About Longitudinal Waves of an Electromagnetic Field

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Abstract

This article presents the solution to the problem of scalar longitudinal waves within the framework of electromagnetic potential $\mathbf{A}_\nu$ without introducing any additional members into the canonical field Lagrangian or the Maxwell equations that ensure the existence of a longitudinal wave component. The Lame equation for the electromagnetic field is obtained, which describes transverse and longitudinal waves, as well as the Fock-Podolsky wave equation for longitudinal electroscalar waves that do not have a magnetic component. It is shown that the terms of the Lorentz gauge condition describe a four-dimensional volume deformation of the electromagnetic field.

Key words: electromagnetic potential, asymmetric tensor, symmetric tensor, antisymmetric tensor, Maxwell equations, longitudinal waves.

The Contents:

1. Introduction

There is a problem of longitudinal waves in electrodynamics. From the perspective of quantum electrodynamics, the transmission of the Coulomb force is produced by longitudinal photons. However, according to Maxwell’s theory of the electromagnetic field (EMF), electromagnetic waves are transverse, and a plane electromagnetic wave in a vacuum does not have a longitudinal component. Therefore, in quantum electrodynamics the transmission of the Coulomb force is performed by zero-spin transverse photons, which are considered «unphysical». However, the longitudinal interaction itself is physically observable, and real longitudinal scalar waves are needed to explain its propagation in space with finite velocity. Hence, numerous attempts to extend the standard electrodynamics have been made since Omura (1956), Aaronov and Bohm (1963), and others until a recent review and analysis of this problem by Modanes (2017). Recent years articles [1-9] are devoted to the problem of the existence of scalar longitudinal waves. The results of experiments on the detection of scalar longitudinal waves of artificial and natural origin are described in [10-13].

All attempts to introduce a scalar component into electrodynamics can be divided into two the path. On the first the path (Omura, Aaronov and Bohm, Fok and Podolsky, and others) new terms are introduced into the Lagrangian of the electromagnetic field, which ensure the appearance of a longitudinal component in the equations of field motion. On the second the path new scalar terms that provide a description of the longitudinal waves are introduced into Maxwell‘s equations, on the basis of their gradient invariance. For this purpose, either the scalar potential of a new physical field [8, 9] is introduced into Maxwell's equations or the already known electromagnetic potential [2-6] is used, Thus, all known attempts to solve the problem are reduced to the introduction of new additional terms into Lagrangian or into the EMF equations.

The existence of longitudinal expansion/contraction waves implies the existence of an elastic continuum (electromagnetic vacuum). However, in Maxwell's theory the electromagnetic vacuum is incompressible. This property is reflected in the EMF description in the form of an antisymmetric tensor the trace of which equals zero. Hence, attempts have been made to introduce an elastic continuum by analogy with a continuous elastic medium. An example would be electrodynamics based on Foka-Podolsky Lagrangian [7], in which an electromagnetic analogue of the Lame equation or the dynamic Navier-Stokes equation is constructed. This representation of an elastic continuum corresponds to the ideas accepted in quantum electrodynamics in regard to electromagnetic vacuum as plasma, consisting of virtual electrons and positrons. In such plasma, transverse and longitudinal waves can propagate.

In electrodynamics, there is also a problem of violation of Newton's third law related to the longitudinal interaction of currents. This led to the appearance of a hypothesis about the existence of a «scalar (potential) magnetic field» [15], introduction of which into Maxwell's electrodynamics makes it possible to preserve the fulfillment of Newton's third law and describe the longitudinal interactions of currents. The existence of the "scalar magnetic field" is confirmed by different authors who have conducted experiments on the longitudinal interaction of currents [14-16].

In the articles [8, 9] the problem of longitudinal interaction is solved by introducing of the four-dimensional scalar potential of a new physical field into the electrodynamics This scalar potential is independent and is not related to Maxwell's electromagnetic potential by differential relations. The connection between this potential and the classical theory is realized at the level of sources of the electromagnetic field - charges and currents. Thus, this is an additive theory in regard to the Maxwell's theory. However, the introduction of new physical entities makes sense only when the solution to the problem is impossible in other ways.

The aim of this article is to solve the problem of scalar longitudinal waves within the framework of the canonical electromagnetic potential $\mathbf{A}_\nu$ without introducing any additional terms into Lagrangian or into the EMF equations that ensure the existence of a longitudinal wave component.

