# Theory Of Discrete Gravity

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***Abstract:*** *Theory of everything (T.O.E), final theory or ultimate theory is a theoretical framework in the field of physics, which holds an ultimate key to unify all the fundamental forces of nature in a single field. In other words such theory can glue quantum mechanics with general relativity into a single framework. Many theories have been postulated over the decades but the dominant one includes string theory and loop quantum gravity. In this paper I would like to present a new framework which can unify quantum mechanics with general relativity by showing that the change in Riemannian metric or the bend in space time is always an integral multiple of planks constant and since gravity is the result due to bend in space-time, gravity itself is a discrete force.*

***Keywords****: Discrete Gravity, General Relativity, Quantum Mechanics, Theory of Everything*

1. **Introduction:** Physicists have been trying over decades to unify quantum mechanics with general relativity in order to define unified field theory. So far, number of theories has been evolved including String Theory and Loop Quantum Gravity. String theory predicted that the fundamental particles are made up of invisible strings, which vibrate at different frequencies and as a result of its vibration different particles are made like proton, electron etc. Whereas on the other hand Loop Quantum Gravity suggested that the whole universe has a fundamental structure or in other words a granular structure which cannot be broken down any further. But this is in complete violation of Einstein's special relativity. However both theories did have a unique approach for unifying the four fundamental forces but none of them have a complete success so as it can be globally accepted. Theory of discrete gravity is another approach to unify the theory of very big and theory of very small by postulating that the change in Riemannian metric is discrete hence the gravity is also discrete.

2. **Planks Length:** The planks length is the scale at which the classical idea about space-time and gravity vanishes and quantum effects take over. The value of planks length is roughly $1.6 \times 10^{-35}$ m and is defined by a series of constants i.e. Gravitational Constant (G), Planks constant (h) and speed of light (c) given by

$$l_p = \sqrt{\frac{\hbar G}{c^3}} \qquad (1)$$

$$\frac{{l_p}^2}{\hbar} = \frac{G}{c^3} \qquad (2)$$

3. **General Relativity & Einstein Field Equation:** General Relativity is the geometric theory of gravitation published by Albert Einstein in 1915. General Relativity generalizes special relativity and Newtonian law of universal gravitation, providing a description of gravity in terms of geometric property of space-time. In other words, the curvature of space-time is related to the energy and momentum of any matter present in space-time. The relation is specified by Einstein's field equation given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (3)$$

4. **Discrete Gravity:** Gravity is the result of bend in space-time due to mass of an object present in it as illustrated by equation (3). In general relativity, the whole mathematics is based on minkowski space-time, which forbids space-time to have a discrete structure but what if bend in space-time is discrete rather than the space-time itself. This property of space-time will explain gravity on a quantum scale without violating any rules that general relativity is based on. This hypothesis can be proved mathematically by showing that change in geometry of space-time is always an integral multiple of planks constant or h-bar. Let us consider the mass M represented by equation (3)

Now, Let us consider a test mass m revolving around a mass M, then the force between the two masses and Energy of mass M is given by

$$F = -\frac{GMm}{r^2} \qquad \text{[- sign represent attractive force]} \qquad (4)$$

Integrating both sides with respect to r

$$\int F.dr = -\int \frac{GMm}{r^2}\, dr$$

$$\int M.\frac{dv}{dt}.dr = -\int \frac{GMm}{r^2}\, dr \qquad \text{[ Since F= Ma]}$$

$$\int M.\frac{dr}{dt}.dv = -\int \frac{GMm}{r^2}\, dr$$

$$M \int v dv = -\int \frac{GMm}{r^2}\, dr \qquad [v = \frac{dr}{dt}]$$

Since, planks length is the boundary up to which classical idea exists, so the limit for integral part of equation (7) can be defined from $l_p$ to $\infty$.For velocity 0 to v. Therefore,

$$M \int_0^v v dv = -\int_{l_p}^{\infty} \frac{GMm}{r^2}.dr$$

$$\frac{1}{2}Mv^2 = \frac{GMm}{l_p}$$

$$\frac{1}{2}Mv^2 = \frac{GMm}{l_p}\frac{4\pi r^2}{4\pi r^2}$$

$$2\pi l_p v^2 M = \left[\frac{GMm}{r^2}\right]4\pi r^2$$

The term on the R.H.S of the above equation represents area integral of force. So above equation can be represented as

$$2\pi l_p v^2 M = \int F.dA$$

$$\int F.dA = 2\pi l_p v^2 M \qquad (5)$$

5. **Divergence Theorem:** According to divergence theorem, the area integral of force is equal to the volume integral of co-variant derivative of force. Mathematically,

$$\int F.dA = \int \nabla F.dV \qquad (6)$$

Therefore, from equation (5) and (6)

$$\int \nabla F.dV = 2\pi l_p v^2 M \qquad (7)$$

The mass can be defined as the volume integral of density. So equation (7) can be re-written as

$$\int \nabla F.dv = 2\pi l_p v^2 \int \rho.dV$$

$$\nabla F = 2\pi l_p v^2 \rho \qquad (8)$$

$$\nabla F = \frac{2\pi l_p v^2 c^2 \rho}{c^2} \qquad (9)$$

Adding and subtracting $2\pi l_p c^2 \rho$ on the L.H.S of equation (9)

$$\nabla F = \frac{2\pi l_p v^2 c^2 \rho}{c^2} + 2\pi l_p c^2 \rho - 2\pi l_p c^2 \rho$$

$$= -2\pi l_p c^2 \rho\left(1 - \frac{v^2}{c^2}\right) + 2\pi l_p c^2 \rho \qquad (10)$$

