# 《Universal and Unified Field Theory》 — 4. Graviton and General Dynamic Fields

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**Abstract**: For the first time, ***Law of Conservation of Gravitation*** is discovered that consists of seven principles, including not only the wave-particle duality but also ***Graviton***, ***Graviton Momentum***, and *Newton's Law*. In addition, a set of ***Gravitational Fields***, known as weak fields, is concisely formulated to coexist with the fields of electromagnetism. Together, they represent the ***Generic Dynamic Fields***, which define or establish a symmetric dynamic environment. As a major part of the unification, the two pairs of *Quantum Fields* give rise to and bring together *Electromagnetic* and *Gravitational* fields through the flux entanglements of conservation and continuity of photons and gravitons.

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## INTRODUCTION

In the world plans $\mathbf{r} \pm i\,\mathbf{k}$, besides the inertial movements, the spiral transportations are cooperated between the two-dimensional manifolds that give rise to the rotational motion dynamics extending into the tetrad coordinates. The derivative to the scalar field is the global $\lambda$ event operations of their motion dynamics, which generate dual vector potentials $V_n^\mu = \partial^\mu \phi_n^+$, $V_m = \partial_m \phi^-$, operate the divergent $\dot{\partial}_\lambda$ to the global density $\rho_n = \phi_n^+ \phi_n^-$, produce the potential transformation $\{\hat{\partial}_\lambda, \check{\partial}^\lambda\}$, and result in the entanglement effects transporting to or projecting from their opponent manifold.

Together with the *Lorentz* generator, the *Torsion Coordinator* forms and projects its spiral potentials to its surrounding space, arisen by or acting on its opponent through a duality of flux continuities, which interacts with both of the Boost transformations and Twist transportations, given by the following [1]:

$$\hat{\partial}_\lambda \psi = \dot{x}_a \left(J_{\mu a}^+ + K_{\mu a}^+\right) \partial^\mu \psi \qquad : K_{\mu a}^+ = \Gamma_{\mu a}^{+\sigma} x_\sigma \tag{3.5}$$

$$\check{\partial}^\lambda \psi = \dot{x}^\alpha \left(J_{m\alpha}^- + K_{m\alpha}^-\right) \partial_m \psi \qquad : K_{m\alpha}^- = \Gamma_{m\alpha}^{-s} x_s \tag{3.7}$$

where $K_{m\alpha}^\pm$ is *Torque Tensor*s. Because of the scaler fields transporting through the rotation entanglements, the rotational communicates give rise to a horizon of vector fields and tensor fields aligning with the stationary or inertial curvatures. Mathematically, a torque can be thought of as a twist to an object and is defined as the rate of change of angular momentum of an object. A torque tensor $\Gamma_{\mu a}^{\pm\sigma} x_\sigma$ is the cross product of the distance vector by which the force's application point is offset relative to the fixed suspension point (distance vector) and the displacement fields $\Gamma_{ik}^l$ to produce rotational motion. If the torsion tensors are known, the distance vectors $x_\sigma$ and $x^\sigma$ can reveal a pair of its rotation centers for an object movements, where exert a pair of the turning forces on, for example, a start or galaxy as the virtual pushing or pulling handles - a pair of the $Y^- Y^+$ dark streaming energies.

The transport communication steams the fluxions in form of the $Y^- Y^+$ entanglements alternating between the world-lines or dual manifolds. The fluxion continuity emerges always as a reciprocal pair, which can be defined as and described by the $Y^-$ or $Y^+$ fields $\dot{\partial}_\lambda \mathbf{f}_n^\pm$ of continuity, given by the *Second Universal Field Equations* [3]:

$$\dot{\partial}_\lambda \mathbf{f}_s^+ = \left[W_0\right]^+ - \left\langle \left(\kappa_1 + \kappa_2 \check{\partial}_\lambda\right)\left(\check{\partial}^\lambda - \hat{\partial}_\lambda\right) \right\rangle^+ + \kappa_2 \zeta^+ \tag{10.3}$$

$$\dot{\partial}_\lambda \mathbf{f}_s^- = \left[W_0\right]^- - \left\langle \left(\kappa_1 + \kappa_2 \check{\partial}^\lambda\right)\left(\hat{\partial}^\lambda - \hat{\partial}_\lambda\right) \right\rangle^- + \kappa_2 \zeta^- \tag{10.8}$$

where the $\kappa_2 \zeta^\pm$ is the asymmetric entanglements, which yields insignificance to the symmetric dynamics of the flux continuities.

## XIII. TORSION TENSORS

Operated by the global events $\lambda$, a universal environment is initially described by a scalar density of dual fields as variable components of the coordinates at the $\lambda$ event: $\psi(\check{x}, \lambda)$ and $\psi(\hat{x}, \lambda)$, which are applicable to or aligned with their respective $Y^- Y^+$ manifolds and representable by both forms of contravariance $Y\{x^\mu\}$ or covariance $Y\{x_m\}$. The torsion fields are mathematically characterizable and quantifiable by the fluxion continuity.

