《Universal and Unified Field Theory》

3. Fluxions, Gravitation, and Thermodynamics of General Symmetric Fields

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Abstract: As a major part of the unification, the quantum fields give rise to a symmetric environment and bring together *all* field entanglements of the flux conservation and continuity. Remarkably, it reveals the natural secrets of:

1) ***General Symmetric Fields*** - Connect a set of generic fluxions unifying *electromagnetism, gravitation, and thermodynamics*.
2) ***Graviton and Gravitational Fields*** - Declare law of conservation of graviton and compose a duality of torque transportations.
3) ***Thermodynamic and Black Body*** - Integrate horizon factors of thermodynamics with the area entropies of *Black Hole* radiations.

Conclusively, this manuscript presents the unification and compliance with the principal theories of classical and contemporary physics.

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INTRODUCTION

A duality nature of virtual and physical coexistences is a universal phenomenon of dynamic entanglements, which always performs a pair of the reciprocal entities. Each of the states cannot be separated independently of the others. Only together do they form a system as a whole although they may not be bound physically. The potential entanglements are a fundamental principle of the real-life streaming such that one constituent cannot be fully described without considering the other. As a consequence, the state of a composite system is always expressible as a sum of products of states of each constituent. Under the *Law of Event Evolutions* and *Universal Topology* [1], they are fully describable by the mathematical framework of the dual manifolds.

This manuscript further represents both of the boost transformation and twist transportations in the generic forms of equations (3.5) and (3.7). As the functional quantity of an object, a set of the scalar fields forms and projects its potentials to its surrounding space, arising from or acting on its opponent through a duality of reciprocal interactions dominated by both *Inertial Boost* $J^+_{\mu a}$ and *Spiral Torque* $K^+_{\mu a}$ generators.

XI. FLUX CONTINUITY AND COMMUTATION

For the entanglement streams between the manifolds, the ensemble of an event λ is in a mix of states such that each pair of the reciprocal states $\{\phi^+_n, \varphi^-_n\}$ or $\{\phi^-_n, \varphi^+_n\}$ is performed in alignment with an integrity of their probability $p^\pm_n = p_n(h^\pm_n)$, where $h^\pm_n$ are the $Y^\pm$ distributive or horizon factors, respectively. The parameter p^-_n is a statistical function of horizon factor $h^-_n(T)$ and fully characterizable by *Thermodynamics*. Under the event operations, the interoperation among four types of scalar fields of $\phi^\pm_n$ and $\varphi^\pm_n$ correlates and entangles an environment of dual densities $\rho^+_\phi{=}\phi^+_n\varphi^-_n$ and $\rho^-_\phi{=}\phi^-_n\varphi^+_n$ by means of the natural derivatives $\dot\lambda$ to form a pair of fluxions $\langle\dot\lambda\rangle^\mp$:

$$\dot\lambda\rho^-_\phi{=}\left\langle\check\lambda,\hat\lambda\right\rangle^- = \langle\dot\lambda\rangle^- = \sum\nolimits_n p^-_n\left(\varphi^+_n\check\lambda\phi^-_n + \phi^-_n\hat\lambda\varphi^+_n\right) \tag{11.1}$$

$$\dot\lambda\rho^+_\phi{=}\left\langle\hat\lambda,\check\lambda\right\rangle^+ = \langle\dot\lambda\rangle^+ = \sum\nolimits_n p^+_n\left(\varphi^-_n\hat\lambda\phi^+_n + \phi^+_n\check\lambda\varphi^-_n\right) \tag{11.2}$$

where the symbols $\langle\ \rangle^\mp$ are called Y^- or Y^+ *Continuity Bracket*. They represent the dual continuities of the Y^-Y^+ scalar densities, each of which extends its meaning to an anti-commutator $\{\ \}$. Considering another pair of the operational symbols $[\dot\lambda]^\mp$ known as commutators or *Lei Bracket*, introduced in 1930s [2], for respective Y^- or Y^+ supremacy, the reciprocal entanglements of density fields are defined as *Commutator Bracket* $[\ ]^\mp$:

$$[\hat\lambda,\check\lambda]^+ = \sum\nolimits_n p^+_n\left(\varphi^-_n\hat\lambda\phi^+_n - \phi^+_n\check\lambda\varphi^-_n\right) \qquad \langle\dot\lambda\rangle^\pm = \varphi^\mp_n\dot\lambda\phi^\pm_n \tag{11.3}$$

$$[\check\lambda,\hat\lambda]^- = \sum\nolimits_n p^-_n\left(\varphi^+_n\hat\lambda\phi^-_n - \phi^-_n\hat\lambda\varphi^+_n\right) \qquad (\dot\lambda\rangle^\pm = \phi^\pm_n\dot\lambda\varphi^\mp_n \tag{11.4}$$

where, in addition, the symbols $\langle\ )^\pm$ and $(\ \rangle^\pm$ are called Y^- or Y^+ *Asymmetry Brackets*. They are essential to cosmological dynamics.

