THE PROOF OF THE FERMAT'S CONJECTURE IN THE CORRECT DOMAIN

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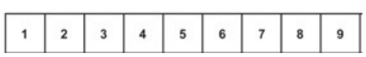
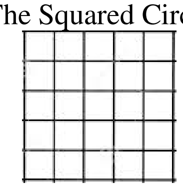
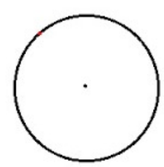
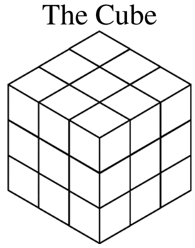
Abstract

The distinction between the Domain of Natural Numbers and the Domain of Line has been highlighted. This distinction provides the new perception to the Fermat's Conjecture, where to place it and how to prove it. The reasons why the Fermat's Conjecture remained unproven for 382 years are summarized. The Fermat's Conjecture has been proved in the Domain of Natural Numbers. The equation $a^n + b^n = c^n$ with positive integers *a, b, c, n* is not the Fermat's Conjecture in the Domain of Line.

Keywords: Fermat's Conjecture; Fermat's Last Theorem; Domain of Natural Numbers; Domain of Line

1. Introduction

There are two fundamental domains in mathematics: The Domain of Natural Numbers (positive whole numbers or positive integers) and the Domain of Line. They are depicted in Table 1.

	Domain of Natural Numbers	**Domain of Line**
One-dimensional filled space	Numbered Unit Squares (Squarits) 	All Kinds of Line Euclidian, Hyperbolic, Elliptic, dashed, etc.
Two-dimensional filled space	The Squared Circle  From the center, it is the same distance equal to a specific integer or number of squarits. A rotation brings the position to the same beginning squarit.	The Lined Circle  From the center, it is the same distance equal to a specific decimal or integer number. A rotation brings the position to the same beginning point.
Three-dimensional filled space	The Cube  From the center, it is the same distance equal to a specific integer number of cubits	The Sphere  From the center, it is the same distance equal to a specific decimal or integer number. A rotation brings the position to

	(unit cubes). A rotation brings the position to the same beginning cubit.	the same beginning point.
Zero	It is the impassable wall at the bottom. Zero refers to none, nothing, no-one. Zero is used for counting the natural numbers to mark the new set of 9. Thus, the numbers that contain zeros can be viewed as multiples of nine plus one digit from 1 to 9, e.g. $10 = 9 + 1$, $103 = 9\times9 + 2\times9 + 4$.	In all accepted combinations and expressions.
Numbers	Whole positive numbers and their ratios (rational numbers) only. The array of even numbers 2n starts at zero: 0, 2, 4, 6, 8, … The array of odd numbers 2n-1 starts at one: 1, 3, 5, 7, 9... On the graph, the positive integers constitute a straight array of dots, with the same pace, stretching at geometrically 45^o, and numbering n-dots.	Real and complex numbers, whole and rational numbers, positive and negative numbers, logarithmic and decimal numbers, irrational and transcendental numbers. On the graph, the function $y = x$ is a continuous line stretching at geometrically 45^o and containing an uncounted number of dots.
Relations	Relations with and for positive integers only.	Algebraic relations and mathematical analyses. Functions and equations of all possible lines, groups, rings, and fields. Euclidian and non-Euclidian geometries. Diophantine and algebraic geometry. Calculus and analytical geometry.

Table 1. The Domain of Natural Numbers versus the Domain of Line

In the Domain of Line, zero is assigned to the origin or the beginning point. In one-dimensional space, the geometry determines the inner distance between two points or one point on the coordinative axis and zero (the origin) and what kinds of lines are passing through those points: parallel (Euclidian), hyperbolic (Lobachevskian) or elliptic. In two-dimensional space, the geometry determines the inner area between three points or one point on each of the two coordinative axes and zero (the origin). In three-dimensional space, the geometry determines the inner volume between four points or one point on each of the two coordinative axes and zero (the origin).

The Domain of Line makes use of the conclusions that come from the Domain of Natural Numbers, but not the opposite. Our existence starts at one. Below zero, $n < 0$, the meaning of life and existence loses. Things, living and self-thinking entities are numbered positively. We exist as numbers and shaped as lines. In the Domain of Natural Numbers, the one- or two- or three-dimensional entities are geometrically unconnected objects. Numbers connect them, because of numbers bond numbers. After squarits or cubits are packed in their respective spaces, there are no void spaces left in between. In one-dimensional space, for example, three connects one and two, because $1 + 2 = 3$.