Because of the $Y^- Y^+$ commutation infrastructure of rising *horizons*, an event generates the twisting entanglements between the manifolds, and performs the operators of $\partial^\mu$ and $\partial_m$, transports the motion vectors of $\dot{x}^\alpha$ and $\dot{x}_a$, and gives rise to the vector potentials of $\dot{x}^\mu \partial^\mu \psi$ or $\dot{x}_m \partial_m \psi$. By expressing those elements into to a pair of the tensors, the above equations provision their commutation fields $\langle T \rangle_{\mu m}^-$ in form of fluxion tensor under dynamics of the physical supremacy:

$$\langle T \rangle_{\mu\alpha}^- = \left\langle \dot{x}^\alpha K_{\mu\alpha}^- \partial_\mu, \dot{x}_\alpha K_{\mu\alpha}^+ \partial^\mu \right\rangle^- \qquad : K_{\mu\alpha}^\pm = \Gamma_{\mu\alpha}^{\pm s} x_s \tag{13.1}$$

Accordingly, the commutative routing represents the basic transportation of the second horizon entangling between the dual manifolds. It naturally constructs a pair of the operational matrices that are completely antisymmetric for the elements in form of 4x4 matrixes:

$$\left\langle T_{ma} \right\rangle^- = \begin{pmatrix} \xi_0 & \pi_1 & \pi_2 & \pi_3 \\ -\pi_1 & \xi_1 & -\vartheta_3 & \vartheta_2 \\ -\pi_2 & \vartheta_3 & \xi_2 & -\vartheta_1 \\ -\pi_3 & -\vartheta_2 & \vartheta_1 & \xi_3 \end{pmatrix} \qquad : \begin{aligned} \xi_m &= \left\langle T_{mm} \right\rangle^- \\ \pi_a &= \left\langle T_{0a} \right\rangle^- \\ \varepsilon_{nam}^- \vartheta_n &= \left\langle T_{ma} \right\rangle^- \end{aligned} \tag{13.2}$$

Named as the $Y^-$ *Torsion Tensor* $\langle T \rangle_{m\alpha}^-$, it constructs a pair of the off-diagonal fields $\langle T \rangle_{m\alpha}^- = -\langle T \rangle_{\alpha m}^-$ as a part of the internal transport infrastructure. The trace of diagonal elements of $\xi_m \equiv \partial_m \Phi_g^-$, the derivative $\partial_m$ of the effective $Y^-$ potential $\Phi_g^-$, are the motion dynamics of the actor or reactors that is traveling or transporting along the world-line infrastructure. They embed a pair of mirrors to each other as of: i) antisymmetric matrix for their off-diagonal entries, and ii) the conjugate function for their trace of diagonal elements. As a consequence, the transport framework institutes an environment where a pair of the commutators compels or exerts their dual fields as a foundational structure to its surrounding area, giving rise to the fields of virtual motion stress ($\pi \mapsto B_g^-$) and physical twist torsion ($\vartheta \mapsto E_g^-$).

In the parallel fashion, the $Y^- Y^+$ commutation infrastructure generates the commutation aligning with the event processes of duality. Under the $Y^+$ primary, it consistently and reciprocally constructs another pair of the operational matrices that are also antisymmetric for the off-diagonal elements in a similar form of 4x4 matrixes:

$$\langle T \rangle_{\nu a}^+ = \left\langle \dot{x}_a K_{\nu a}^+ \partial^\nu, \dot{x}^\alpha K_{\nu a}^- \partial_\nu \right\rangle^+ \qquad : K_{\mu a}^\pm = \Gamma_{\mu a}^{\pm s} x_s \tag{13.3}$$

$$\left\langle T_{\nu a} \right\rangle^+ = \begin{pmatrix} -\xi^0 & -\chi^1 & -\chi^2 & -\chi^3 \\ \chi^1 & \xi^1 & \omega^3 & -\omega^2 \\ \chi^2 & -\omega^3 & \xi^2 & \omega^1 \\ \chi^3 & \omega^2 & -\omega^1 & \xi^3 \end{pmatrix} \qquad : \begin{aligned} \xi^\mu &= \left\langle T_{\nu\nu} \right\rangle^+ \\ \chi^\alpha &= \left\langle T_{0\alpha} \right\rangle^+ \\ \varepsilon_{\nu\alpha\mu}^+ \omega^\nu &= \left\langle T_{\nu a} \right\rangle^+ \end{aligned} \tag{13.4}$$