Similarly, a set of the reciprocal vector fields of $V^\pm_m = -\partial\phi^\pm_m$ $\Lambda^\pm_\mu \equiv -\partial\varphi^\pm_\mu$, has the brackets of Y^- or Y^+ continuity and commutation:

$$\left\langle\hat\lambda,\check\lambda\right\rangle^\pm_v \equiv \sum\nolimits_n p^\pm_n\left(\varphi^\mp_n\hat\lambda V^\pm_n + \phi^\pm_n\check\lambda\Lambda^\mp_n\right) \qquad \langle\dot\lambda)^\pm_v = \varphi^\mp_n\dot\lambda V^\pm_n \tag{11.5}$$

$$\left[\hat\lambda,\check\lambda\right]^\mp_v \equiv \sum\nolimits_n p^\mp_n\left(\varphi^\pm_n\hat\lambda V^\mp_n - \phi^\mp_n\check\lambda\Lambda^\pm_n\right) \qquad (\dot\lambda\rangle^\pm_v = \phi^\pm_n\dot\lambda\Lambda^\mp_n \tag{11.6}$$

where the index n is corresponds to each type of particle, and v indicates entanglements of vector potentials, which respectively give rise to or balance each other's horizon environment.

A measure of the specific operations of ways is called entropy in which states of a universe system could be arranged and balanced towards its equilibrium. As an operational duality, the entropy tends towards both extrema alternately to maintain a continuity of energy conservations, operated by each of the opponents. When a total entropy decreases, the intrinsic order, or Y^- development, of virtual into physical regime $\hat\partial_\lambda\hat\partial_\lambda$ is more dominant than the reverse process. This philosophy states that for the central quantity of *Lagrangian,* conversely, when a total entropy increases, the extrinsic disorder, or Y^+ annihilation $\check\partial^\lambda\check\partial^\lambda$, becomes dominant and conceals physical resources into virtual regime.

Artifact 11.1: Entropy of Fluxions. There are two types of thermodynamic entropies. Each yields its own laws respectively. In *Classic Thermodynamics*, during the physical observations, the "internal" or "physical" operations result in its local effects parallel to the global domain with the entropy events of physical supremacy PdV, functions of the special transportation $\mu_n dN^\pm_n$, and states of symmetric property dE/T. In *Cosmological Thermodynamics*, as the virtual generators, "external" or "virtual" operations result in the projection or transform from its local effects to the neighbor domain with the events of virtual supremacy $\Omega\,dJ$, functions of general commutations $\kappa\,dA/(8\pi)$, and states of asymmetric property $\Phi\,dQ$, where κ is the surface gravity, A the horizon area, Ω the angular velocity, J the angular momentum, Φ the electrostatic potential, and Q the electric charge. For an observation at long range, the commutation becomes a conservation of Y^-Y^+ thermodynamics, or is known as black hole radiations, which yields law of the area fluxions. The total entropy $\mathcal{S}^\pm$ represent law of conservation of area fluxions and defined by the following expression:

$$\mathcal{S}^\pm = 4\mathcal{S}^\pm_A = \dot\partial_\lambda\mathbf{f}_s \qquad :\dot\partial_\lambda\mathbf{f}^\pm_s = \kappa_s\left\langle\hat\partial_\lambda\hat\partial_\lambda,\check\partial^\lambda\check\partial^\lambda\right\rangle^\pm \tag{11.7}$$

where κ_s is factored by normalization of the potential fields. For a triplet quark system, the total entropy is at $2\phi^+_a(\phi^-_b + \phi^-_c) \approx 4\phi^+_a\phi^-_b$ fluxions.

