbar}\right)\right]$$
$$\Rightarrow G_N \cong \left\{\exp\left[\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{m_p R_p c}{\hbar}\right)\right]\right\}^{-1}\frac{e^2}{4\pi\varepsilon_0 m_p^2} \qquad (36)$$

PDG recommended [55] value of RMS radius of proton is, $R_p \cong \left(0.8751 \pm 0.0061\right)$ fm. It is very interesting to note that, recommended value of G_N seems to be fitted with lower limit of the rms radius of proton i.e. $R_p \cong \left(0.8751 - 0.0061\right) \cong 0.869$ fm. From relation (36), estimated $G_N \cong 7.2092\times10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2}$.

Considering the recommended value of $G_N \cong 6.67408 \times 10^{-11}$ m^{3}kg^{-1}sec^{-2}, from relation (35), estimated $R_p \cong 0.8698073$ fm.

Based on the values estimated from relations (35) and (36), it is possible to say that, there exists a very tight correlation in between Newtonian gravitational constant and RMS radius of proton.

Alternatively, independent of proton rest mass and reduced Planck's constant, we noticed that,

$$2\pi R_p \cong 2\ln\left(\frac{G_s}{G_N}\right)\sqrt{\frac{e^2 G_s}{4\pi\varepsilon_0 c^4}} \cong 0.8746 \text{ fm}$$
$$\rightarrow R_p \cong \frac{1}{\pi}\ln\left(\frac{G_s}{G_N}\right)\sqrt{\frac{e^2 G_s}{4\pi\varepsilon_0 c^4}} \quad (37)$$

Based on this relation,

$$\ln\left(\frac{G_s}{G_N}\right) \cong \pi\sqrt{\frac{4\pi\varepsilon_0 c^4 R_p^2}{e^2 G_s}} \text{ and}$$
$$G_N \cong \left\{\exp\left(\pi\sqrt{\frac{4\pi\varepsilon_0 c^4 R_p^2}{e^2 G_s}}\right)\right\}^{-1} G_s \quad (38)$$

If, recommended $R_p \cong 0.8751$ fm, estimated $G_N \cong 6.3793785 \times 10^{-11}$ m^{3}kg^{-1}sec^{-2}.

22. To understand neutron life time controversy and to fit G_N

With reference to (Gs, G_N) and with reference to bottle experiments and beam experiments, it is also possible to express t_n in the following way.

To fit with bottle experiments,

$$(t_n)_{bottle} \cong \left(\frac{G_s}{G_N}\right)^{\frac{2}{3}}\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{\hbar(1-\alpha)}{m_n c^2}\right) \cong 878.985 \text{ sec} \quad (39)$$

To fit with beam experiments,

$$(t_n)_{beam} \cong \left(\frac{G_s}{G_N}\right)^{\frac{2}{3}}\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{\hbar}{m_n c^2}\right) \cong 885.45 \text{ sec} \quad (40)$$

Interesting point to be noted is that, results of bottle experiments and beam experiments can be correlated with a factor of the kind, $(1-\alpha)$.

$$\frac{(t_n)_{bottle}}{(t_n)_{beam}} \cong (1-\alpha) \quad (41)$$

Based on relations (39) and (40), magnitude of G_N can be estimated with the following relations.

$$G_N \cong \left\{\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{\hbar}{m_n c^2 (t_n)_{bottle}}\right)\right\}^{\frac{3}{2}} G_s \quad (42)$$
$$\cong 6.753392 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}.$$

$$G_N \cong \left\{\sqrt{\frac{4\pi\varepsilon_0 G_s m_p^2}{e^2}}\left(\frac{\hbar}{m_n c^2 (t_n)_{beam}}\right)\right\}^{\frac{3}{2}} G_s \quad (43)$$
$$\cong 6.648678 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}.$$

Based on relations (11) and (39),

$$G_N \cong \left\{\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_s m_p^2}} \times \exp\left(\frac{E_Y}{(m_n - m_p)c^2}\right)\right\}^{-\frac{3}{2}} G_s$$
$$\cong 6.76075 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2}. \quad (44)$$