In two-dimensional space, 5^2 connects 3^2 and 4^2, because $9 + 16 = 25$. This heavenly-existed set 3-4-5 is the first square set in the unique sequence commonly called the Pythagorean Triples. These can, for example, be generated by the Fibonacci's method (since the year 1225), by the Michael Stifel's method (since the year 1544) and Jacques Ozanam (since the year 1694) of the progressions of whole and fractional numbers, by the Leonard Eugene Dickson's method (since the year 1920), using Euclid's algebraic quadratic equation, using matrices and linear transformations, etc. The first set of positive integers 3-4-5 is followed by 6-8-10, 5-12-13, 9-12-15, 8-15-17, 12-16-20, 15-20-25, 7-24-25, 10-24-26, 20-21-29, 18-24-30, and so on. [1] Any relationship in the Pythagorean Triples can be proved using squared circles. Only for the Pythagorean Triples, the three squared circles form in between the geometrical shape of the right-angled triangle with sides taking integer numbers. Otherwise, the right-angled triangles are geometrical lines and have the length at least one of their sides taking a non-integer number.

In three-dimensional space, 6^3 connects 3^3, 4^3 and 5^3, because 27 + 64 + 125 = 216. This essentially natural cubic set is the first in the unique cubic sequence 3-4-5-6, 6-8-01-9, 6-8-10-12, 12-16-02-18, 9-12-15-18, 12-16-20-24, 18-24-03-27, and so on.

A lined circle cannot take positive integers and be converted to a lined (geometrical) square with positive integers. Because, a lined square consists of four equal sides with either an odd or an even integer number of steps, which so produce either an odd or an even integer number of squarits. Thus, a lined square essentially falls into the Domain of Natural Numbers at a time when the lined circle divided into an irrational number π = 3.14159265358… of steps remains in the Domain of Line. The geometric irrational number π = 3.14159265358… mirrors the ratio 22/7 in the Domain of Natural Numbers. A lined circle and a lined square bond only when they have an equal geometrical inner area or by inscribing the lined circle inside the lined square and vice versa.

2. The Fermat's Last Theorem

In the year of 1635, Pierre de Fermat (1607-1665) wrote a comment in the margin of a page in a copy of 1621 edition of the book *Arithmetica*, that translations from the century III A.D. had brought as written by Diophantus of Alexandria. The first part of the comment stated that four positive integers or natural numbers a, b, c, n when $n > 2$ cannot be a solution to the following equation:

$$a^n + b^n = c^n \tag{1}$$

The second part of the comment stated that he, Pierre de Fermat, had the proof for Eq. (1) but he could not write it because the page margin did not have enough space for it. Likely, Pierre de Fermat had a flash that could prove Eq. (1), because he did not write anytime later a general proof for Eq. (1). What he communicated in detail was the use of an original logic known as "The Infinite Descent" to derive a contradiction to an invented counterexample from himself. [1-10] He stated that if the area of a right-angled triangle were equal to the square of an integer, e.g. r^2, then there would exist two numbers p, q in the fourth power the difference of which equals r^2. [3, 10, 11] And without his assertion what the numbers p and q were, the following was his equation:

$$p^4 - q^4 = r^2 \tag{2}$$

In the Domain of Line, if by desire r^2 is chosen equal to s, then Eq. (2) appears as $p^4 - q^4 = s$. If by desire $r = t^2$, then $p^4 - q^4 = t^4$ which is a form of Eq. (1) for $n = 4$. If by desire $t = u^2$ then $p^4 - q^4 = u^8$, and so on.

Eq. (2) is inaccurately taken as the specific case of $n = 4$ for Eq. (1), because it conditions by desire $r = t^2$. Also, the counterexample built by Pierre de Fermat or his Eq. (2) falls in the Domain of Line, while the mathematical relationship bodied in Eq. (1) falls in the Domain of Natural Numbers.

As a sort of indirect proof, the technique of Infinite Descent is more a wording logic looking for a contradiction to its start than a mathematical method of proof. Though it relies on geometry and numbers, the purpose of this technique is to decide by language. The contradiction emerges since the start is either nonexistent or untrue or unproven. The Infinite Descent by Pierre de Fermat trailed the logic of *reductio ad absurdum* (reduction to absurdity) by ancient Aristotle. Though *reductio ad absurdum* has full power in philosophical perception, it is not enough in the mathematics of numbers. It is so because reasoning is subjective (coming or accepted from thinking) and numbers are objective (existing independently of thinking).