Artifact 11.2: General Field Commutations. As an integral set of the tensors, both boost and torque elements can be provisioned in forms of fluxions of commutation fields:

$$[\mathcal{F}_{\mu\alpha}]^\mp = [\dot x^\alpha\mathcal{F}^-_{\mu\alpha}\partial_\mu, \dot x_\alpha\mathcal{F}^+_{\mu\alpha}\partial^\mu]^\mp = [F_{\mu\alpha}]^\mp + [T_{\mu\alpha}]^\mp \tag{11.8}$$

$$[F_{\mu\alpha}]^\mp = \pm\,[\dot x^\alpha J^-_{\mu\alpha}\partial_\mu, \dot x_\alpha J^+_{\mu\alpha}\partial^\mu]^\mp \tag{11.9}$$

$$[T_{\mu\alpha}]^\mp = \pm\,[\dot x^\alpha K^-_{\mu\alpha}\partial_\mu, \dot x_\alpha K^+_{\mu\alpha}\partial^\mu]^\mp \tag{11.10}$$

As a consequence, the fluxion framework institutes an environment where a pair of commutators compels or exerts two pairs of the dual fields as a foundational structure to its surrounding area, giving rise to the four fields of the virtual $[\mathcal{F}_{\mu\alpha}]^+$ and physical $[\mathcal{F}_{\mu\alpha}]^-$ dynamics.

Together, the *Transform Tensors* provoke *Electromagnetism* and the *Torsion Tensors* generate *Gravitation*, cohesively and simultaneously.

XII. SECOND UNIVERSAL DYNAMIC EQUATIONS

Symmetry is the law of natural conservations that a system is preserved or remains unchanged or invariant under some transformations or transportations. As a duality, there is always a pair of intrinsics reciprocal conjugation: Y^-Y^+ symmetry. The basic principles of symmetry and anti-symmetry are as the following:

1) Associated with its opponent potentials of either scalar or vector fields, symmetry is a fluxion system cohesively and completely balanced such that it is invariant among all composite fields.
2) As a duality, an Y^-Y^+ anti-symmetry is a reciprocal component of its symmetric system to which it has a mirroring similarity physically and can annihilate into nonexistence virtually.
3) Without a pair of Y^-Y^+ objects, no symmetry can be delivered to its surroundings consistently and perpetually sustainable as resources to a life streaming of entanglements at zero net momentum.
4) Both Y^-Y^+ symmetries preserve the laws of conservation consistently and distinctively, which orchestrate their local continuity respectively and harmonize each other dynamically.

In mathematics, *World Equations* of (5.7) can be written in terms of the scalar, vector, and higher orders tensors, shown as the following:

$$W_b = W_0^{\pm} + \sum_n h_n \left\{ \kappa_1 \left\langle \dot{\partial}_\lambda \right\rangle^{\pm} + \kappa_2 \dot{\partial}_{\lambda_2} \left\langle \dot{\partial}_{\lambda 1} \right\rangle_s^{\pm} + \kappa_3 \dot{\partial}_{\lambda_3} \left\langle \dot{\partial}_{\lambda 2} \right\rangle_v^{\pm} \cdots \right\} \tag{12.1}$$

where κ_n is the coefficient of each order n of the event $\lambda^n = \lambda_1 \lambda_2 \cdots \lambda_n$ aggregation. The above equations are constituted by the scalar fields: $\phi^{\pm}$ and $\varphi^{\mp}$ at the first horizon (index s), their tangent vector fields at the second horizon (index v), and their tensor fields at higher horizons.

Add φ_n^- times (6.7) and ϕ_n^+ times (6.13), we constitute a density continuity of the Y^+ fluxion in forms of symmetric formulation:

$$\boxed{\dot{\partial}_\lambda \mathbf{f}_s^+ = \left\langle W_0 \right\rangle^+ - \left[\left(\kappa_1 - \kappa_2 \check{\partial}_{\lambda_3} \right) \left(\check{\partial}^{\lambda_2} - \hat{\partial}_{\lambda_2} \right) \right]^+ + \kappa_2 \zeta^+ \qquad \mathbf{(12.2)}}$$

$$\kappa_1 = \frac{\hbar c^2}{2}, \quad \kappa_2 = -\frac{(\hbar c)^2}{2E^+}, \quad \zeta^+ = \left(\hat{\partial}_{\lambda_2} \check{\partial}^{\lambda_2} - \check{\partial}_{\lambda_2} \hat{\partial}_{\lambda_2} \right\rangle^+ \tag{12.3}$$

where the entangle bracket $\left\langle \hat{\partial}_\lambda \hat{\partial}_\lambda, \check{\partial}^\lambda \check{\partial}^\lambda \right\rangle^+ = \dot{\partial}_\lambda \mathbf{f}_s^+$ features the Y^+ ***Transforming Continuity***. As one set of the universal laws, the events incepted in the virtual world not only generate its opponent reactions but also create the real-life objects in the physical world.