23. Discussion

It is true that, unless stringent requirements are met, in general, speculative alternatives to currently accepted theories cannot be accepted. Scientific papers having content that lie outside the mainstream of current research must justify by including a clear, detailed discussion of the motivation for the new speculation, with reasons for introducing any new concepts. If the new formulation results are in contradiction with the accepted theory, then there must both be a discussion of which experiments could be done to verify that the conventional theory needs improvement, and also an analysis showing the consistency of the new theory with the existing experiments. In this context, we would like to appeal that, in this paper, we presented a variety of relations pertaining to nuclear and electroweak coupling constants. It is clear from the above relations that we could satisfactorily fit the nuclear data through semi-empirical relations. This sincere attempt is to be ascertained by the scientific community. The problem is with "our understanding" and "our perception" by using which the current 'scientific standards' and 'procedures' can be reviewed for a better understanding of nature. We would like to appeal that,

1) With respect to currently believed String theory and Quantum gravity models - proposed assumption and proposed semi empirical relations, can be given some consideration in developing a 'workable model' of TOE.
2) Magnitude of G_N can be estimated from microscopic elementary constants by considering expressions like

$\left(\frac{G_s}{G_N}\right)$ or $\ln\left(\frac{G_s}{G_N}\right)$ or $\ln\left(\frac{e^2}{4\pi\varepsilon_0 G_N m_p^2}\right)$. We are working in this new direction.

3) Magnitude of G_s can be estimated with any of the following three relations:

$$\left.\begin{array}{l} hc \cong \sqrt{\left(\frac{m_p}{m_e}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)\left(G_s m_p^2\right)} \\ \left(\frac{m_n c^2 - m_p c^2}{m_e c^2}\right) \cong \ln\sqrt{\frac{4e^2 G_s m_p^3}{4\pi\varepsilon_0 \hbar^2 m_e c^2}} \\ \left(\frac{e^2 G_s m_p^3}{4\pi\varepsilon_0 (\hbar/2)^2 (m_n - m_p) c^2}\right) \cong \ln\left(\frac{(m_n c^2) t_n}{\hbar}\right) \end{array}\right\}$$

24. Conclusion

We would like to appeal the science community that:

1) So far, the whole subject of nuclear physics and particle physics is being studied independent of 'gravity'.
2) Background of whole experimental apparatus of nuclear and particle physics is 'gravity' only.
3) As of today, 'string theory and its sister models' seem to be completely theoretical in nature and beyond the scope of observed four dimensions.
4) To have a 'theory of everything, it is inevitable to unite gravity and other three atomic interactions and is beyond the scope of current experimental physics.

Based on these points, even though it is in its budding stage, in a broad view, our work can be recommended for further research.

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Table 1: To estimate nuclear binding energy close to beta stability line

Proton number	Mass number	Neutron number	Estimated binding energy Relation (15) (MeV)	(1 to 4) terms of SEMF binding energy (MeV)
2	4	2	14.1	16.0
3	6	3	31.8	31.7
4	8	4	49.8	48.5

5	10	5	68.1	66.0
6	12	6	86.5	83.8
7	14	7	105.0	101.9
8	16	8	123.5	120.1
9	19	10	151.3	148.7
10	21	11	169.8	167.3
11	23	12	188.2	185.9
12	25	13	206.7	204.5
13	27	14	225.1	223.0
14	29	15	243.5	241.4
15	31	16	261.8	259.7
16	34	18	289.3	288.4
17	36	19	307.5	306.7
18	38	20	325.7	324.9
19	40	21	343.8	343.0
20	43	23	370.8	371.1
21	45	24	388.8	389.1
22	47	25	406.7	407.0
23	49	26	424.6	424.7
24	52	28	451.2	452.4
25	54	29	468.9	470.0
26	56	30	486.5	487.4
27	59	32	512.9	514.6
28	61	33	530.3	531.9
29	63	34	547.7	549.0
30	66	36	573.7	575.8
31	68	37	590.9	592.8
32	71	39	616.7	619.1
33	73	40	633.7	635.9
34	75	41	650.7	652.5
35	78	43	676.1	678.5
36	80	44	692.9	694.9
37	83	46	718.1	720.5
38	85	47	734.7	736.7
39	88	49	759.6	762.0
40	90	50	776.1	778.0
41	93	52	800.7	803.0
42	95	53	817.0	818.8
43	98	55	841.3	843.4
44	100	56	857.4	859.1
45	103	58	881.6	883.4
46	106	60	905.5	907.4
47	108	61	921.3	922.8
48	111	63	945.0	946.6
49	113	64	960.7	961.7
50	116	66	984.1	985.2
51	119	68	1007.4	1008.5
52	121	69	1022.7	1023.4
53	124	71	1045.7	1046.4
54	127	73	1068.6	1069.2
55	129	74	1083.6	1083.8
56	132	76	1106.2	1106.4
57	135	78	1128.7	1128.7
58	138	80	1151.0	1150.9
59	140	81	1165.6	1165.2