Called as the $Y^+$ *Torsion Tensor* $\left\langle T_{\nu a} \right\rangle^+$, it constructs additional pair of the off-diagonal fields $\left\langle T_{\nu a} \right\rangle^+ = -\left\langle T_{a\nu} \right\rangle^+$ as a part of the internal transport infrastructure. The trace of diagonal elements of $\xi^\mu \equiv \partial^\mu \Phi_g^+$, the derivative $\partial^\mu$ of the effective $Y^+$ potential $\Phi_g^+$, are the motion dynamics of the actor or reactors traveling or transporting along the world-line infrastructure. As a second pair of the commuters from the $Y^+$ manifold, they embed a pair of mirrors to each other, compelling or exerting its dual fields to its surrounding area or into the reciprocal or physical regime: foundational fields of the virtual strength displacement ($\gamma \mapsto D_g^+$) and physical twisting torsion ($\omega \mapsto H_g^+$).

## XIV. LAW OF CONSERVATION OF GRAVITATION

Similar to the light continuity equations, the $Y^-$ fluxion of density continuity (10.8) has the acceleration tensors in the diagonal elements and symmetric formulation:

$$\mathbf{g}_d^- = \frac{\hbar c}{\tilde{E}_g^-} \dot{x}_\kappa \partial_\kappa \xi_m - \frac{c^2 \tilde{E}_g^+}{\hbar c} + ic \left\langle \dot{x}_\kappa \partial_\kappa \right\rangle_s^- \qquad : \left\langle \dot{x}_\kappa \partial_\kappa \right\rangle_s^- \mapsto 0 \quad (14.1)$$

As the trace of diagonal elements of the continuity equation, the time invariance $\left\langle \dot{x}_\kappa \partial_\kappa \right\rangle_s^-$ is undetectable or equivalent to zero, which is know as time translation symmetry. For the motion speed at $\mathbf{u}$, the acceleration tensor $\mathbf{g}_d^-$ of equation (14.1) for the trace of diagonal elements can be expressed as the regular vector and matrix form:

$$\mathbf{g}_d^- = \frac{\hbar c}{\tilde{E}_g^-} \begin{pmatrix} ic \frac{\partial}{\partial x_0} & \mathbf{u}\nabla \end{pmatrix} \begin{pmatrix} -ic \frac{\partial}{\partial x^0} \\ \mathbf{u}\nabla \end{pmatrix} \Phi_g^- - \frac{c^2 \tilde{E}_g^+}{\hbar c} = 0 \quad (14.2)$$

$$\frac{\partial^2 \Phi_g^-}{\partial t^2} + (\mathbf{u}\nabla)(\mathbf{u}\nabla)\Phi_g^- = \frac{\tilde{E}_g^- \tilde{E}_g^+}{\hbar^2} \quad (14.3)$$

where $\Phi_g^-$ is named as *Potential Density of Gravitation*. Considering the wave function at a constant speed $\mathbf{u} \mapsto c_g$, the above equation of the acceleration tensor $\mathbf{g}_d^-$ represents that the potential density $\Phi_g^-$ of gravitation is a wave function , which is conserved or remains constant at the quantum states of a system, given by the following equation:

$$\frac{1}{c_g^2} \frac{\partial^2 \Phi_g^-}{\partial t^2} + \nabla^2 \Phi_g^- = \left( \frac{E_g}{\hbar c_g} \right)^2 \qquad : E_g^2 = \tilde{E}_g^+ \tilde{E}_g^- \quad (14.4)$$

The dual energies $E_g^\pm$ maintain the gravity transformable between virtual and physical states by the conversion equation: $E_g^2 = \tilde{E}_g^+ \tilde{E}_g^-$ where $E_g$ is the total energy. Since $E_g$ is a constant, the gravitational wave of the potential energy is conserved. It states that, at the constant speed $c_g$, gravitation has the characteristics, shown as the following:

1) Gravitation remains constant and conserves over time during its transportation.
2) Gravitation transports in wave formation virtually and acts on objects physically.
3) Neither can a gravitation energy of potential density be created nor destroyed.
4) Gravitation is consisted of virtual energy with $\tilde{E}_g^\pm$ as an irreducible photon unit.
5) Gravitation has at least two gravitons for entanglement at zero net momentum.
6) Gravitation transforms from one form to another carrying potential messages.
7) Without an energy supply, no gravitation can be delivered to its surroundings.

Therefore, the above equation is named as the ***Law of Conservation of Gravitation***, discovered at 0:00am August 21, 2017, Washington DC USA.

Under the assumption of the constant speed $c_g$ for its torque torsion, this conservation law is rigorously derived by a direct consequence of $Y^-$ fluxion continuity balanced at $Y^+$ time translation symmetry. Its fundamental principle is the $Y^- Y^+$ transportation coordinators that generates matrixes of a *torque tensors* $K_{\mu m}^\pm$ as a duality of the twist rotations.