EMF and electric charges are considered in a vacuum. The space-time geometry is taken in the form of pseudo-Euclidean Minkowski space (ct, ix, iy, iz). The four-dimensional electromagnetic potential is defined as $\mathbf{A}_\nu(\varphi / c, i\mathbf{A})$, where φ and A are the scalar and vector potentials of the EMF. The four-dimensional current density is defined as $\mathbf{J}_\nu(\rho \cdot c, i\mathbf{J})$, where ρ and J are the electric charges density and current density.

2. Antisymmetric tensor of the electromagnetic field and Maxwell's equations

EMF in four-dimensional form is described by the canonical antisymmetric tensor of the second rank:

$$F_{[\mu\nu]} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu \qquad (1)$$

This antisymmetric tensor is a four-dimensional (covariant) rotor the components of the tensor $F_{[\mu\nu]}$ are the derivatives of the scalar φ and vector A of EMF potentials, which are defined as the strength components of the electric field **E** and the magnetic field induction **B**:

$$\mathbf{E} = -\nabla\varphi - \partial_t \mathbf{A} \qquad \mathbf{B} = \nabla \times \mathbf{A} = (\partial_y A_z - \partial_z A_y)_x + (\partial_z A_x - \partial_x A_z)_y + (\partial_x A_y - \partial_y A_x)_z$$

The Maxwell's equations are obtained from the tensor (1) in the form of its four-dimensional divergence under one of the indices to which the field source is equated [17] $\partial^\mu F_{[\mu\nu]} = \mathbf{J}_\nu$. Let us write these equations in vector form:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \qquad \text{or} \qquad -\partial_t \nabla \cdot \mathbf{A} - \Delta\varphi = \rho / \varepsilon_0 \qquad (2)$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\partial_t \mathbf{E} = \mu_0 \cdot \mathbf{J} \qquad \text{or} \qquad \frac{1}{c^2}\partial_{tt}\mathbf{A} + \frac{1}{c^2}\partial_t \nabla\varphi + \nabla \times \nabla \times \mathbf{A} = \mu_0 \cdot \mathbf{J} \qquad (3)$$

From the EMF tensor (1), in the form of the well-known tensor identity $\partial_\eta F_{\mu\nu} + \partial_\nu F_{\eta\mu} + \partial_\mu F_{\nu\eta} = 0$ for an antisymmetric tensor, follow two more of Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{or} \qquad \nabla \cdot (\nabla \times \mathbf{A}) = 0 \qquad (4)$$

$$\nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0 \qquad \text{or} \qquad -\nabla \times \partial_t \mathbf{A} + \partial_t \nabla \times \mathbf{A} = 0 \tag{5}$$

3. Asymmetric and symmetric tensors and Lagrangians of the electromagnetic field

For the existence of longitudinal waves of expansion/contraction of EMF, there must be an elastic continuum (electromagnetic vacuum). In the theory of continuous media, an elastic medium is described by a symmetric tensor. Let us consider the possibility of describing such an environment for EMF The definition of a canonical antisymmetric tensor $F_{[\mu\nu]} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$ includes an asymmetric tensor of the second rank $\partial_\mu \mathbf{A}_\nu$, which is a four-dimensional derivative of the electromagnetic potential. Let us denote it as $F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu$ and write this tensor in the matrix form:

$$F_{\mu\nu} = \partial_\mu \mathbf{A}_\nu = \begin{pmatrix} \frac{1}{c^2}\partial_t \varphi & \frac{1}{c} i \cdot \partial_t A_x & \frac{1}{c} i \cdot \partial_t A_y & \frac{1}{c} i \cdot \partial_t A_z \\ -\frac{1}{c} i \cdot \partial_x \varphi & \partial_x A_x & \partial_x A_y & \partial_x A_z \\ -\frac{1}{c} i \cdot \partial_y \varphi & \partial_y A_x & \partial_y A_y & \partial_y A_z \\ -\frac{1}{c} i \cdot \partial_z \varphi & \partial_z A_x & \partial_z A_y & \partial_z A_z \end{pmatrix} \tag{6}$$