60	143	83	1187.6	1187.1
61	146	85	1209.5	1208.9
62	149	87	1231.2	1230.5
63	151	88	1245.4	1244.4
64	154	90	1266.9	1265.7
65	157	92	1288.1	1286.9
66	160	94	1309.3	1307.9
67	163	96	1330.2	1328.8
68	166	98	1351.0	1349.5
69	168	99	1364.6	1362.9
70	171	101	1385.2	1383.4
71	174	103	1405.5	1403.8
72	177	105	1425.8	1424.0
73	180	107	1445.8	1444.0
74	183	109	1465.7	1463.9
75	186	111	1485.5	1483.7
76	189	113	1505.1	1503.3
77	192	115	1524.5	1522.8
78	195	117	1543.8	1542.1
79	198	119	1562.9	1561.3
80	201	121	1581.9	1580.4
81	204	123	1600.7	1599.4
82	207	125	1619.3	1618.2
83	210	127	1637.8	1636.9
84	213	129	1656.1	1655.5
85	216	131	1674.3	1673.9
86	219	133	1692.3	1692.2
87	222	135	1710.1	1710.4
88	226	138	1734.0	1734.5
89	229	140	1751.4	1752.4
90	232	142	1768.8	1770.2
91	235	144	1785.9	1787.9
92	238	146	1802.9	1805.5
93	241	148	1819.7	1823.0
94	245	151	1842.3	1846.0
95	248	153	1858.8	1863.2
96	251	155	1875.1	1880.3
97	254	157	1891.3	1897.2
98	257	159	1907.3	1914.1
99	261	162	1928.7	1936.3
100	264	164	1944.4	1952.9