***Artifacts 22: Newton's Law***. For a micro or macro system of homogeneous fields, *Law of Conservation of Gravitation* represents that the physical divergence of the $Y^-$ dark fluxions remains constant and conserves over time. For time independent observations, it leads to the gravitational potential in the following form:

$$\nabla^2 \Phi_g^- = \frac{4\pi G}{c_g^2} \rho_g \qquad : \frac{\partial^2 \Phi_g^-}{\partial t^2} = 0, \ \left( \frac{E_g}{\hbar} \right)^2 = 4\pi G \rho_g \quad (14.5)$$

known as *Gauss's* Law for gravitation, discovered in 1773 [4] and followed by *Poisson's* equation in 1813 [5]. As the trace of diagonal elements of the acceleration tensor, it is natural that $\mathbf{g}_d^-$ is at physical primacy, a conservative force, irrotational $\nabla \times \mathbf{g}_d^- = 0$, and external to the point mass. It is equivalent to *Newton's* law [6] for gravitational field:

$$\mathbf{F}^- = -m c_g^2 \nabla \Phi_g^- = -m G \rho_g \frac{\mathbf{r}}{r^2} \qquad : c_g^2 \nabla \Phi_g = G \rho_g \frac{\mathbf{r}}{r^2} \quad (14.6)$$

where $G$ is a universal gravitational constant, and $\rho_g$ is the mass density.

Newton's law of universal gravitation states that A particle attracts on every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This general physical law was derived from empirical observations by what Isaac Newton called inductive reasoning. It is a part of classical mechanics and was formulated in Newton's work *Philosophiæ Naturalis Principia Mathematica* ("the Principia"), published on 5 July 1687.

### GRAVITON

According to the law of conservation of gravitation, gravitation exhibits wave–particle duality such that its properties must acquire characteristics of both waves and particles. Integration with Newton's empirical law, a duality of the energy formations of gravitation has both of its convertible form to physical $\rho_g c^2$ and its transportable form at virtual unit $\rho_g c^2 \rightleftharpoons \frac{2\hbar}{V_g} \sqrt{\pi G \rho_g}$, where $V_g$ is the volume of the gravitational sources. It is conservation of energy $\mathbf{P} \pm i \tilde{E}_g^\pm = M_g c^2$ and invariance of momentum $\mathbf{P} = ic\hat{\mathbf{p}}$ that maintain the gravitation transformable between virtual and physical states:

$$\frac{4\pi G}{V_g} \left( \frac{\hbar}{c} \right)^2 \rightleftharpoons M_g c^2 \qquad : \tilde{E}_g \mapsto \rho_g c^2 \rightleftharpoons \frac{2\hbar}{V_g} \sqrt{\pi G \rho_g} \quad (14.7)$$

where $M_g$ is the total mass of its $V_g$ volume. As a pair of irreducible virtual unit, the energy densities become a pair of the complex forms for the virtual and physical duality, shown by the following:

$$\tilde{E}_g^\pm = \frac{4\pi G}{V_g} \left( \frac{\hbar}{c} \right)^2 \mp i M_g c^2 \qquad : E_g^2 = \tilde{E}_g^+ \tilde{E}_g^- \quad (14.8)$$

named as Graviton Energy - a fundamental property of gravitation. The total energy $\tilde{E}_g$ contains naturally not only a complex formula but also an irreducible virtual unit, $4\pi G \hbar^2/(cV_g)^2$, defined as ***Graviton***, introduced at 9:30pm August 24 2017 *Metropolitan Area of Washington, DC USA*. As a constant, a graviton defines an irreducible unit of energy state either at virtual $4\pi G \hbar^2/(cV_g)^2$ or at physical $\rho_g c^2$, but not at both.

Graviton represents a transportation duality between a physical $\rho_g c^2$ and a virtual $4\pi G \hbar^2/(cV_g)^2$ energy. Not only has this model accounted for the mass density dependence of gravity's energy and explained the ability of matter and graviton radiation to be in thermal equilibrium, but also deemed for anomalous observations, including the properties of black-body gravitational radiations. It may follow conservation of energy momentum that the physical and virtual states exist and include both of the dual manifolds:

$$E_g^2 = \tilde{E}_g^- \tilde{E}_g^+ = M_g^2 c^4 + \left( \frac{4\pi G}{V_g} \right)^2 \left( \frac{\hbar}{c} \right)^4 \quad (14.9)$$

where the total $E_g$ energy includes both gravitons in the dual manifolds. Therefore, the complex form of the graviton energy derives the gravitational momentum $p_g$ as the following expression:

$$p_g = 4\pi \frac{G\hbar^2}{c^3 V_g} \tag{14.10}$$

In the center of entanglement, the colliding duality has no net momentum. Whereas gravitons always have source volume, conservation of momentum (equivalently, transportation invariance) requires that at least two gravitons are created for entanglements transporting at their zero net momentum.