This asymmetric tensor can be uniquely decomposed into symmetric and antisymmetric tensors $F_{\mu\nu} = F_{[\mu\nu]}/2 + F_{(\mu\nu)}/2$. Then the canonical antisymmetric EMF tensor can be written in the form $F_{[\mu\nu]} = 2F_{\mu\nu} - F_{(\mu\nu)}$. It is clear from this decomposition that, in addition to the antisymmetric tensor $F_{[\mu\nu]} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu$, one can justifiably include a symmetric tensor $F_{(\mu\nu)} = \partial_\mu \mathbf{A}_\nu + \partial_\nu \mathbf{A}_\mu$ into the EMF description, more precisely, this tensor is implicitly contained in the canonical description of EMF $F_{[\mu\nu]} = 2F_{\mu\nu} - F_{(\mu\nu)}$. In the theory of continuous media, the antisymmetric displacement tensor of a medium is associated with its rotation as a whole, and the symmetric tensor is connected by longitudinal and shear deformations of the medium. Using this analogy, the antisymmetric tensor $F_{[\mu\nu]}$, which is a four-dimensional covariant rotor, can be associated with the four-dimensional rotation of the EMF, and the symmetric tensor $F_{(\mu\nu)}$ can be associated with four-dimensional deformations of the EMF. Let us write the symmetric EMF tensor $F_{(\mu\nu)}$ in the matrix form:

$$F_{(\mu\nu)} = \begin{pmatrix} 2\frac{1}{c^2}\partial_t \varphi & \frac{1}{c} i \cdot (\partial_t A_x - \partial_x \varphi) & \frac{1}{c} i \cdot (\partial_t A_y - \partial_y \varphi) & \frac{1}{c} i \cdot (\partial_t A_z - \partial_z \varphi) \\ \frac{1}{c} i \cdot (\partial_t A_x - \partial_x \varphi) & 2\partial_x A_x & (\partial_x A_y + \partial_y A_x) & (\partial_x A_z + \partial_z A_x) \\ \frac{1}{c} i \cdot (\partial_t A_y - \partial_y \varphi) & (\partial_x A_y + \partial_y A_x) & 2\partial_y A_y & (\partial_y A_z + \partial_z A_y) \\ \frac{1}{c} i \cdot (\partial_t A_z - \partial_z \varphi) & (\partial_x A_z + \partial_z A_x) & (\partial_y A_z + \partial_z A_y) & 2\partial_z A_z \end{pmatrix} \tag{7}$$

The diagonal components of this tensor describe a four-dimensional volumetric deformation of the EMF and represent a four-dimensional divergence of the electromagnetic potential $\partial^{\nu}\mathbf{A}_{\nu}$. The Lorentz gauge condition $\partial^{\nu}\mathbf{A}_{\nu}=0$ is widely used in electrodynamics. Thus, the physical essence of the Lorentz gauge condition is the elimination of four-dimensional volume deformation from the EMF equations. Naturally, when the condition $\partial^{\nu}\mathbf{A}_{\nu}=0$, is imposed, longitudinal waves and interactions of longitudinal currents are excluded from electrodynamics.

The divergence of the electromagnetic potential $\partial^{\nu}\mathbf{A}_{\nu}$ is a scalar and is used in most research papers to modify the Maxwell's equations. This is done by introducing its three-dimensional terms, in the form of additional terms, into Maxwell's equations. Since this divergence is already contained in the EMF equations, its reintroduction is pointless.

The energy-momentum tensor $T^{AS}_{\mu\eta}=F_{[\mu\nu]}\cdot F_{[\nu\eta]}$ corresponds to the antisymmetric tensor $F_{[\mu\nu]}=\partial_{\mu}\mathbf{A}_{\nu}-\partial_{\nu}\mathbf{A}_{\mu}$ obtained in [18]. From this energy-momentum tensor, in the form of its linear invariant, follows the canonical Lagrangian of free EMF:

$$L_{AS}=\frac{1}{4}[2(-\partial_t\mathbf{A}-\nabla\varphi)^2/c^2-2(\nabla\times\mathbf{A})^2]=\frac{1}{2}(E^2/c^2-B^2)$$

