On the Sum of the Squares of the Particle Masses

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Abstract

We observe that the sum of the squares of the three known fundamental massive bosons is within 0.4% of the square of the vacuum expectation value divided by 2. It is also well known that the top quark mass squared is also slightly less than this value. We put forth the conjecture that this is not a coincidence, and that these two facts are a result of a general principle for the Standard Model that the sum of the squares of the boson masses and the sum of the squares of the fermion masses actually equals the vacuum expectation value squared divided by two. Furthermore this foreshadows a coming particle desert at TeV energies, as the available reservoir of mass couplings has already been allocated to known particles.

Introduction

Our current theory of fundamental particles and interactions is given by the Standard Model of Particle Physics. Under the Standard Model, particles gain mass from their coupling to the Higgs field. Early in the history of the universe, as it cooled, the symmetry of the potential underlying the Higgs field was spontaneously broken, yielding a non-zero vacuum expectation value (VEV) for the field. In addition this broken symmetry produced a set of four Nambu-Goldstone bosons / degrees of freedom. In the Standard Model three of these degrees of freedom are "eaten" by the weak vector bosons W+, W-, and Z0. The fourth of these appears as a scalar, the Higgs boson.

Of the currently known bosons, only the W+/-, Z0, and Higgs have a non-zero mass, unlike the photon and gluons which are massless. The W and Z boson masses were correctly predicted before their discovery in 1983 from the strength of the electroweak coupling constants and their relationship to the VEV. The Higgs mass, on the other hand, remained a mystery until its experimental discovery in 2012. Accurate measurements of all three massive bosons have now been made. The value of the VEV, derived from muon decay measurements, has long been known to great accuracy.

Calculation

We now calculate that the sum of the squares of the three known fundamental massive bosons is within 0.4% of the square of the vacuum expectation value divided by 2.

v [VEV]	246.22 GeV
v/rt(2)	174.104

We'll divide each mass by the VEV divided by the square root of 2, then square that.

	M (GeV)	r	r^2
W+/-	80.385	0.4617	0.2131
Z0	91.1876	0.5237	0.2743
H	125.09	0.7185	0.5162
Sum			1.0036

It is also well known that the top quark mass squared is also very near this value (vev^2 / 2).
(Note that the ratio of a fermion's mass to the VEV/rt(2) is its Yukawa coupling.)

	M (GeV)	y	y^2
top	173.34	0.9956	0.9912

We observe that top quark on its own gets us to within 0.9% of the target value. Contributions by the other known fermions can account for less than 0.06% of the remainder, however.

The Conjecture

We put forth the conjecture that this is not a coincidence, and that these two facts are a result of a general principle which might exist beyond the Standard Model.

$$\sum_f M_f^2 = \sum_b M_b^2 = \frac{v^2}{2}$$

Conjecture - The sum of the squares of the boson masses and the sum of the squares of the fermion masses equals the vacuum expectation value squared divided by two.

A theoretical motivation as to why this might be true comes from a statistical view of the particle masses / Yukawa couplings. Unitarity demands that amplitudes, when squared, should sum to unity. Perhaps Yukawa couplings can be understood as primordial probability amplitudes of the different modes excited by the Mexican Hat potential.

Conclusion

In the years to come experimental results from the Large Hadron Collider and other experiments will give us a better understanding of what awaits us at higher energies. However, it is quite possible that what we face over the next several decades as we explore the TeV range is a particle desert. If particle mass is only determined by the coupling with the Higgs field, and that these couplings come from a finite pie, that pie may already be accounted for.

References

C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update.
A. Pich, “Electroweak Symmetry Breaking and the Higgs Boson” [arXiv:1512.08749].
A. Pich, “The Standard Model of Electroweak Interactions”, arXiv:1201.0537 [hep-ph].
P. Binétruy, Supersymmetry: Theory, Experiment, and Cosmology, Oxford U Press, New York, 2006.
D. Griffiths, Introduction to Elementary Particles, Wiley-VCH, New York, 2008.
G. Kane, Modern Elementary Particle Physics, Addison-Wesley Publishing, New York, 1987.