Figure 1: Binding energy per nucleon close to beta stability line of Z= 2 to 100

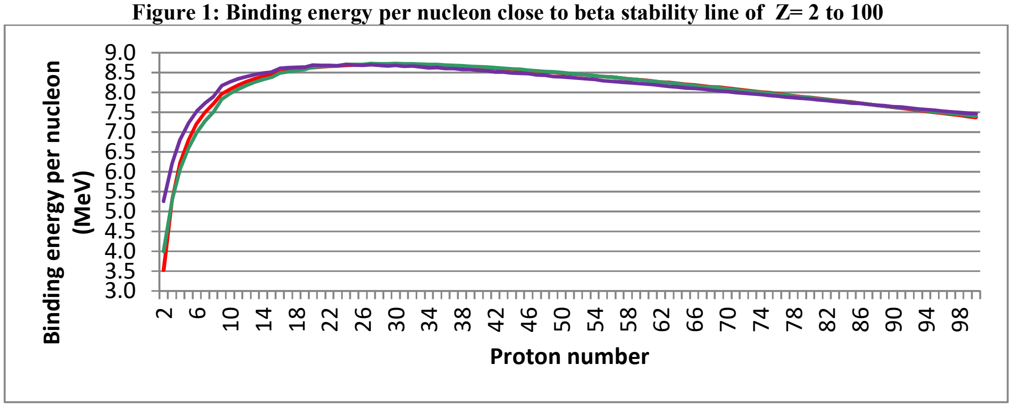

Table-2: To fit nuclear charge radii

Proton number	Mass number	Neutron number	Estimated charge radii from relation (18) fm	Charge radii from relation (19) fm
2	4	2	1.5623	1.5900
3	6	3	1.7884	1.8201
4	8	4	1.9684	2.0033
5	10	5	2.1204	2.1580
6	12	6	2.2532	2.2932
7	14	7	2.3720	2.4141
8	16	8	2.4800	2.5240
9	19	10	2.6022	2.6240
10	21	11	2.6929	2.7176
11	23	12	2.7779	2.8052
12	25	13	2.8580	2.8877
13	27	14	2.9338	2.9658
14	29	15	3.0059	3.0399
15	31	16	3.0746	3.1107
16	34	18	3.1556	3.1791
17	36	19	3.2182	3.2438
18	38	20	3.2785	3.3060
19	40	21	3.3366	3.3660
20	43	23	3.4055	3.4256
21	45	24	3.4596	3.4814
22	47	25	3.5119	3.5356
23	49	26	3.5628	3.5881
24	52	28	3.6233	3.6411
25	54	29	3.6712	3.6906
26	56	30	3.7178	3.7389
27	59	32	3.7734	3.7881
28	61	33	3.8176	3.8339
29	63	34	3.8608	3.8786
30	66	36	3.9124	3.9246
31	68	37	3.9536	3.9673
32	71	39	4.0027	4.0116
33	73	40	4.0421	4.0524
34	75	41	4.0808	4.0924
35	78	43	4.1269	4.1342
36	80	44	4.1640	4.1726
37	83	46	4.2083	4.2130
38	85	47	4.2440	4.2500
39	88	49	4.2866	4.2891
40	90	50	4.3211	4.3247
41	93	52	4.3622	4.3627
42	95	53	4.3955	4.3971
43	98	55	4.4352	4.4339
44	100	56	4.4674	4.4672
45	103	58	4.5058	4.5029
46	106	60	4.5436	4.5382
47	108	61	4.5743	4.5699
48	111	63	4.6109	4.6043
49	113	64	4.6408	4.6351
50	116	66	4.6764	4.6686
51	119	68	4.7114	4.7016
52	121	69	4.7400	4.7311
53	124	71	4.7741	4.7633
54	127	73	4.8077	4.7952
55	129	74	4.8352	4.8235
56	132	76	4.8679	4.8547
57	135	78	4.9002	4.8854
58	138	80	4.9320	4.9159
59	140	81	4.9582	4.9428

60	143	83	4.9893	4.9725
61	146	85	5.0200	5.0020
62	149	87	5.0503	5.0312
63	151	88	5.0753	5.0568
64	154	90	5.1050	5.0853
65	157	92	5.1343	5.1136
66	160	94	5.1633	5.1416
67	163	96	5.1919	5.1693
68	166	98	5.2202	5.1968
69	168	99	5.2436	5.2207
70	171	101	5.2714	5.2476
71	174	103	5.2988	5.2743
72	177	105	5.3260	5.3007
73	180	107	5.3529	5.3269
74	183	109	5.3795	5.3528
75	186	111	5.4058	5.3785
76	189	113	5.4319	5.4040
77	192	115	5.4577	5.4293
78	195	117	5.4833	5.4544
79	198	119	5.5086	5.4792
80	201	121	5.5337	5.5038
81	204	123	5.5586	5.5282
82	207	125	5.5832	5.5524
83	210	127	5.6077	5.5765
84	213	129	5.6319	5.6003
85	216	131	5.6558	5.6239
86	219	133	5.6796	5.6474
87	222	135	5.7032	5.6706
88	226	138	5.7302	5.6969
89	229	140	5.7534	5.7198
90	232	142	5.7763	5.7425
91	235	144	5.7991	5.7651
92	238	146	5.8217	5.7875
93	241	148	5.8442	5.8097
94	245	151	5.8698	5.8349
95	248	153	5.8919	5.8568
96	251	155	5.9138	5.8785
97	254	157	5.9355	5.9001
98	257	159	5.9570	5.9215
99	261	162	5.9817	5.9459
100	264	164	6.0029	5.9670