The symmetric tensor $F_{(\mu\nu)}=\partial_{\mu}\mathbf{A}_{\nu}+\partial_{\nu}\mathbf{A}_{\mu}$ describes the four-dimensional deformation of the EMF. This tensor corresponds to the energy-momentum tensor describing this deformation energy. The energy-momentum tensor is obtained from the tensor $F_{(\mu\nu)}$ by the method described in [18] $T^{S}_{\mu\eta}=F_{(\mu\nu)}\cdot F_{(\nu\eta)}$. From this energy-momentum tensor, in the form of its linear invariant, follows the EMF Lagrangian associated with the four-dimensional deformation of EMF:

$$L_S=\frac{1}{4}[(2\frac{1}{c^2}\partial_t\varphi)^2-2\frac{1}{c^2}(\partial_t\mathbf{A}-\nabla\varphi)^2+(\partial_i A_k+\partial_k A_i)^2]$$

The description of the total energy of the EMF can be obtained in the form of an energy-momentum tensor following from an asymmetric tensor $F_{\mu\nu}=\partial_{\mu}\mathbf{A}_{\nu}$. A linear invariant of the energy-momentum tensor $T^{NS}_{\mu\eta}=F_{\mu\nu}\cdot F_{\nu\eta}$ is the total Lagrangian of EMF associated with the four-dimensional deformation and the EMF rotation:

$$L_{NS}=(\frac{1}{c^2}\partial_t\varphi)^2+2\frac{1}{c^2}\cdot\partial_t A\cdot\nabla\varphi+(\partial_i A_k\cdot\partial_k A_i)$$

It is obvious that $L_{NS}=L_{AS}+L_S$. The components of the Lagrangians L_S, L_{AS} and L_{NS} present the components of the EMF energy. The components of the L_{AS} represent the energy of the four-dimensional rotation of the EMF. The components L_S represent the energy of four-dimensional deformation of EMF. The components of the L_{AS} are components of the total energy of the EMF. It is interesting to note that in the complete Lagrangian L_{NS} there are no constituent parts of the total energy $(\partial_t \mathbf{A})^2$ and $(\nabla\varphi)^2$. This is due to the fact that these types of energy are included into Lagrangians L_S and L_{AS} with different signs hence, these types of energy are fictitious. Lagrangians L_S, L_{AS} and L_{NS} can be applied in quantum electrodynamics.

4. The equations of the electromagnetic field and longitudinal waves

The total four-dimensional divergence of the symmetric tensor $F_{(\mu\nu)}$ can be nonzero, so we equate it with the four-dimensional source of EMF $(\partial^\mu F_{\mu\nu} + \partial^\nu F_{\mu\nu})/2 = \mathbf{J}_\nu$ This equation is equivalent to the equations $\partial^\mu(\partial_\mu \mathbf{A}_\nu + \partial_\nu \mathbf{A}_\mu) = \mathbf{J}_\nu$ and $\partial^\mu F_{(\mu\nu)} = \mathbf{J}_\nu$. Let us write down this four-dimensional divergence of the symmetric tensor in three-dimensional form:

$$2\frac{1}{c^2}\partial_{tt}\varphi + \partial_t \nabla\cdot\mathbf{A} - \Delta\varphi = \rho/\varepsilon_0 \tag{8}$$

$$\frac{1}{c^2}\partial_{tt}\mathbf{A} - \nabla(\nabla\cdot\mathbf{A}) - \Delta\mathbf{A} - \frac{1}{c^2}\partial_t\nabla\varphi = \mu_0\cdot\mathbf{J} \tag{9}$$

Eq. (8) is scalar and analogous to the Maxwell's Eq. (2). Eq. (9) is a vector equation and analogous to the Maxwell's Eq. (3). Eq. (9) can be written in the form:

$$\partial_{tt}\mathbf{A} - 2\cdot c^2\cdot\nabla(\nabla\cdot\mathbf{A}) + c^2\cdot\nabla\times\nabla\times\mathbf{A} = \mu_0 c^2\cdot\mathbf{J} + \partial_t\nabla\varphi \tag{10}$$

