Conventional Definition of Potential Energy is Controversial

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Abstract-The conventional definition of potential energy leads, considered from a fundamental scientific point of view, to absurd consequences. It is strongly advised to eliminate the intermediate variable 'work' in the definition of potential energy and, as a result, to reverse the boundaries in the definite integral.

Conventional definition of potential energy and its relation with 'work'

Potential energy *can*, in the most general way, be defined as $E_p = \int F(r)\, dr$, with F(r) a force along the path r, of which the boundaries have yet to be defined.
Considering the situation of two masses M and m at a mutual distance r, $F(r)=GMm/r^2$.
In the situation of an electron at distance r from Z protons $F(r)=k_eZq^2/r^2$.
G is the so-called gravitational constant, k_e the so-called Coulomb's constant ($1/4\pi\varepsilon_0$).
The symbol q represents the electrical charge of an electron as well as of one proton.

For ease of reading, F(r) will be written as C/r^2 for both orbital systems.
The result of the indefinite integral is: $E_p = - C/r$. The definite integral is $- C/r\, |_l^u$.
The symbol $|_l^u$ is meant to express that the boundaries l and u have yet to be applied.
Traditionally these boundaries are defined as ($l=\infty$, $u=r$) in order to calculate the *work* to be done <u>in</u>, or the *potential energy* <u>of</u> an orbital system. The result is (also): $E_p = -C/r$.

In Wikipedia at https://en.wikipedia.org/wiki/Potential_energy the negative sign is explained as follows in: "Potential energy for gravitational forces between two bodies":
The negative sign follows the convention that work is gained from a loss of potential energy
The origin of the confusion is found in the explanation, shown in that reference under 'Work and potential energy': the supposed importance of the concept 'work'. Quoted:

"*Potential energy is closely linked with forces. If the work done by a force on a body that moves from A to B does not depend on the path between these points, then the work of this force measured from A assigns a scalar value to every other point in space and defines a scalar potential field. In this case, the force can be defined as the negative of the vector gradient of the potential field.*

If the work for an applied force is independent of the path, then the work done by the force is evaluated at the start and end of the trajectory of the point of application. This means that there is a function U(x), called a "potential," that can be evaluated at the two points x_A and x_B to obtain the work over any trajectory between these two points.
<u>It is tradition to define this function with a negative sign so that positive work is a reduction in the potential</u>, that is $W = \int_C F.dx = U(x_A) - U(x_B)$ where C is the trajectory taken from A to B. Because the work done is independent of the path taken, then this expression is true for any trajectory from A to B.

The function U(x) is called the potential energy associated with the applied force. Examples of forces that have potential energies are gravity and spring forces."

Alternative direct and common sense definition of potential energy

Back to the alternative and direct definition of potential energy $E_p = \int F(r)dr$, the boundaries in the definite integral of $\int C/r^2 dr$ will be chosen ∞ for the upper and r for the lower one. The result then is $E_p = C/r$. Effectively this potential energy is calculated with respect to zero ($E_p(\infty)$), fully in accordance with common sense and thus always positive. So more generally: the upper boundary in $\int C/r^2 dr$ should constitute the reference for the calculation of the potential energy, at the position: 'lower boundary'.

In order to explain further why $E_p = C/r$ is a better result than $E_p = -C/r$, the kinetic energy in an orbital system will be taken into consideration.
The kinetic energy E_k of an object with mass m and velocity v equals $\frac{1}{2} mv^2$.
The property of a circular orbit is that the centrifugal force mv^2/r equals the gravitational respectively Coulomb force. Both forces have, for ease of reading, been taken C, so $mv^2/r = C/r^2$. If $E_p = C/r$ then it follows from $C/r^2 = mv^2/r$ that $E_p = 2*E_k$.

Kinetic energy is by definition positive and can in an orbital system also be represented by $\frac{1}{2}C/r$, just like C/r can represent its potential energy.
This is another reason to define potential energy, at least in such a system, as positive.

The total energy $E_k + E_p$ would be, if E_p would be defined as negative, in such a configuration $-E_k$, given the relation $|E_p| = 2*E_k$.
It is an absurd consequence that in atoms the total energy would be negative!

Energy should always be defined as positive, except in situations where energy levels are compared in order to show, by means of a negative sign, that one level is less positive than the other.

Another absurd consequence of the conventional definition of potential energy is demonstrated in a description found in http://en.wikipedia.org/wiki/Atomic_orbital
The wrong words have been scratched out and the correct words, obeying the alternative definition of potential energy, written behind them in italics.
Orbital energy
In atoms with a single electron ...., the energy of an orbital ...... is determined exclusively by n. The n=1 orbital has the ~~lowest~~ *highest* possible energy in the atom. Each successively higher value of n has a ~~higher~~ *lower* level of energy, but the difference decreases as n increases *. For high n, the level of energy becomes so ~~high~~ *low* that the electron can easily escape from the atom.

* The Rydberg expression is meant here: $E = hf = hc * R_\infty (1/n_1^2 - 1/n_2^2)$.
The example below shows the correctness of the statement: ".... the difference decreases as n increases."

n_1	n_2	$1/n_1^2 - 1/n_2^2$
1	2	0,75
5	6	0,012
10	11	0,0017

N.B. *This correctness emphasizes the incorrectness of the three other statements!*

Other consequences of the alternative definition of potential energy

In the previous chapter it has been argued that negative potential energy should not exist, except to show that a certain energy level is less than the other.

Suppose m moves from r_1 to r_2 with $r_2 > r_1$.The question is: what is the difference in potential energy in both situations?
In order to answer this question, another question arises: which difference is meant: $E_p(r_1) - E_p(r_2)$, or the opposite?

The alternative potential energy is calculated with respect to zero as a positive value if the upper boundary in $\int C/r^2 dr$ is chosen infinite.
Similarly, if ΔE_p is meant to be $E_p(r_2) - E_p(r_1)$ then seemingly $E_p(r_1)$ is the reference, leading to r_1 as upper boundary.
The result is $\Delta E_p = - C(1/r_1 - 1/r_2)$, which indeed is negative, because $E_p(r_2) < E_p(r_1)$.

Special attention has to be paid to the calculation of the "potential energy" of a mass m at height h with respect to earth's surface. The mass of the earth is represented by M, its radius by r_e. The generally accepted answer is "E_p"= mgh, with $g = G.M/r_e^2$.
Such an answer makes sense in only one way: the larger h the higher the kinetic energy of m, after having been released, just before it reaches earth.
However mgh is not a potential energy but a difference between two (kind of absolute) potential energies as shown hereafter.

$$E_{ph} = GMm/(r_e+h)$$

$$E_{p0} = GMm/r_e$$

For $h << r_e$

$$E_{ph} \approx GMm(1/r_e - h/r_e^2).$$

In case ΔE_p is taken as $E_{ph} - E_{p0}$, $\Delta E_p \approx -GMmh/r_e^2 = - mgh$.
In case ΔE_p is taken as $E_{p0} - E_{ph}$, $\Delta E_p \approx +GMmh/r_e^2 =+mgh$.

The choice is up to the "user".

A remarkable fact is that ΔE_p = mgh suggests that potential energy increases when the distance increases. But mind the Δ in the equation!

See also the quoted **Orbital energy** above, where potential energy is described as increasing with an increasing orbital radius!

Conclusion

Based on the above presented argumentations it is strongly recommended to calculate potential energy as $E_p = \int F(r)dr$, by using as upper boundary ∞ and lower boundary r.