In a parallel fashion, there is another dual state fields $\{\phi_n^-, \varphi_n^+\}$ in the dynamic equilibrium, given by $\varphi_n^+ E_n^-$ times (6.12) and $\phi_n^- E_n^+$ times (6.8). Adding the two formulae, we institute Y^- fluxion of density continuity $\dot{\partial}_\lambda \mathbf{f}_s^- = \kappa_2 \left\langle \check{\partial}_\lambda \check{\partial}_\lambda, \hat{\partial}^\lambda \hat{\partial}^\lambda \right\rangle^-$ of symmetric dynamics:

$$\boxed{\dot{\partial}_\lambda \mathbf{f}_s^- = \left\langle W_0 \right\rangle^- + \kappa_1 \left[\check{\partial}_{\lambda_1} - \hat{\partial}^{\lambda_1} \right]^- + \kappa_2 \left\langle \left(\hat{\partial}^{\lambda_2} - \check{\partial}^{\lambda_2} \right) \check{\partial}_{\lambda_1} \right\rangle^- + \kappa_2 \zeta^- \qquad \mathbf{(12.4)}}$$

$$\kappa_1 = \frac{\hbar c^2}{2}, \quad \kappa_2 = \frac{(\hbar c)^2}{2E^-}, \quad \zeta^- = \left(\hat{\partial}^{\lambda_1} \check{\partial}^{\lambda_2} - \hat{\partial}^{\lambda_2} \check{\partial}_{\lambda_1} \right\rangle^- \tag{12.5}$$

The entangle bracket $\left\langle \check{\partial}_\lambda \check{\partial}_\lambda, \hat{\partial}^\lambda \hat{\partial}^\lambda \right\rangle^-$ of the Y^- general dynamics features the Y^- ***Mirroring Continuity***. As another set of the laws, the events initiated in the physical world have to leave a life copy of its mirrored images in the virtual world without an intrusive effect in the virtual world. In other words, the virtual world is aware of and immune to the physical world.

Artifact 12.1: Acceleration Tensors. Under the Y^+ environment, it contains the Y^- effective energy $\tilde{E}_c^-$ of a physical object. This continuity equation of the Y^+ fluxion $\dot{\partial}_\lambda \mathbf{f}^+$ gives rise to the next horizon of acceleration tensor $\mathbf{g}^+ = \dot{\partial}_\lambda \mathbf{f}^+ / (\hbar c)$ for dynamics and interactions balancing the physical forces.:

$$\mathbf{g}^+ = \frac{1}{\hbar c} \dot{\partial}_\lambda \mathbf{f}^+ = \frac{\hbar c}{2E^+} \left[\left(\frac{E^-}{\hbar} + \check{\partial}_\lambda \right) \left(\hat{\partial}_\lambda - \check{\partial}^\lambda \right) \right]^+ + \frac{\hbar c}{2E^+} \zeta^+ \tag{12.6}$$

$$\mathbf{g}^- = \frac{1}{\hbar c} \dot{\partial}_\lambda \mathbf{f}^- = \frac{c}{2} \left[\check{\partial}_\lambda - \hat{\partial}^\lambda \right]^- + \frac{\hbar c}{2E^-} \left\langle \check{\partial}_\lambda \left(\hat{\partial}^\lambda - \check{\partial}^\lambda \right) \right\rangle^- + \frac{\hbar c}{2E^-} \zeta^- \tag{12.7}$$

where the static fluxions are balanced $\left\langle W_0^{\pm} \right\rangle = 0$. It represents a duality of the Y^-Y^+ bi-directional entanglements. Because the virtual resources are massless and appear as if it were nothing or at zero resources 0^+, the Y^- supremacy of flux continuity equation is given by $\mathbf{g}^- = 0^+$.