In this form Eq. (10), represents a complete electromagnetic analogue of the Lame equation or the Navier-Stokes dynamic equation describing the wave motion of a continuous medium in the linear theory of elasticity [19]:

$$\partial_{tt}\mathbf{U} - \upsilon_1^2\cdot\nabla(\nabla\cdot\mathbf{U}) + \upsilon_2^2\cdot\nabla\times\nabla\times\mathbf{U} = \mathbf{G}$$

where U is the displacement vector of the medium, υ1 is the velocity of longitudinal waves, υ2 is the velocity of transverse waves, and G is the external forces. A comparison of the Lame equation with Eq. (10) shows that the propagation velocity of longitudinal EMF waves is $\upsilon_1 = \sqrt{2}\cdot c$. By the methods adopted in the theory of elasticity [19], the waves described by Eq. (10) can be decomposed into longitudinal and transverse waves. In the articles [8-9], the Lame equation was taken as the basis for constructing electrodynamics with a longitudinal component. For this purpose a new independent scalar potential is introduced in electrodynamics. In our case, the electromagnetic analogue of the Lame equation for EMF automatically follows from the EMF tensor without resorting to any additional physical entities and hypotheses, Thus, Eq. (10) shows that the electromagnetic analogue of the Lame equation already exists in electrodynamics and its introduction with the help of field Lagrangian is not required.

For the static case, Eqs. (8) and (10) can be written in the form:

$$-\Delta\varphi = \rho/\varepsilon_0 \tag{11}$$

$$-2\cdot\nabla(\nabla\cdot\mathbf{A}) + \nabla\times\nabla\times\mathbf{A} = \mu_0\cdot\mathbf{J} \tag{12}$$

Eq. (11) for the static case coincides with the Maxwell's Eq. (2) and describes the Gaussian law for a constant potential electric field. The Eq. (12) differs from the Maxwell's Eq. (3) for the stationary case by the presence of the first term with the divergence of the vector potential. This term represents the gradient of the hypothetical «scalar (potential) magnetic field» introduced by Nikolaev [15] into Maxwell's Eq. (3) in order to explain the longitudinal interaction between steady currents and the observance of Newton's third law in electrodynamics. Eq. (12) coincides with the equation given in [16]. In this paper, Eq. (12) was constructed empirically on the basis of experimental results by supplementing the Maxwell's equation. In this article Eq. (12) is obtained strictly mathematically, as a consequence of the divergence of the symmetric EMF tensor $F_{(\mu\nu)}$. Thus, Eq. (9) eliminates the problem of violating of Newton's third law in electrodynamics.

Eqs. (8) and (9) can be written in the form:

$$\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi + \partial_t\left(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}\right) = \rho/\varepsilon_0 \quad \text{and} \quad \frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A} - \nabla\left(\frac{1}{c^2}\partial_t\varphi + \nabla\cdot\mathbf{A}\right) = \mu_0\cdot\mathbf{J}$$

Applying the Lorentz gauge $\partial_t\varphi/c^2 + \nabla\cdot\mathbf{A} = 0$ to them, we will obtain Maxwell's canonical equations in the Lorentz gauge [17]:

$$\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi = \rho/\varepsilon_0 \qquad \frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A} = \mu_0\cdot\mathbf{J}$$

Let us take the rotor from both sides of Eq. (9) and obtain the canonical wave equation for the magnetic induction **B**:

$$\frac{1}{c^2}\partial_{tt}(\nabla\times\mathbf{A}) - \Delta(\nabla\times\mathbf{A}) = \mu_0\cdot\nabla\times\mathbf{J} \quad \text{or} \quad \frac{1}{c^2}\partial_{tt}\mathbf{B} - \Delta\mathbf{B} = \mu_0\cdot\nabla\times\mathbf{J} \tag{13}$$

Let us take the divergence from both sides of Eq. (8) and the time derivative of Eq. (9). In the result of summation of these equations and applying simple the transformations we will obtain the canonical wave equation for the electric field intensity **E:**

$$\partial_t\left(\frac{1}{c^2}\partial_{tt}\mathbf{A} - \Delta\mathbf{A}\right) + \nabla\left(\frac{1}{c^2}\partial_{tt}\varphi - \Delta\varphi\right) = \nabla\rho/\varepsilon_0 + \mu_0\cdot\partial_t\mathbf{J} \quad \text{or} \quad \Delta\mathbf{E} - \frac{1}{c^2}\partial_{tt}\mathbf{E} = \nabla\rho/\varepsilon_0 + \mu_0\cdot\partial_t\mathbf{J} \tag{14}$$