Similar to the energy equation $E_c^{\mp}$ at the virtual space for a photon, a graviton exhibits wave–particle duality, transports under the $Y^-Y^+$ entanglements, and obeys the *Law of Conservation of Gravitation*. However, unlike a photon, the total energy of a pair of gravitons is conserved under a volume density, and mediated by *Spiral Torque* $\Gamma_{\mu\alpha}^{+\sigma} x_\sigma$. It might behave like a particle with definite and finite measurable position or momentum, though not both at the same time. As a density streaming, graviton waves may be refracted by an object or interfered with themselves.

## XV. GRAVITATIONAL FIELDS

As the off-diagonal elements, the fluxion generates acceleration tensor $\mathbf{g}_\times^+$ under virtual primacy for physical forces, and $\mathbf{g}_\times^-$ under physical primacy for virtual forces $0^+$. For a micro or macro system, both acceleration tensors represent the time divergence of the $Y^+$ or $Y^-$ dark flexions acting on a physical object. With the substitution of equation (5.9) and (5.11), the formulae of (10.8) and (10.3) are translatable at approximately to the first order:

$$\mathbf{g}_x^- = \frac{\hbar c}{\tilde{E}_g^-} \dot{x}_\kappa \partial_\kappa \langle T_{m\alpha} \rangle_\times^- \propto \mathbf{K}_s^+ \qquad : \mathbf{K}_s^+ \mapsto 0^+ \tag{15.1}$$

$$\mathbf{g}_\times^+ = i\frac{c}{2}\left\langle \hat{\partial}_\lambda - \check{\partial}^\lambda \right\rangle_\times^+ = ic\langle T_{m\alpha}\rangle_\times^+ \qquad : \frac{\hbar c}{2\tilde{E}_c^+}\check{\partial}_\lambda \ll i\frac{c}{2} \tag{15.2}$$

where the symbol $\langle\ \rangle_\times^-$ indicates the off-diagonal elements of a tensor. Since the derivative to the potential fields is equivalent to a fluxion $\mathbf{g}_\times^+ \mapsto \mathbf{f}_\times^+$, the $Y^+$ flux continuity $\dot{\partial}\mathbf{g}_\times^+$ is balanced to its resources $\mathbf{K}_s^- \neq 0$.

***Artifacts 23:*** *$Y^-$* ***Gravitational Fields***. For the off-diagonal elements $\mathbf{g}_\times^-$ of the $Y^-$ acceleration tensor $\mathbf{g}^-$, the dark fluxions of $Y^-$ *Transforming Continuity Equation* (12.1) have the conservation equation: $\check{\partial}_\lambda \mathbf{f}^- = 0^+$, sourced by the virtual time operation $\lambda = t$. Substituted by the $Y^-$ *Torsion Tensor* $\langle T\rangle_{m\alpha}^-$ of the equations (13.2), the conservation equation forms up an acceleration tensor for the gravitational fields balanced by the $Y^+$ forces:

$$\mathbf{g}_x^- = \begin{pmatrix} ic\frac{\partial}{\partial x_0} & \mathbf{u}\nabla \end{pmatrix} \begin{pmatrix} 0 & c\mathbf{B}_g^- \\ -c\mathbf{B}_g^- & \mathbf{b}\times\mathbf{E}_g^- \end{pmatrix}_\times = 0^+ \tag{15.3}$$

where $0^+$ represents the virtual force for its zero mass, $\mathbf{b}$ is the coordinate basis of the $Y^-$ manifold, $\mathbf{E}_g^-$ is named as ***Strength Field***, and $\mathbf{B}_g^-$ is titled as ***Twisting Field*** defined as the following:

$$\mathbf{B}_g^- = \hbar \sum_n \frac{p_n^-}{\tilde{E}_n^-}\pi_\alpha \qquad : \alpha \in \{1,2,3\} \tag{15.4}$$

$$\mathbf{b}\times\mathbf{E}_g^- = \hbar c \sum_n \frac{p_n^-}{\tilde{E}_n^-}\varepsilon_{iam}^- \vartheta_i \qquad : i \neq m \neq a \in \{1,2,3\} \tag{15.5}$$

Therefore, the acceleration tensor $\mathbf{g}_\times^-$ for the off-diagonal components is equivalent to the following equations:

$$(\mathbf{u}\nabla)\cdot\mathbf{B}_g^- = 0 \tag{15.6}$$

$$\frac{\partial \mathbf{B}_g^-}{\partial t} + \left(\frac{\mathbf{u}}{c}\nabla\right)\times\mathbf{E}_g^- = 0 \tag{15.7}$$

The first equation is conservation of fluxion similar to *Gauss's Law* for gravitation, and the second is for induction between the fields. They are the basic laws of gravitational torsion entangling strength with twisting fields to produce a gravity or a phenomenon called gravitational induction. It is also a part of the fundamental operating principle of *Lorentz*-invariant theory of gravitation (LTG) [7-8].