He activated his proving approach using the formula of the Pythagorean Triples, where the sides of the right-angle triangles are sets of specific positive integers and belong to the Domain of Natural Numbers. Also, he guessed that the sides of such triangles were relatively prime numbers. Then through further calculations and assumptions, e.g. anytime the difference of two integers in fourth power was assumed a squared integer, a descending spiral of infinite smaller and smaller such right-angled triangles emerged. The only way to stop the descending loop or the Infinite Descent was by the wording, as Pierre de Fermat wrote, "…this is impossible since there is not an infinitude of positive integers than a given one". Thus, in accord with Pierre de Fermat, the Infinite Descent was in contradiction to the original counterexample and so it proved that a right triangle could not have an area equal to a squared integer. [3, 5, 11]

The proof for a problem that stays within the Domain of Natural Numbers is not enough or valid to become a valid proof for the Domain of Line. Reversibly, a general proof extracted in the Domain of Line is bigger than the gate of the Domain of Natural Numbers, and thus unacceptable there.

The Infinite Descent or the descending spiral did not produce anything new, except the need to stop it verbally on purpose. The Infinite Descent generated right-angled triangles with decreasing size and headed to infinitely small such triangles. This is equivalent to the direction of the Infinite Ascent, which generates right-angled triangles with increasing size and heading to infinitely big such triangles. Geometrically, as Pierre de Fermat created his counterexample and the procedure for finding its contradiction, there are not any contradictions going down to infinitely small or up to infinitely big right-angled triangles. As such, both the Infinite Descent and Infinite Ascent cannot be stopped verbally except than on purpose.

In the Domain of Line, the area of a right-angled triangle equal to a squared integer is possible and can be only when the lengths of the adjacent sides to the right angle relates in the ratio 2:1. In which case, the length of the side opposite the right angle equals the unit number multiplying $\sqrt{5}$. Which means that such a right-angled triangle is not one of the Pythagorean Triples and precisely it appears within the Domain of Line.

In the Domain of Natural Numbers, the sequence of natural numbers begins at one and has zero its bottom limit. The chain of natural numbers has no top limit and increases infinitely by an increment of one. The existence of the bottom limit cannot constitute a contradiction in the process of the Infinite Descent for the invented counterexample because it is just an arrival at the lower limit. It is just a trial in the engineering optimization.

After the death of Pierre de Fermat, his son Clément-Samuel examined his father's papers, letters, and notes and published them as a book in 1670. [8] Then, Eq. (1) came into sight for other mathematicians who began a pursuit to prove it. Equation (1) is known as the Fermat's Last Theorem or the Fermat's Conjecture, because since then in century XVII it has not been proved in a general form.

3. The endeavors for proving the Fermat's Last Theorem

The first effort for the specific case $n = 4$ to prove the relation embodied in Eq. (1) appeared in 1676 and accelerated in century XIX and early century XX. Due to its outward ease, Eq. (1) attracted all mathematicians and leaders in mathematics. [1, 2, 6, 8, 12-15] The diving efforts of brilliant minds into the ocean of mathematics for solving the Fermat's Conjecture advanced the science of mathematics in new directions. [10, 16, 17]

There have been many publications related to the efforts for proving the Fermat's Conjecture. They cover a range of peer-reviewed top mathematical journals to the simplest personal trials and progress reports posted on the Internet. Such relevant publications keep coming into the scientific view. [7-9, 13, 14, 18-29] It is impossible to cite for reference all of them. However, it is possible to praise all researchers for the time spent for searching to prove the Fermat's Last Theorem.

The shared characteristics of the efforts exerted in proving the Fermat's Conjecture and the root reasons why not a final general proof has been reached will be examined below.

FIRST – The proofs have been searched geometrically (e.g. by means of elliptic curves) or algebraically (e.g. by means of Bernoulli or complex numbers) in the Domain of Line at a time that Eq. (1) is inside the Domain of Natural Numbers; please refer to Table 1 and associated elucidations. Likewise, the proof for Eq. (1) has been examined on algebraic equations, abstract functions, and conditions noticeable other than Eq. (1). [2, 4, 6, 8, 18, 19, 23-26, 29-41]

SECOND – The logic of conclusion has been the logic of contradiction to the one assumed either counterexample or new starting conditions; please refer to Figure 1.