Let us take the divergence of both sides of Eq. (10) and we will obtain the wave equation:

$$\frac{1}{2\cdot c^2}\partial_{tt}\nabla\cdot\mathbf{A} - \Delta(\nabla\cdot\mathbf{A}) = \left(\mu_0\nabla\cdot\mathbf{J} + \frac{1}{c^2}\partial_t\Delta\varphi\right)/2 \tag{15}$$

This equation describes the longitudinal waves of the divergence of the vector potential or the wave of the hypothetical «longitudinal (scalar) magnetic field» of Nikolaev. A distinct feature of these waves is the absence of the component of magnetic induction **B** in them. Therefore, these waves correspond to the name of longitudinal electroscalar waves.

Taking the time derivatives of both sides of Eq. (8) and the divergence from both sides of Eq. (9) and adding the results, we will obtain the equation:

$$\frac{1}{\mu_0}2(\nabla\cdot(\frac{1}{c^2}\partial_{tt}\mathbf{A}-\Delta\mathbf{A})+\frac{1}{c^2}\partial_t(\frac{1}{c^2}\partial_{tt}\varphi-\Delta\varphi))=\nabla\cdot\mathbf{J}+\partial_t\rho \quad \text{или} \quad \frac{1}{\mu_0}2\cdot(\frac{1}{c^2}\partial_{tt}-\Delta)\partial^\mu\mathbf{A}_\mu=\partial^\mu\mathbf{J}_\mu \qquad (16)$$

This equation is the equation of longitudinal (electroscalar) waves of the divergence of the electromagnetic potential known in the electrodynamics of Foka-Podolsky and others [7-9]. The left-hand side of Eq. (16) can be zero for an electromagnetic potential that is not equal zero. Then Eq. (16) can be written in the form of two free-standing equations:

$$\frac{1}{\mu_0}2\cdot(\frac{1}{c^2}\partial_{tt}-\Delta)\partial^\mu\mathbf{A}_\mu=0=\partial^\mu\mathbf{J}_\mu \quad \text{or} \quad (\frac{1}{c^2}\partial_{tt}-\Delta)\partial^\mu\mathbf{A}_\mu=0 \quad \text{and} \quad \partial^\mu\mathbf{J}_\mu=0$$

From this equation it follows that the wave equation of electroscalar waves $(\partial_{tt}/c^2-\Delta)\partial^\mu\mathbf{A}_\mu=0$ is just as fundamental as the current density conservation equation $\partial^\mu\mathbf{J}_\mu=0$. This means that the motion of charges in accordance with the equation $\partial^\mu\mathbf{J}_\mu=0$ always excites the longitudinal waves $(\partial_{tt}/c^2-\Delta)\partial^\mu\mathbf{A}_\mu=0$.

Thus, Eqs. (8) and (9) describe longitudinal interactions of charges and currents. All the basic canonical wave equations of electrodynamics follow from them. Eq. (10), which is an electromagnetic analogue of Lame or Navier-Stokes equation of the isotropic elastic medium motion, shows the general laws of motion of all kinds of matter. This analogy allows us to consider the field of the four-dimensional electromagnetic potential $\mathbf{A}_\nu$ as a physical medium in which electromagnetic waves propagate due to the dynamic deformation of this medium. Consequently, the field of electromagnetic potential $\mathbf{A}_\nu$ can be identified as a «physical vacuum» or «electromagnetic vacuum», which is the source of virtual photons and other elementary particles.

5. Conclusion

The problem of longitudinal interaction in electrodynamics can be solved on the basis of the canonical electromagnetic potential $\mathbf{A}_\nu$ without involving additional physical entities and hypotheses. To solve this problem, it is proposed to apply the expansion of the four-dimensional derivative of the electromagnetic potential into symmetric and antisymmetric tensors. From the antisymmetric tensor follow Maxwell's canonical equations. From the symmetric tensor follow the Lame equation for the electromagnetic field, which describes the transverse and longitudinal interactions from the symmetric tensor follow all the basic canonical EMF equations, as well as the Fock-Podolsky wave equation for longitudinal electroscalar waves that do not have a magnetic component. It follows from the symmetric tensor that the terms of the Lorentz gauge condition describe a four-dimensional volume deformation of the EMF.

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