***Artifacts 24: Gravitational Force***. For an integrity of microscopic effects at the third horizon, a time derivative to the fluxion fields formulates an acceleration tensor to give rise to the vector fields from *Virtual Torsion Tensors*. As a micro or macro system, the off-diagonal elements of $Y^+$ transforming continuity represents the time divergence of the $Y^+$ dark fluxions for symmetry system. With the substitution of equation (13.4), the above formula can be approximated to the first order as the following:

$$\mathbf{g}_\times^+ = i\frac{c}{2}\left\langle \hat{\partial}_\lambda - \check{\partial}^\lambda \right\rangle_\times^+ = ic\left\langle T_{\mu a}\right\rangle_\times^+ = ic\begin{pmatrix} 0 & c^2\mathbf{D}_g^+ \\ -c^2\mathbf{D}_g^+ & \mathbf{u}\times\mathbf{H}_g^+ \end{pmatrix} \tag{15.8}$$

$$\mathbf{D}_g^+ = \frac{1}{c^2}\sum_n p_n^+ \chi^\alpha \qquad : \alpha \in \{1,2,3\} \tag{15.9}$$

$$\mathbf{u}\times\mathbf{H}_g^+ = \sum_n p_n^+ \varepsilon_{\nu\alpha\mu}^+ \omega^\nu \qquad : \nu \neq \alpha \neq \mu \in \{1,2,3\} \tag{15.10}$$

where $\mathbf{D}_g^+$ is named as ***Displacing Stress Feld*** and $\mathbf{H}_g^+$ as ***Materializing Torsion Feld***. For a point charge moving with velocity $\mathbf{u}$, the above acceleration field represents the gravitational force:

$$\mathbf{F}_g^+ = \kappa_g^+ \mathbf{g}_\times^+ = M\mu_g\left(c^2\mathbf{D}_g^+ + \mathbf{u}\times\mathbf{H}_g^+\right) \tag{15.11}$$

$$\kappa_g^+ = -\frac{i}{c}M\mu_g,\ c^2 = \frac{1}{\varepsilon_g\mu_g} \tag{15.12}$$

where $M$ is the mass, $\varepsilon_g$ is the gravitational permittivity and $\mu_g$ the gravitational permeability of the materials. Without polarization and materialization, the constitutive relations results in the summation of the stress and torsion forces in the free space:

$$\mathbf{F}_g^+ = M\left(\mathbf{E}_g^- + \mathbf{u}\times\mathbf{B}_g^-\right) \tag{15.13}$$

$$\mathbf{D}_g^+ = \varepsilon_g \mathbf{E}_g^-, \qquad \mathbf{H}_g^+ = \mu_g \mathbf{B}_g^- \tag{15.14}$$

Similar to *Lorentz Force*, a particle of mass $M$ moving with velocity $\mathbf{u}$ in the presence of a stressing strengthen field $\mathbf{E}_g^-$ and a twisting torsion field $\mathbf{B}_g^-$ experiences a force $\mathbf{F}_g^+$. A mass particle is accelerated in the same linear orientation as the $\mathbf{E}_g^-$ field, but will curve perpendicularly to both the instantaneous velocity vector $\mathbf{u}$ and the torsion $\mathbf{B}_g^-$ field according to the right-hand rule. Therefore, the total force of gravitation has two components, one of which is proportional to the vector distance to the attracting body, and the other is associated with the velocity vector circle to the closed body.

***Artifacts 25:*** *$Y^+$* ***Gravitational Fields***. Continuing to operate the gravitational force, the time events, $\lambda = t$, evolve and give rise to the next horizon fields:

$$\dot{\partial}_\lambda \mathbf{g}_\times^+ = ic\dot{\partial}_\lambda \begin{pmatrix} 0 & c^2\mathbf{D}_g^+ \\ -c^2\mathbf{D}_g^+ & \mathbf{u}\times\mathbf{H}_g^+ \end{pmatrix} \quad : \dot{\partial}_\lambda \in \left\{ic\frac{\partial}{\partial x_0}, \mathbf{u}\nabla\right\}, \tag{15.15}$$

For the macro density and current, the contravariant vector combines or entangles the gravitational mass density $\rho_g$ and mass current density $\mathbf{J}_g$ that can be balanced by the physical resources and defined by the following four-vector as the continuity equation:

$$\dot{\partial}_\lambda \mathbf{g}_\times^+ = \mathbf{K}_s^- \qquad : \mathbf{K}_s^- \in -ic^3 4\pi G\{\mathbf{u}\rho_g, \mathbf{J}_g\} \tag{15.16}$$