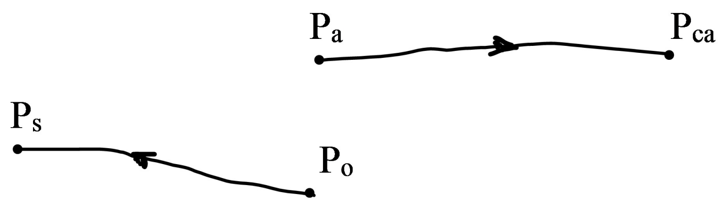


Figure 1. The two paths of the solution, where: P_o is the original point of conditions of the problem. P_s is the solution point of the problem. P_a is the point of the assumed to-be-original-conditions of the problem. P_{ca} is the contradicting point to P_a.

The path or vector of solution P_oP_s is the path that preserves the original conditions of the problem. While the imaginary path P_aP_{ca} starts with an assumed identity of conditions at point P_a that is detached from the point P_o, the original identity of conditions. And sometimes, one counterexample or invented supposition is planted at point P_a. Then, a solution is accepted if a contradiction to point P_a comes across in the path P_aP_{ca}. The rejection by contradiction at point P_{ca} proves only that the assumed to-be-original-conditions or the counterexample at point P_a were not true or could not exist. That is, an encounter at a point P_{ca} will undoubtedly contradict its self-non-existence that rooted at point P_a. The emerged contradiction relates to the false assumption made at point P_a and ruins only the characteristics of point P_a, which is detached from point P_o. Thus, the emerged contradiction at point P_{ca} has no connection with path P_oP_s and conditions of the solution at the point P_s. In addition, a counterexample is specific and there is not any general counterexample.

THIRD – The proofs have progressed on steps that incorporated the assumption or supposition of specific conditions or properties for variables, equations, and functions. [1, 2, 4-8, 10, 12, 14, 16-18, 22, 23, 25, 27, 29, 31-35, 37-48, 50] That is, the conditions or properties or counterexamples have been created on purpose, taken for granted, personally accepted or assigned, thought or imagined to be that way. Analyzing the natural Eq. (1) imaginarily or trying to reach its proof with tools of the imaginary mathematics brings imaginary results. This reaffirms Figure 1.

FOURTH – The proofs of the Fermat's Conjecture have been researched for isolated power numbers, for example, n = 3, 4, 5, 7, 6, 10, 14 or ideal numbers, and especially for prime numbers. [1, 2, 4, 6, 8, 12, 14, 18, 24-27, 30-39, 41, 42, 46-49]

The trail of attempts for proving Fermat's Conjecture selecting prime numbers for the exponent in Eq. (1) that started with Sophie Germain in 1823 needs to be addressed. Sophie Germain grouped in Case One the prime numbers p that cannot divide a, b, c in Eq. (1) and in Case Two those that do. Moreover, she reformulated Eq. (1) into the following equation, which both had different conditions from Eq. (1) and it was not the Fermat's Last Theorem anymore [18, 24, 31-36, 47]:

$$a^p + b^p + c^p = 0 \tag{3}$$

In 1847, Gabriel Lamé tried unsuccessfully to factorize the Fermat's Last Theorem in the cyclotomic field of complex numbers. Based on that experience, Ernst E. Kummer developed the theory of ideal numbers in 1849. Within that and using the complicated Bernoulli numbers, Ernst E. Kummer defined the set of regular prime numbers. He used them to prove the first case of Fermat's Last Theorem. [1, 2,4, 6, 8, 18, 31, 33, 38, 39, 47-49]

The ideal numbers are algebraic integers, which means they are complex numbers. They are part of the ring theory studied in the Abstract Algebra. They represent ideals (subsets) in the rings of integers of algebraic number fields, which have finite dimensions. As such, the Bernoulli, complex and ideal numbers differ totally from natural numbers and do not reside in the Domain of Natural Numbers. Their incorporation in the form of regular prime numbers for proving Eq. (1) cannot give the proof or at least a general solution. Above all, the past and modern researchers that try to find a proof for Sophie Germain's First Case embodied in Eq. (3) have tried to find a proof for a relationship which is not the Fermat's Conjecture embodied in Eq. (1).

