Theorem of prime pair distribution

Let

\[ S_n = \{(A_1, B_1), (A_2, B_2), \ldots, (A_n, B_n)\} \]

\[ A_n = a_1 n + a_2 \]

\[ B_n = b_1 n + b_2 \]

If \( A_n, B_n \) are not obviously composite,

\( S_p \) contains 2 pair that contains factor \( p \)

Lenth of 3,3,5,5,\( \ldots \),p,p is about \( \frac{2p}{\ln p} \)

3 continuous pair \((A_n, B_n)\) contains 2 pair that contains factor 3

\((3,3,5), (3,3,5), \ldots, (3,3, p), (3,3, p), (3,3)\)

It’s lenth not greater than \( \frac{2p}{\ln p} \cdot \frac{3+2}{3-2} < \frac{2p}{\ln p} \cdot \left(\frac{3}{3-1}\right)^4 \)

For \( p_i, p_i \) continuous pair \((A_n, B_n)\) contains 2 pair that contains factor \( p_i \) makes it’s lenth not greater than \( \frac{2p}{\ln p} \cdot \left(\frac{p_i}{p_i-1}\right)^4 \)
Hence if \( \frac{2p \cdot \left(\frac{3}{3-1}\right)^4 \cdot \left(\frac{5}{5-1}\right)^4 \cdots \left(\frac{p}{p-1}\right)^4}{\ln p} < n \),

\( S_n \) doesn’t contain \((A_n, B_n)\) has factor \( p_i \leq n \)

but \( \left(\frac{2-1}{2}\right) \cdot \left(\frac{3-1}{3}\right) \cdots \left(\frac{p-1}{p}\right) \) is about \( \frac{1}{\ln p} \),

\( \left(\frac{3}{3-1}\right)^4 \cdot \left(\frac{5}{5-1}\right)^4 \cdots \left(\frac{p}{p-1}\right)^4 \) is about \( \left(\frac{\ln p}{2}\right)^4 \)

Hence lence of \((A_1, B_1), (A_2, B_2), \cdots, (A_n, B_n)\) that isn’t have factor \( p_i \leq n \) is not greater than

\[
\frac{2p \cdot \left(\frac{\ln p}{2}\right)^4}{\ln p} = p \cdot \left(\frac{\ln p}{2}\right)^3
\]

Hence, if \( n > p \cdot \left(\frac{\ln p}{2}\right)^3 \), every \( A_n, B_n < p^2 \),

\( S_n \) contains \((A_k, B_k)\) that both \( A_k \ and \ B_k \) are prime.

From that, we can solve

1. \[
A_n = 2n + 1
\]
\[
B_n = -2n + 2N - 1
\]

Goldbach’s conjecture
2.

\[ A_n = 2n - 1 + 2N \]
\[ B_n = 2n + 1 + 2N \]

Twin prime conjecture,

And polignac’s conjecture, so on