Therefore, the above relationships derive the general equations of displacing $\mathbf{D}_g^+$ and materializing $\mathbf{H}_g^+$ fields as the following:

$$\nabla\cdot\mathbf{D}_g^+ = 4\pi G\rho_g \tag{15.17}$$

$$\frac{\mathbf{u}\cdot\mathbf{u}}{c^2}\nabla\times\mathbf{H}_g^+ - \frac{\partial\mathbf{D}_g^+}{\partial t} = 4\pi G\mathbf{J}_g + \mathbf{H}_g^+\cdot\left(\frac{\mathbf{u}}{c}\cdot\nabla\right)\times\frac{\mathbf{u}}{c} \tag{15.18}$$

where the formula, $\nabla\cdot(\mathbf{u}\times\mathbf{H}) = \mathbf{H}\cdot(\nabla\times\mathbf{u}) - \mathbf{u}\cdot(\nabla\times\mathbf{H})$, is applied. The first equation is inhomogeneous and describes how the fields vary in space due to their physical sources of $4\pi G\rho_g$ as an integrity effects. The second equation demonstrates how the stress and torsion fields entangle with their respective movements.

As a result of torsion field fluxions, its entanglements drives a property of gravitational field to act upon matters or objects, which defines gravitational induction. Gravitational field equations in the dual manifolds consist of six vector differential equations. Among them for five strengths of the gravitational fields are parallel to and accompany with electromagnetism. Inducted by torsion tensors $\{\Gamma^{-\sigma}_{\mu\nu} x_\sigma, \Gamma^{+\sigma}_{\mu\nu} x^\sigma\}$, the $Y^- Y^+$ entanglements produce another set of the reciprocal fields as the general equations of the weak gravitational fields. Their virtual torsion $\langle T \rangle^+_{\mu\alpha}$ conforms with the strength of displacing stress $\mathbf{D}^+_g$ and materializing torsion $\mathbf{H}^+_g$ fields, and their physical torsion $\langle T \rangle^-_{\mu\alpha}$ corresponds to the motion stress $\mathbf{E}^-_g$ and twist torsion $\mathbf{B}^-_g$ fields.

## GRAVITATIONAL EQUATIONS

In summary, the energy attractions exerts a pair of reciprocal actors: $Y^-$ and $Y^+$ entanglements that formulates a set of equations for the stress and torsion field formulation. Under the symmetric fluxions, graviton entanglements at a constant speed $c_g$ characterize gravitational behaviors describable mathematically by *Law of Conservation of Graviton, Graviton Energy*, and *Gravitational Field Tensor*, as a group of the equations:

$$\tilde{E}^\pm_g = \frac{4\pi G}{V_g}\left(\frac{\hbar}{c}\right)^2 \mp i M_g c^2 \quad \text{: Graviton Energy} \quad (15.19)$$

$$\frac{1}{c_g^2}\frac{\partial^2 \Phi^-_g}{\partial t^2} + \nabla^2 \Phi^-_g = \left(\frac{\tilde{E}_g}{\hbar c_g}\right)^2 \quad \text{: Law of Conservation} \quad (15.20)$$

$$\nabla^2 \Phi^-_g = \frac{4\pi}{c_g^2} G \rho_g \quad \text{; Time Independent Fluxion} \quad (15.21)$$

$$\mathbf{F}^- = -m c_g^2 \nabla \Phi^-_g \quad \text{: Newton's Law of Gravity} \quad (15.22)$$

$$\mathbf{g}^+_g = \mathbf{E}^-_g + \mathbf{u} \times \mathbf{B}^-_g \quad \text{: Gravitational Field Tensor} \quad (15.23)$$

$$\nabla \cdot \mathbf{B}^-_g = 0^+ \quad \text{: Conservation of } Y^- \text{ Fluxions} \quad (15.24)$$

$$\nabla \cdot \mathbf{D}^+_g = 4\pi G \rho_g \quad \text{: Conservation of } Y^+ \text{ Fluxions} \quad (15.25)$$

$$\frac{\partial \mathbf{B}^-_g}{\partial t} + \nabla \times \mathbf{E}^-_g = 0 \quad \text{: } Y^- \text{ Continuity Equation} \quad (15.26)$$

$$\frac{\partial \mathbf{D}^+_g}{\partial t} - \frac{c_g^2}{c^2} \nabla \times \mathbf{H}^+_g = -4\pi G \mathbf{J}_g \quad \text{: } Y^+ \text{ Continuity Equation} \quad (15.27)$$

The *Potential Density of Gravitation* $\Phi^-_g$ illustrates the Newton's fields in homogeneous environment for an external observation, appearing as the long range force from massive objects. Apparently, in the centre of a black hole, it appears as if there were a gravitational singularity in an infinitely small space, where masses are converted into virtual dark energies. At the center of gravitation, a set of partial differential equations forms the entanglements of gravitational fields and emerges the short range force from massive objects for internal communications.