Table-3: To fit the mass dependent magnetic dipole moments of hot Jupiters

Hot Jupiter	**Mass** (kg)	**Magnetic dipole moment data taken from ref. [48] (J/tesla)**	**Estimated magnetic dipole moment from relation (23) (J/tesla)**	**Ratio of estimated value and ref. value**
HD 160691 d	7.98E+25	1.89E+24	1.14E+24	0.60
55 CnC e	8.55E+25	7.91E+24	1.22E+24	0.15
GJ 436 b	1.27E+26	1.31E+25	1.81E+24	0.14
HD 49674 b	2.28E+26	1.25E+25	3.26E+24	0.26
HD 76700 b	3.74E+26	2.76E+25	5.34E+24	0.19
HD 88133 b	4.18E+26	3.69E+25	5.97E+24	0.16
HD 168746 b	4.37E+26	1.93E+25	6.24E+24	0.32
HD 46375 b	4.73E+26	4.84E+25	6.75E+24	0.14
HD 63454 b	7.22E+26	8.35E+25	1.03E+25	0.12
HD 83443 b	7.79E+26	8.52E+25	1.11E+25	0.13
HD 75289 b	7.98E+26	7.31E+25	1.14E+25	0.16
51 Peg b	8.89E+26	6.71E+25	1.27E+25	0.19
BD -10 3166 b	9.12E+26	8.54E+25	1.30E+25	0.15

HD 2638 b	9.12E+26	8.66E+25	1.30E+25	0.15
HD 187123 b	9.88E+26	1.06E+26	1.41E+25	0.13
OGLE-TR-111 b	1.01E+27	8.15E+25	1.44E+25	0.18
OGLE-TR-10 b	1.08E+27	1.18E+26	1.54E+25	0.13
TrES-1	1.16E+27	1.30E+26	1.66E+25	0.13
ups-And b	1.31E+27	9.35E+25	1.87E+25	0.20
HD 209458 b	1.31E+27	1.26E+26	1.87E+25	0.15
HD 330075 b	1.44E+27	1.48E+26	2.06E+25	0.14
HD 179949 b	1.86E+27	2.15E+26	2.66E+25	0.12
HD 130322 b	2.05E+27	6.04E+25	2.93E+25	0.48
OGLE-TR-132 b	2.26E+27	5.19E+26	3.23E+25	0.06
HD 217107 b	2.43E+27	1.15E+26	3.47E+25	0.30
OGLE-TR-113 b	2.56E+27	7.17E+26	3.66E+25	0.05
OGLE-TR-56 b	2.75E+27	9.34E+26	3.93E+25	0.04
HD 73256 b	3.55E+27	5.44E+26	5.07E+25	0.09
HD 68988 b	3.61E+27	2.04E+26	5.16E+25	0.25
Tau-Boo	7.84E+27	9.77E+26	1.12E+26	0.11
HD 162020 b	2.61E+28	1.32E+27	3.73E+26	0.28