Since relativistic mass density is given by $\rho = \gamma^2 \rho' = \frac{\rho'}{(1-\frac{v^2}{c^2})}$, therefore

$$\rho\left(1 - \frac{v^2}{c^2}\right) = \rho' \qquad (11)$$

Substituting the value of equation (11) in equation (10)

$$\nabla F = -2\pi l_p c^2 \rho' + 2\pi l_p c^2 \rho$$

$$\nabla F = 2\pi l_p c^2 (\rho - \rho')$$

In general, $$\nabla F = 2\pi l_p c^2 \rho \qquad (12)$$

6. **Christoffel Symbol:** The christoffel symbol denoted by $\Gamma$ are a form of tensor derived from Riemannian metric and is given in form of metric tensor in general relativity represented by

$$\Gamma = \frac{1}{2} g^{ad} [\frac{\partial g_{dc}}{\partial x^b} + \frac{\partial g_{db}}{\partial x^c} + \frac{\partial g_{bc}}{\partial x^b}] \qquad (13)$$

For Newtonian mechanics the above equation or christoffel symbol reduced down to $\frac{1}{2}\frac{\partial g_{00}}{\partial x}$ as the value of other derivatives are very small and can be neglected. Therefore

$$\Gamma = \frac{1}{2}\frac{\partial g_{00}}{\partial x} \qquad (14)$$

Christoffel symbol ($\Gamma = \frac{dx^\mu}{d\tau^2}$) is the representation of force in Newtonian Mechanics thus the above equation can be treated equivalent to force.

$$F = \frac{1}{2}\frac{\partial g_{00}}{\partial x} \qquad (15)$$

Also, $$F = -\frac{d\Phi}{dx} \text{ where } \Phi \text{ is potential} \qquad (16)$$

From equation (15) and (16)

$$\frac{1}{2}\frac{\partial g_{00}}{\partial x} = -\frac{d\Phi}{dx}$$

$$\frac{1}{2} g_{00} + const = -\Phi \qquad (17)$$

For a three dimensional analysis instead of normal differentiation we need to represent force as covariant derivative of potential. Therefore,

$$F = -\nabla\Phi$$

$$\nabla F = -\nabla^2\Phi \qquad (18)$$

Substituting the value of $\nabla F$ and $\Phi$ from equation (12) and (17) respectively,

$$\nabla^2 \frac{1}{2} g_{00} = 2\pi l_p c^2 \rho$$

$$\nabla^2 g_{00} = 4\pi l_p c^2 \rho \tag{19}$$

The mass density on the R.H.S of the equation (15) can be represented as a tensor in the form of stress-energy tensor whose derivative will always be equal to zero as energy can neither be created nor destroyed. Thus,

$$4\pi l_p c^2 \rho = 4\pi l_p c^2 T_{\mu\nu} \quad \text{and,}$$

$$\nabla\left(4\pi l_p c^2 T_{\mu\nu}\right) = 0 \tag{20}$$

Multiplying and dividing by $\frac{2l_p c}{\hbar^2}$ on the L.H.S of equation 20, we have

$$\nabla\left(\frac{8\pi {l_p}^2 c^3}{\hbar^2}\left(\frac{\hbar^2}{2cl_p}T_{\mu\nu}\right)\right) = 0 \tag{21}$$

Let $\frac{\hbar^2}{2cl_p}T_{\mu\nu}$ be $T'_{\mu\nu}$ So, equation (21) can be written as

$$\nabla\left(\frac{8\pi {l_p}^2 c^3}{\hbar^2}T'_{\mu\nu}\right) = 0 \tag{22}$$

Also, from general relativity, $\nabla\left(R_{\mu\nu} - (\frac{1}{2}R - \Lambda)g_{\mu\nu}\right) = 0$ (23)

From equation (22) and (23)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi {l_p}^2 c^3}{\hbar^2}T'_{\mu\nu} \tag{24}$$

Since we are dealing with four dimensions, that is one dimension of time and three dimension of space we will divide the R.H.S of equation (24) with $c^4$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi {l_p}^2 c^3}{c^4\hbar^2}T'_{\mu\nu} \tag{25}$$

Substituting the value of $\frac{{l_p}^2}{\hbar}$ from equation (2) in above equation

$$\hbar[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu}] = \frac{8\pi G}{c^4}T'_{\mu\nu} \tag{26}$$

Therefore, comparing equation (3) and (26) it can be clearly seen that, when the stress-energy tensor changes by a factor of $\frac{\hbar^2}{4cl_p}$ the geometry of space time changes by a factor of $\hbar$.

$$n\hbar[R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu}] = \frac{8\pi G}{c^4}T_{\mu\nu} \quad \text{[where n= 1, 2, 3….]} \tag{27}$$

7. **Conclusion:** Due to the change in the mass of a body, the Riemannian metric changes accordingly. When there is a change in mass by a factor of $\frac{\hbar^2}{2cl_p}$ where,$\frac{\hbar^2}{2cl_p}$ is called Mass factor (denoted by $M^v$) the Riemannian metric changes discreetly. In general, the change in mass by an integral multiple of mass factor leads to chance in space-time discretely. This can be clearly observed from equation (27). And since, gravity is the result of bend in Space-time and the bend of space-time is discrete in nature, this must conclude that the gravitational force itself is discrete in nature.

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