FIFTH – A wording instrument linked to integer numbers, known as *modulus operandi,* has been used in algebraic or number formulas. [2, 4, 7, 18, 24-26, 30, 31, 33-35,37, 38, 42-44, 47, 49]

The modulo operation depicts the integer remaining after one integer is divided by another integer number. Thus, for two integers x, y that give the same remainder R after divided by another shared integer z, it is worded that both x and y are congruent modulo z and $x - y$ is a multiple of z. Putting it differently, it is written as the following phrase:

$$x \equiv y \pmod{z} \tag{4}$$

Arithmetically, the relations among the integers x, y, z are generalized as the following:

$$\frac{x}{z} = v + R \tag{5}$$

$$\frac{y}{z} = w + R \tag{6}$$

$$\frac{(x - y)}{z} = v - w \tag{7}$$

The wording phrase (4) is not a numeral operator, a *numeralis operandi*, and only describes the ratio $(x - y)/z$ in Eq. (7) by implying that it is equal to an integer number. As only a notation, the wording phrase (4) does not display the values of $v - w$ and R. It is not some kind of mathematical formula or line equation or numerical function. The wording phrase (4) is a *verbum operandi* and does not bring mathematically anything new. The complete explicit information is given in Eq. (5-7). In the Domain of Natural Numbers, mathematics is explicitly expressed through numeral operators of plus, minus, multiplication, division (ratio), power, equal and sum.

The use of the *verbum operandi* (4) in *numeralis operandi* for proving the Fermat's Conjecture does not fit. It does not offer explicit sets of natural numbers that can be applied as examples to Eq. (1). [19, 31, 34, 35] The Arithmetic is an explicit and exact science, while modulo operation is both a wording phrase and an implying operator. The modulo itself deals with cyclic numbers and all integers, while the natural numbers a, b, c, n in Eq. (1) are only positive integers and not cyclic. A modulo solution used for proving Eq. (1) must be congruent with a proof using arithmetic operators and mathematical formulas. It just complicates a mathematical expression, e.g. Eq. (7), by making invisible and undetermined the integers v-w and R in Eqs. (5-7).

Even when Eq. (1) is arranged in the following rational-number form,

$$\left(\frac{a}{c}\right)^n + \left(\frac{b}{c}\right)^n = 1 \tag{8}$$

there is not any condition in the Fermat's Conjecture that the first term is congruent to the second term or a is congruent to b modulo c in Eq. (8). Anyway, a solution must keep or provide the variables a, b, c, n as positive integers.

SIXTH – The effort to use the elliptic curves and imaginary Galois representations to prove the Fermat's Conjecture is separately examined here. Between 1955 and 1967, Goro Shimura, Yutaka Taniyama, and André Weil set forth the modularity theorem, known also as the Taniyama-Shimura-Weil conjecture. It claimed that all elliptic curves in the field of rational numbers (at rational number coordinates) were associated with the modular forms; that is, they were modular. [2, 4, 6, 12, 16, 42, 50]

Yves Hellegouarch in 1974 and Gerhard Frey in 1982 claimed that the following algebraic equation of the geometrical semi-stable elliptic curves, where p is an odd prime number, is correlated with Fermat's Last Theorem or Eq. (1). [2, 6, 12, 42, 51]

$$y^2 = x\left(x - a^p\right)\left(x + b^p\right) \tag{9}$$

Gerhard Frey proposed that if a solution for a, b, c, p exists from Eq. (1) then a, b of it would give a semi-stable elliptic curve from Eq. (9), referred to as the Frey-Hellegouarch curve, which would not be modular. Thus, referring to point P_a in Figure 1, Gerhard Frey established a counterexample to Fermat's Conjecture. In 1985, Gerhard Frey deepened the mathematical abstraction by articulating that the Taniyama-Shimura-Weil conjecture implied Fermat's Last Theorem. [2, 4, 6, 7, 12] So, referring to point P_{ca} in Figure 1, Gerhard Frey laid down the imaginary path of solution P_aP_{ca}. On it, someone could investigate for a proof of the Taniyama-Shimura-Weil conjecture that would contradict the counterexample flagged at point P_a, thus proving the Fermat's Last Theorem. [46]

In 1985, Jean-Pierre Serre wrote that a Frey-Hellegouarch curve could not be modular and since he did not offer a solid proof for his own proposition it turned to be known as the Epsilon Conjecture. In the summer of 1986, Kenneth A. Ribet proved the Epsilon conjecture for a semi-stable elliptic curve, which meant that the Taniyama-Shimura-Weil conjecture implied the Fermat's Last Theorem. [2, 4, 6, 13, 22, 39, 41, 46]

A highlighted effort for proving Eq. (1) emerged when Andrew J. Wiles published a final article 108-page-long in parallel with a supportive article co-authored with Richard Taylor 19-page-long in the Annals of Mathematics in 1995. [43, 44] By means of those two articles, Andrew J. Wiles confirmed the modularity theorem for semistable elliptic curves to be adequate for contradicting the Gerhard Frey's proposition and thus implying the truth of Fermat's Last Theorem. Very a few mathematicians seem to understand the depths of abstract mathematics contained in those two published papers and the connection to the proof of Fermat's Last Theorem. [2, 7, 13] The whole approach is summarized in the following Figure 2:

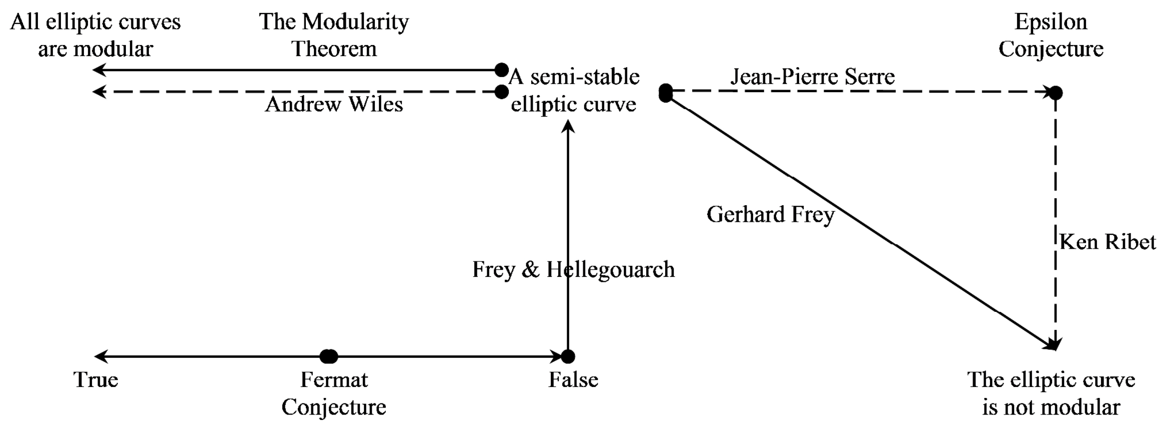


Figure 2. The paths associated with the efforts to prove the Fermat's Conjecture by means of geometric elliptic curves.

As a preface, the proposed solution first guessed by Gerhard Frey and later laid out by Andrew J. Wiles did not provide a general proof because they treated prime numbers instead of the natural numbers for the exponent in Eq. (1). Also, the elliptic curves, modular forms or Galois representations incorporated by them are tools for inside the Domain of Line while the Fermat's Conjecture is inside the Domain of Natural Numbers.

The counterexample proposed by Yves Hellegouarch and Gerhard Frey was a false assumption because the solution to Fermat's Conjecture never existed. Generally, something cannot exist and at the same time exist at the same place, under the same conditions. Ancient Aristotle had summarized this in his principle of non-contradiction, as well. That is, a solution cannot be known and at the same time be known at the same place and conditions. That is, it was and is impossible to find a set of four natural numbers *a, b, c, n* that can prove Eq. (1).

Figure 2 confirms Figure 1 and both Figures confirm the principle of explosion *ex contradictione sequitur quodlibet* (from a contradiction, anything follows). Since both the right and left paths started from a false point or non-existing key, their time-shifted conclusive points had neither connection with nor an authority on the true point of the Fermat's Conjecture. Even if both branches are opposite, their disagreement is dual and not general. Both right and left routes were not combined with the Gottfried W. Leibniz's principle of the Truth of Reasoning, in which an object is resolved into its simplest ideas and truths, into its primitives, to prove it.

As brilliant mathematicians, Yves Hellegouarch, Gerhard Frey, Jean-Pierre Serre and Kenneth A. Ribet on the right route and Yutaka Taniyama, Goro Shimura, André Weil, Andrew J. Wiles and Richard Taylor on the left route were correct in their conclusions about the modularity of geometrical semis-stable elliptic curves. They built their conjectures on detached assumptions, conditions, and tools, independently. Therefore, they produced various products (conclusions). Otherwise, they should have reached the same conclusions. Their right and left approaches to exploration were not even contradicting. Their conclusions in the conceptual mathematics were only different in seeing the geometrical semi-stable elliptic curves from diverse viewpoints. Their research brought highlighted advancements in the theoretical mathematics.