Table-4: To fit and estimate the photo electric work functions

Proton number	**Atomic symbol**	**Radius (m)**	$\left(\frac{G_s m_p m_e}{2r_Z}\right)$ eV	**Estimated** W_Z **from rel. (33) (eV)**	**Estimated** W_Z **from rel. (34) (eV)**	**Exp.** W_Z **(eV)**
3	Li	1.28E-10	1.237	1.78	2.25	2.9
4	Be	9.60E-11	1.649	2.62	3.3	4.98
5	B	8.40E-11	1.885	3.22	4.06	4.45
6	C	7.60E-11	2.083	3.79	4.77	5
11	Na	1.66E-10	0.954	2.12	2.67	2.36
12	Mg	1.41E-10	1.123	2.57	3.24	3.66
13	Al	1.21E-10	1.308	3.08	3.87	4.28
14	Si	1.11E-10	1.426	3.44	4.33	4.85
19	K	2.03E-10	0.78	2.08	2.62	2.29
20	Ca	1.76E-10	0.9	2.44	3.08	2.87
21	Sc	1.70E-10	0.931	2.57	3.24	3.5
22	Ti	1.60E-10	0.99	2.77	3.5	4.33
23	V	1.53E-10	1.035	2.94	3.71	4.3
24	Cr	1.39E-10	1.139	3.29	4.14	4.5
25	Mn	1.39E-10	1.139	3.33	4.2	4.1
26	Fe	1.32E-10	1.199	3.55	4.48	4.5
27	Co	1.26E-10	1.257	3.77	4.75	5
28	Ni	1.24E-10	1.277	3.88	4.89	5.15
29	Cu	1.32E-10	1.199	3.68	4.64	4.51
31	Ga	1.22E-10	1.298	4.08	5.14	4.32
33	As	1.19E-10	1.33	4.27	5.37	3.75
34	Se	1.20E-10	1.319	4.27	5.38	5.9
37	Rb	2.20E-10	0.72	2.4	3.02	2.26
38	Sr	1.95E-10	0.812	2.73	3.44	2.59
39	Y	1.90E-10	0.833	2.82	3.56	3.1
40	Zr	1.75E-10	0.905	3.1	3.9	4.05
41	Nb	1.64E-10	0.965	3.33	4.19	4.3
42	Mo	1.54E-10	1.028	3.57	4.5	4.6
44	Ru	1.46E-10	1.084	3.83	4.82	4.71
45	Rh	1.42E-10	1.115	3.97	5	4.98
46	Pd	1.39E-10	1.139	4.08	5.14	5.12
47	Ag	1.45E-10	1.092	3.94	4.97	4.26
48	Cd	1.44E-10	1.099	3.99	5.03	4.08
49	In	1.42E-10	1.115	4.08	5.14	4.09
50	Sn	1.39E-10	1.139	4.2	5.29	4.42
51	Sb	1.39E-10	1.139	4.22	5.32	4.55
52	Te	1.39E-10	1.147	4.28	5.39	4.95
55	Cs	2.44E-10	0.649	2.47	3.11	1.95

56	Ba	2.15E-10	0.736	2.82	3.55	2.7
57	La	2.07E-10	0.765	2.94	3.71	3.5
58	Ce	2.04E-10	0.776	3	3.78	2.9
60	Nd	2.01E-10	0.788	3.08	3.89	3.2
62	Sm	1.98E-10	0.8	3.17	3.99	2.7
63	Eu	1.98E-10	0.8	3.18	4.01	2.5
64	Gd	1.96E-10	0.808	3.23	4.07	3.17
65	Tb	1.94E-10	0.816	3.28	4.13	3.15
66	Dy	1.92E-10	0.825	3.33	4.2	3.25
67	Ho	1.92E-10	0.825	3.35	4.22	3.22
68	Er	1.89E-10	0.838	3.42	4.31	3.25
69	Tm	1.89E-10	0.833	3.42	4.3	3.1
70	Yb	1.87E-10	0.847	3.49	4.4	3
71	Lu	1.87E-10	0.847	3.51	4.42	3.3
72	Hf	1.75E-10	0.905	3.76	4.74	3.9
73	Ta	1.70E-10	0.931	3.89	4.9	4.25
74	W	1.62E-10	0.977	4.1	5.17	4.55
75	Re	1.51E-10	1.048	4.42	5.57	4.72
76	Os	1.44E-10	1.099	4.66	5.87	5.93
77	Ir	1.41E-10	1.123	4.78	6.02	5.27
78	Pt	1.36E-10	1.164	4.97	6.27	5.65
79	Au	1.36E-10	1.164	4.99	6.29	5.1
80	Hg	1.32E-10	1.199	5.17	6.51	4.49
81	Tl	1.45E-10	1.092	4.72	5.95	3.84
82	Pb	1.46E-10	1.084	4.71	5.93	4.25
83	Bi	1.48E-10	1.07	4.67	5.88	4.34
84	Po	1.40E-10	1.131	4.95	6.24	5
90	Th	2.06E-10	0.769	3.45	4.34	3.4
91	Pa	2.00E-10	0.792	3.56	4.49	3.7
92	U	1.96E-10	0.808	3.65	4.6	3.63
93	Np	1.90E-10	0.833	3.77	4.75	3.9
94	Pu	1.87E-10	0.847	3.85	4.85	3.6
95	Am	1.87E-10	0.88	4.02	5.06	3.7
96	Cm	1.69E-10	0.937	4.29	5.41	3.9