Since a pair of the equations (12.2) and (12.4) is generic or universal, it is called ***Second Universal Dynamic Equations***, representing the conservations of symmetric $\zeta^{\pm} = 0$ dynamics, and of asymmetric $\zeta^{\pm} \neq 0$ motions. As a precise duality, the asymmetry coexists with symmetric continuity to extend discrete subgroups, and exhibits additional dynamics to operate world-line motions and to carry on the symmetric system as a whole. Throughout the rest of this manuscript, the fluxions satisfy the "local" conditions of Y^-Y^+ *Symmetric Entanglements*, $\zeta^{\pm} = 0$, known as a system without asymmetric entanglements or symmetric breaking that does not have additional flex transportation asynchronously.

XIII. GENERAL SYMMETRIC FIELDS

For the symmetric fluxions, the entangling invariance requires that their fluxions are either conserved at zero net momentum or maintained by energy resource. Normally, the divergence of Y^- fluxion is conserved by the virtual forces 0^+ and the divergence of Y^+ fluxion is balanced by physical resources. Together, they maintain each other's conservations and continuities cohesively and complementarily.

Under physical primacy, the Y^- fluxion generates acceleration tensor $\mathbf{g}_\times^-$ and represents the time divergence of the forces acting on the opponent objects. This divergence, $\check{\partial}_{\lambda=t} = \left(ic\partial_\kappa \;\; \mathbf{u}^- \nabla \right)$, is at the *Two-Dimensional* world plane acting on the 2x2 tensors. Substituting the equations (11.8) into (12.7), we have the matrix formula in a vector formulation for the off-diagonal fields:

$$\mathbf{g}_x^- = \frac{c}{2} \check{\partial}_\lambda \left\langle \check{\mathscr{F}}_{\mu a} \right\rangle_\times^- = \frac{c}{2} \left(ic\partial_\kappa \;\; \mathbf{u}^- \nabla \right) \begin{pmatrix} 0 & \mathbf{B}^- \\ -\mathbf{B}^- & \frac{\check{\mathbf{b}}}{c} \times \mathbf{E}^- \end{pmatrix} \tag{13.1}$$

$$\mathbf{B}^- = \mathbf{B}_q^- + \mathbf{B}_g^- \qquad \mathbf{E}^- = \mathbf{E}_q^- + \mathbf{E}_g^- \tag{13.2}$$

where $\mathbf{u}_q$ is speed of a charged object, and $\mathbf{u}_g$ is speed of a gravitational mass. Given by the Y^- *Transform* (7.9) and *Transport* (7.15) *Tensors*, the $\mathbf{E}_q^-$ and $\mathbf{E}_g^-$ are the *Electric* and *Torsion Strength* fields, and the $\mathbf{B}_q^-$ and $\mathbf{B}_g^-$ are the *Magnetic* and *Twist* fields.

In a parallel fashion, the Y^+ fluxion (12.6) generates acceleration tensor $\mathbf{g}_\times^+$ under virtual primacy for the off-diagonal $\{\ \}_\times$ elements.

$$\mathbf{g}_\times^+ = -c \left(\frac{E^+}{\hbar} + \check{\partial}_\lambda \right) \begin{pmatrix} 0 & \mathbf{D}^+ \\ -\mathbf{D}^+ & \frac{\mathbf{u}_q}{c^2} \times \mathbf{H}_q^+ + \frac{\mathbf{u}_g}{c^2} \times \mathbf{H}_g^+ \end{pmatrix} \tag{13.3}$$

$$\mathbf{D}^- = \mathbf{D}_q^- + \mathbf{D}_g^- \qquad : \check{\partial}_{\lambda=t} = \left(ic\partial_\kappa \;\; \mathbf{u}^- \nabla \right) \tag{13.4}$$

Given by the Y^+ *Transform* and *Transport Tensors* of (7.17), the $\mathbf{D}_q^+$ and $\mathbf{D}_g^+$ are the *Electric* and *Torsion Displacing* fields, and the $\mathbf{H}_q^+$ and $\mathbf{H}_g^+$ are the *Magnetic* and *Twist Polarizing* fields.