As a natural science, mathematics is an explicitly exact science that makes unfit the implying proposition that the Modularity Theorem can imply the Fermat's Last Theorem. Both routes do not end at the true point of the Fermat's Conjecture. The route for going to the true point of the Fermat's Conjecture is explicitly obvious. Eq. (1) was not born from Eq. (9) or some modular forms, or *vice versa*. There is no genetic connection between Eq. (1) and Eq. (9), independently that the two pairs a^n, b^n and a^p, b^p seem of the same gender. Whatever solution that the values a^p, b^p can for elliptic curves in the field of rational numbers, the pair a^p, b^p does not deliver the pair a^n, b^n. And this, at a time that c^n is not known, and so even the sum $a^p + b^p$ cannot be evaluated. Along with Eq. (9), a solution to any other elliptic or non-elliptic equation $y = f(x)$ that combines a^n, b^n, c^n is not a condition of eligibility for giving any hint how to prove Eq. (1). In addition, a Galois Field is a theoretical finite field enclosing a finite number of elements, while the array of natural numbers is a chain without end. Therefore, any discovery on Eq. (9) has no sway on Eq. (1).

The elliptic Eq. (9) is a specific equation and other elliptic curves are geometrical two-dimensional functions $y^2 = f(x^3)$ that give continuous geometrical lines, which contain an incalculable amount of numbers of all kinds. The properties that the elliptic curves might have at rational number coordinates have no link to Eq. (1), which contains only four arrays of positive integers. While Eq. (1) has as variables the natural number *a, b, c, n,* Eq. (9) has geometrical variables *x, y, a, b* and prime number variable *p*. [4, 6, 22, 46] A solution of Eq. (9) is an optimum solution that incorporates and belongs to the set of the geometrical variables *x, y, a, b* and prime number variable *p*. That is, even when *a, b* in Eq. (9) are positive integers they are processed and so have lost their originality and individuality as positive integers. Therefore, such a solution has no authority to the solution of Eq. (1).

In addition, by definition, a modular form is a complex analytic function (a holomorphic function) on the upper half-plane, which itself is a set of complex numbers with positive imaginary part. Furthermore, a meromorphic function, expressed as a ratio between two holomorphic functions, is a complex-valued function and unlinked to the chain of natural numbers. A modular form is a function that has superior symmetries and complexity on a unit disk. [7, 22, 42, 46, 51] Which means that a modular form is not an array of natural numbers. A function can be symmetric. On the other side, the collection of natural numbers has no symmetries because it is a chain of increasing positive integers. The modular forms are absolutely part of the Domain of Line and not part of the Domain of Natural Numbers.

In the article by Andrew J. Wiles, there is no conclusive formula where any substitution with concrete natural numbers *a, b, c, n* would confirm the Fermat's Conjecture. Except mentioning the Fermat's Last Theorem by name six times in the title and introduction, Eq. (1) was not engaged in the article. It was so because Andrew J. Wiles theoretically proved using related Galois representations only that the semi-stable elliptic curves were modular. [4, 12, 39, 43, 46, 51] Christophe Breuil, Brian Conrad, Fred Diamond and Richard Taylor advanced the path laid down by Andrew J. Wiles and proved the modularity theorem for all elliptic curves in 2001 [45]. Both right and left paths in Figure 2 constitute a non-constructive proving endeavor for the Fermat's Conjecture because they provide no numeral examples for Eq. (1).

4. The proof of the Fermat's Last Theorem

To prove the Fermat's Conjecture expressed in Eq. (1) initially, means to assume (to bear the error) that Eq. (1) will remain the same for all $n > 2$: two terms on the left and one term on the right. Because the unique cubic sequence 3-4-5-6, 6-8-01-9, 6-8-10-12… is the example just at the beginning $n > 2$ that Eq. (1) does not exist with two terms on the left and one term on the right when *a, b, c* are positive integers. Which means that the effort for proving the Fermat's Conjecture have conveyed the untruth that Eq. (1) with natural numbers, *a, b, c, n* had only two terms on the left and one term on the right. With the knowledge of this error, summing up Eq. (1) side by side for all *n* gives the following:

$$\sum_{n=1}^{n} a^n + \sum_{n=1}^{n} b^n = \sum_{n=1}^{n} c^n \tag{10}$$

$$\frac{a^{n+1} - a}{a - 1} + \frac{b^{n+1} - b}{b - 1} = \frac{c^{n+1} - c}{c - 1} \tag{11}$$

For $n = 1$, Eq. (1) or Eq. (11) becomes $a + b = c$ that is true for unlimited cases in which the numbers a, b, c form the bond $a + b = c$. This relation also tells that always $c > \{a, b\}$. The naturalness and conditions of natural numbers a, b, c, n of Eq. (1) are kept undisturbed in Eq. (11). However, Eq. (11) cannot be used for proving the Fermat's Conjecture because it is untrue that Eq. (1) will remain with only two terms on the left and one term on the right for all n.