Especially, the nature context of torsion tensors $K^\pm_{\mu\alpha} = \Gamma^{\pm s}_{\mu\alpha} x_s$ constitute and act as the sources of "graviton" fields being operated at the heart of energy formulations of stress strengths and twist torsions, which are driven by the events descending from the two-dimensional world lines of the dual manifolds.

## XVI. GENERIC DYNAMIC FIELDS

Under the continuities of symmetric fluxions, motions in the dual manifolds characterize the entangle fields including both *inertial boost* $J^\pm_{\mu\alpha}$ of transformations and *spiral torque* $K^\pm_{\mu\alpha}$ of transportations. Because the linear relationship is embedded in their common potential fields of the equations (3.5) and (3.7), the fields of their combined effects can be derived straightforwardly, similar to the same approach in deriving electromagnetic formulae.

$$\nabla \cdot \mathbf{B}^- = 0^+ \quad : \mathbf{B}^- = \mathbf{B}^-_c + \mathbf{B}^-_g \quad (16.1)$$

$$\nabla \cdot \mathbf{D}^+ = \rho_q + 4\pi G \rho_g \quad : \mathbf{D}^+ = \mathbf{D}^+_c + \mathbf{D}^+_g \quad (16.2)$$

$$\frac{\partial \mathbf{B}^-}{\partial t} + \nabla \times \mathbf{E}^- = 0 \quad : \mathbf{E}^- = \mathbf{E}^-_c + \mathbf{E}^-_g \quad (16.3)$$

$$\frac{\partial \mathbf{D}^+}{\partial t} - \nabla \times \mathbf{H}^+ = -\mathbf{J}_q - 4\pi G \mathbf{J}_g \quad : \mathbf{H}^+ = \mathbf{H}^+_c + \frac{c_g^2}{c^2}\mathbf{H}^+_g \quad (16.4)$$

$$\mathbf{F}^-_c = Q\left(\mathbf{E}^-_c + \mathbf{u} \times \mathbf{B}^-_c\right) \quad \text{: Electromagnetic Force} \quad (16.5)$$

$$\mathbf{F}^-_g = M\left(\mathbf{E}^-_g + \mathbf{u} \times \mathbf{B}^-_g\right) \quad \text{: Gravitational Force} \quad (16.6)$$

where $\rho_q$ or $\rho_g$ is the total charge or mass density of the system, $\mathbf{E}^-$ is the total *Stressing Fields*, and $\mathbf{B}^-$ is the total *Twisting Fields,* $\mathbf{D}^+$ is the total *Displacing Fields,* and $\mathbf{H}^+$ is the total *Materializing Fields*. The lower index c represents the electromagnetic fields while the index g is for the gravitational fields.

These equations are generic to both of electromagnetism and gravitation, which characterize the symmetric motion dynamics. Generally, the fields of gravitation are weaker than electromagnetism in a microscopic or short range, and stronger than electromagnetism in a macroscopic or long range.

## CONCLUSION

From *First Universal Field Equations* of (8.7), (8.8), (8.12) and (8.13) of reference [2], the $Y^- Y^+$ fluxions are operated to give rise to the horizons where a set of density continuities is instituted symmetrically to function as the horizons of *Second Universal Dynamic Equations*, unifying the fields of photon, light, electromagnetism, graviton and gravitation from the potential quantum fields.

More remarkably, the *Law of Conservation of Gravitation* demonstrates, that, besides the constant speed at $c_g$, gravitation is conserved to the symmetric invariance of any boost transformations and twist transportations during its graviton entanglements:

$$\frac{1}{c_g^2}\frac{\partial^2 \Phi^-_g}{\partial t^2} + \nabla^2 \Phi^-_g = \left(\frac{\tilde{E}_g}{\hbar c_g}\right)^2 \quad \text{: Law of Conservation}$$

$$\tilde{E}^\pm_g = \frac{4\pi G}{V_g}\left(\frac{\hbar}{c}\right)^2 \mp i M_g c^2 \quad \text{: Duality of Graviton}$$

$$p_g = 4\pi \frac{G \hbar^2}{c^3 V_g} \quad \text{: Graviton Momentum}$$

Defined as the ***Law of Conservation of Gravitation***, it states that, in any given frame of reference, the effective potential density of gravitation remains constant and conserves over time during its transportation. The potential density can neither be created nor destroyed; rather, it transforms from one form to another between the virtual wave and physical mass objects by its graviton energies of $\tilde{E}^\mp_g$. Besides, their physical interruptions of gravitation obey *Newton's Law* at macroscopic long range for external observation and *Lorentz Force* at microscopic or short range for internal cognition whenever the resource is distinguishable by the characteristics of mass distributions.

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