Apparently, the first field of equation (13.3) has a force that aggregates *Dirac* matrixes and gives rise to the second field of the next horizon. Projecting on the two-dimensional world-planes, it emerges and acts as the micro forces on objects.

$$\mathbf{F}^+ = -\kappa_x^+ \mathbf{g}_\times^+ = \kappa_x^+ \frac{cE^+}{\hbar} \begin{pmatrix} 0 & \mathbf{D}^+ \\ -\mathbf{D}^+ & \frac{\mathbf{u}_q}{c^2} \times \mathbf{H}_q^+ + \frac{\mathbf{u}_g}{c^2} \times \mathbf{H}_g^+ \end{pmatrix} \tag{13.5}$$

With charges or masses, this force is projecting to and imposed on the physical lines of the world planes with the following expressions:

$$\mathbf{F}_q^+ = Q\mu_e \left(c^2 \mathbf{D}_q^+ + \mathbf{u}_q \times \mathbf{H}_q^+ \right) \quad : \kappa_q^+ = \frac{Qc\mu_e \hbar}{E^+}, \; c_q^2 = \frac{1}{\varepsilon_q \mu_q} \tag{13.6}$$

$$\mathbf{F}_g^+ = M\mu_g\left(c_g^2\mathbf{D}_g^+ + \mathbf{u}_g \times \mathbf{H}_g^+\right) \qquad : \kappa_g^+ = Mc\mu_g,\ c_g^2 = \frac{1}{\varepsilon_g\mu_g} \tag{13.7}$$

where Q is a charge, M is a mass, ε_q or ε_g is the permittivity, μ_q or μ_g is the permeability of the materials.

Artifacts 12.1: Lorentz Force. In a free space, the constitutive relation (13.6) results in a summation of electric and magnetic forces:

$$\mathbf{F}_q = Q\left(\mathbf{E}_q^- + \mathbf{u}_q \times \mathbf{B}_q^-\right) \qquad : \mathbf{D}_q^+ = \varepsilon_e\mathbf{E}_q^- \quad \mathbf{B}_q^- = \mu_e\mathbf{H}_q^+ \tag{13.8}$$

known as *Lorentz Force*, discovered in 1889. Apparently, the forces are aggregated from or developed by *Dirac Spin Generators*.

Artifacts 12.2: Resources. Balanced by the physical sources of the macroscopic fluxion density $\rho_\sigma\mathbf{u}_\sigma$ and current density $\mathbf{J}_\sigma$, the Y^+ continuity institutes a general expression of local conservations:

$$\mathbf{g}_\times^+ = K_q^- - K_g^- \quad : K_q^- = \left\{\mathbf{u}_q\rho_q, \mathbf{J}_q\right\}, K_g^- = 4\pi G\left\{\mathbf{u}_g\rho_g, \mathbf{J}_g\right\} \tag{13.9}$$

$$\mathbf{g}_\times^- \mapsto 0^+ \tag{13.10}$$

where the $\mathbf{u}_q$ is a negative charged object and $\mathbf{u}_g$ appears moving in an opposite direction, and G is Newton's gravitational constant. For the first equation (13.9), the total source might comprise multiple components $K^- = \sum_n p_n^-\left\{\mathbf{u}\rho_n^-, \mathbf{j}_n^-\right\} \propto \left\{\mathbf{u}\rho, \mathbf{J}\right\}$ to include the horizon forces, thermodynamics, as well as other asymmetric suppliers ζ^+. Likewise, since the virtual sources are massless, it yields the local conservations such that the equation (13.10) has a continuity balancing at its virtual source or massless zero 0^+.

Artifacts 12.3: Y^- Symmetric Fields. Sourced by the virtual time operation $\lambda = t$, the dark fluxion of Y^- boost fields has the conservation equation: $\check{\partial}_\lambda\mathbf{f}_\times^- = 0$. Therefore, the equation (13.1) is equivalent to a pair of the expressions:

$$\left(\mathbf{u}_q\nabla\right)\cdot\mathbf{B}_q^- + \left(\mathbf{u}_g\nabla\right)\cdot\mathbf{B}_g^- = 0 \tag{13.11}$$

$$\frac{\partial}{\partial t}\left(\mathbf{B}_q^- + \frac{c_g}{c}\mathbf{B}_g^-\right) + \left(\frac{\mathbf{u}_q}{c}\nabla\right)\times\mathbf{E}_q^- + \left(\frac{\mathbf{u}_g}{c}\nabla\right)\times\mathbf{E}_g^- = 0 \tag{13.12}$$

It represents the cohesive equations of gravitational and electromagnetic fields under the Y^- symmetric dynamics.