For $n = 1$ and $a = b$ Eq. (1) becomes $2a = c$, which is true for all cases when $c = 2a$. When $n \geq 2$ and $a = b$ then Eq. (1) becomes $2a^n = c^n$, which is not true with natural numbers (positive integers) because $2^{1/n}$ cannot be a positive integer.

For $n = 2$, $a \neq b$, $a + b \neq c$ and $c > \{a, b\}$, Eq. (1) is true only for the Pythagorean Triples. Those are generated when a, b, c relate through, for example, the Euclid's algebraic quadratic equation with $a = p^2 - q^2$, $b = 2pq$, $c = p^2 - q^2$ and where p, q are coprime.

The Fermat's Last Theorem provides only one equation, the Eq. (1), with four variables and no link among them. As such, the Eq. (1) cannot be measured because the pair distances in the set $\{a, b, c, n\}$ are not fixed. The use of *modulus operandi* does not help either because the bonds among a, b, c, n are undefined and unconditioned. Staying in the Domain of Natural Numbers and without disturbing the identity of natural numbers, the only equations that can be used to prove the Fermat's Conjecture are Eq. (1) and Eq. (8).

Eq. (1) can be directly proved in the Domain of Natural Numbers by generally relating $b = f(a)$ and $c = f(b)$. Thus, the reframed Eq. (1) appears in the following form:

$$c = \left(a^n + b^n\right)^{1/n} \qquad (12)$$

When a is a positive integer, c will be integer only when $a^n + b^n = m^n a^n$ with m being a positive integer. Then:

$$b = a\left(m^n - 1\right)^{1/n} \qquad (13)$$

Eq. (13) shows that b cannot be a positive integer because $(m^n - 1)^{1/n}$ cannot be a positive integer for $n > 2$. This explicitly proves the Fermat's Conjecture in the Domain of Natural Numbers. That is Eq. (1) cannot have a solution for $n > 2$ when a, b, c, n are all positive integers.

Another way to find the answer for the Fermat's Conjecture is to adjust Eq. (1) as follows:

$$\left(a^{n/2}\right)^2 + \left(b^{n/2}\right)^2 = \left(c^{n/2}\right)^2 \qquad (14)$$

Only when the three squared terms are bonded in the Domain of Natural Numbers in the form of the Pythagorean Triples through Euclid's algebraic quadratic equation, it can be written as follows:

$$a = \left(p^2 - q^2\right)^{2/n} \qquad (15)$$

$$b = \left(2\,p\,q\right)^{2/n} \qquad (16)$$

$$c = \left(p^2 + q^2\right)^{2/n} \qquad (17)$$

For $n > 2$, the values of a, b, c calculated with Eqs. (15-17) will not be positive integers because of $2/n < 1$. Thus, the Fermat's Conjecture holds true in the Domain of Natural Numbers that Eq. (1) does have a solution for positive integer values of a, b, c, n when $n > 2$.

The proof of the Fermat's Conjecture using Eq. (13) and Eqs. (15-17) makes obvious that for $n > 2$ the Eq. (1) has its field of the degrees of freedom in the Domain of Line where a, b, c, n can be real or complex numbers. In the Domain of Line, Eq. (1) can be analyzed with all possible mathematical, geometrical, algebraic, analytical, complex and imaginary tools. In the Domain of Line, Eq. (1) is not the Fermat's Conjecture anymore.

5. Conclusion

A mathematical conjecture or any formula and equation needs be first defined to which Domain it belongs: to the Domain of Natural Numbers or to the Domain of Line. Then, this will determine the point of view and tools directed to the analyzed conjecture or equation. If a conjecture or equation is fully on natural numbers (it is inside the Domain of Natural Numbers), then the mathematical tools should be extracted from the Domain of Natural Numbers. If a conjecture or equation is defined for the Domain of Line, then the mathematical tools should be extracted from the Domain of Line and/or from the Domain of Natural Numbers if they fit.

The Fermat's Last Theorem preserves its original identity if it is proved within the Domain of Natural Numbers and with mathematical tools from this Domain. Pierre de Fermat was correct that Eq. (1) having a, b, c, n as positive integers cannot be true for $n > 2$. However, he missed defining both in which Domain he was conjuring the Eq. (1) and any relationship among numbers a, b, c, n. It took 382 years to correctly outline and prove the Fermat Last Theorem.

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