Artifacts 12.4: Y^+ Symmetric Fields. Continuing to operate through the time events $\lambda = t$, sustained by the resources (13.9), the second field $\check{\partial}_{\lambda=t}$ of (13.3) evolves and gives rise to the fields for next horizon, shown by the Y^+ field relationships:

$$\mathbf{u}_q\nabla\cdot\mathbf{D}_q^+ + \mathbf{u}_g\nabla\cdot\mathbf{D}_g^+ = \mathbf{u}_q\rho_q - 4\pi G\mathbf{u}_g\rho_g \tag{13.13}$$

$$\begin{aligned}&\frac{\mathbf{u}_q\cdot\mathbf{u}_q}{c^2}\nabla\times\mathbf{H}_q^+ + \frac{\mathbf{u}_g\cdot\mathbf{u}_g}{c^2}\nabla\times\mathbf{H}_g^+ - \frac{\partial\mathbf{D}_q^+}{\partial t} - \left(\frac{c_g}{c}\right)^2\frac{\partial\mathbf{D}_g^+}{\partial t} \\ &= \mathbf{J}_q - 4\pi G\mathbf{J}_g + \mathbf{H}_q^+\cdot\left(\frac{\mathbf{u}_q}{c}\nabla\right)\times\frac{\mathbf{u}_q}{c} + \mathbf{H}_g^+\cdot\left(\frac{\mathbf{u}_g}{c}\nabla\right)\times\frac{\mathbf{u}_g}{c}\end{aligned} \tag{13.14}$$

where the formula, $\nabla\cdot(\mathbf{u}\times\mathbf{H}) = \mathbf{H}\cdot(\nabla\times\mathbf{u}) - \mathbf{u}\cdot(\nabla\times\mathbf{H})$, is applied.

Artifacts 12.5: General Symmetric Fields. At the constant speeds, a set of the formulations (13.11)-(13.14) is further simplified to and collected as:

$$\nabla\cdot\mathbf{B}_s^- = 0^+ \qquad : \eta = c_g/c,\ \mathbf{B}_s^- = \mathbf{B}_q^- + \eta\mathbf{B}_g^- \tag{13.15}$$

$$\nabla\cdot\mathbf{D}_s^+ = \rho_q - 4\pi G\eta\rho_g \qquad : \mathbf{D}_s^+ = \mathbf{D}_q^+ + \eta\mathbf{D}_g^+ \tag{13.16}$$

$$\nabla\times\mathbf{E}_s^- + \frac{\partial\mathbf{B}_s^-}{\partial t} = 0^+ \qquad : \mathbf{E}_s^- = \mathbf{E}_q^- + \eta\mathbf{E}_g^- \tag{13.17}$$

$$\nabla\times\left(\mathbf{H}_q^+ + \eta^2\mathbf{H}_g^+\right) - \frac{\partial}{\partial t}\left(\mathbf{D}_q^+ + \eta^2\mathbf{D}_g^+\right) = \mathbf{J}_q - 4\pi G\mathbf{J}_g \tag{13.18}$$

Because gravitational tensors are in proportion to *Torques* $K_{m\alpha}^\pm = \Gamma_{m\alpha}^{\pm s}x_s$, *Gravitational* fields at short range might appear weak when the charge fields are dominant by electrons. Vise versa, at long range, electromagnetic fields become weak while gravitational fields can be significant if an object is massive. For any charged objects, both fields are hardly separable although their intensive effects can be weighted differently by the range of distance and quantity of charges and masses.

Artifacts 12.6: General Symmetric Conservation. Considering a homogeneous environment and following the methodological approach in deriving the formula (8.21), we institute a *continuity field* for trace of the diagonal elements of the density $\Phi = \varphi^+\phi^-$. Travelling on the two-dimensional word planes $\{\mathbf{r} \pm i\,\mathbf{k}\}$, the trace of diagonal elements formulates its density fluxion Φ scaling as an area wave-function:

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \nabla^2\Phi = 4\frac{E^-E^+}{(\hbar c)^2} \qquad : \Phi = \Phi_c + \gamma^2\Phi_g \tag{13.19}$$

Introduced at 20:00 August 19 of 2017, it illustrates that it is the intrinsic radiance of its potential elements that are entangling and transforming between physical and virtual instances. The potential density Φ transports as the waves, conserves to the constant energies, carries the potential information, maintains its continuity states of the area density, and essentially bounds on an area entropy S_A in radiance possessing two-degrees of freedom and propagating long-range of energy fluxions.

XIV. GRAVITON AND GRAVITATION

Graviton represents a torque duality between the virtual and the physical energies of manifolds. Not only has this model accounted for the mass volume independence of gravitational energy and explained the ability of matter and graviton radiation to be in thermal equilibrium, but also reveals anomalies in thermodynamics, including the properties of black-body gravitational radiance.

Artifacts 13.1: Gravitational Fields. For the charge neutral objects $E^-E^+ \mapsto \eta^2E_g^-E_g^+$, the equations (13.15)-(13.18) become a group of the pure *Gravitational Fields*, shown straightforwardly by:

$$\left(\mathbf{u}_g\nabla\right)\cdot\mathbf{B}_g^- = 0 \tag{14.1}$$

$$\mathbf{u}_g\nabla\cdot\mathbf{D}_g^+ = -4\pi G\mathbf{u}_g\rho_g \tag{14.2}$$

$$\frac{\partial}{\partial t}\mathbf{B}_g^- + \left(\frac{\mathbf{u}_g}{c_g}\nabla\right)\times\mathbf{E}_g^- = 0 \tag{14.3}$$

$$\frac{\mathbf{u}_g\cdot\mathbf{u}_g}{c^2}\nabla\times\mathbf{H}_g^+ - \left(\frac{c_g}{c}\right)^2\frac{\partial\mathbf{D}_g^+}{\partial t} = -4\pi G\mathbf{J}_g + \mathbf{H}_g^+\cdot\left(\frac{\mathbf{u}_g}{c}\nabla\right)\times\frac{\mathbf{u}_g}{c} \tag{14.4}$$

These equations coincide with *Lorentz invariant Theory* of gravitation [5], introduced in 1893.

Artifacts 13.2: Gravitational Force. Following the same methodology, the *Torsion* forces emerges as gravitation given by the off-diagonal elements of Y^+ dark fluxions of the symmetric system.

$$\mathbf{F}_g = M\mu_g\left(c_g^2\mathbf{D}_g^+ + \mathbf{u}_g\times\mathbf{H}_g^+\right) = M\left(\mathbf{E}_g^- + \mathbf{u}_g\times\mathbf{B}_g^-\right) \tag{14.5}$$

where $c^2 = 1/(\varepsilon_g\mu_g)$, ε_g is gravitational permittivity and μ_g gravitational permeability of the materials.

Artifacts 13.3: Newton's Law. For the charge neutral objects, the equations (14.2) become straightforwardly as:

$$\nabla^2\phi_g = 4\pi G\rho_g \qquad : \mathbf{D}_g^+ = -\nabla\phi_g \tag{14.6}$$

$$\mathbf{F}^- = -m\nabla\phi_g = -mG\rho_g\frac{\mathbf{r}}{r^2} \tag{14.7}$$

known as *Newton's Law* of Gravity for a homogeneous environment where, external to an observer, source of the fields appears as a point object and has the uniform property at every point without irregularities in field strength and direction, regardless of how the source itself is constituted with or without its internal or surface twisting torsions.

Artifact 13.4: Continuity of Gravitation. Conserved to invariance of the *Torque Transportations*, the equation (13.19) at the index g only illustrates conservation of gravitational potentials:

$$-\frac{1}{c_g^2}\frac{\partial^2\Phi_g}{\partial t^2} + \nabla^2\Phi_g = 4\frac{E_g^-E_g^+}{(\hbar c_g)^2} \tag{14.8}$$

Similar to the energy equation $E_c^\mp$ at the virtual space for a photon, a graviton exhibits wave–particle duality and transports under the Y^-Y^+ entanglements. Unlike a photon, a part of the total energy of a pair of gravitons is mediated by *Spiral Torque* $\Gamma_{\mu\alpha}^{+\sigma}x_\sigma$ as well as conserved under an area density. It might behave like a particle with definite and finite measurable position or momentum, though not both at the same time. As an area density streaming, graviton waves may be refracted by an object or interfered with themselves.

Artifacts 13.5: Graviton. Gravitation exhibits wave–particle duality such that its properties must acquire characteristics of both waves and particles. Integrating with the black hole thermal radiance, gravitational fluxion (14.8) has the transportable commutation of area entropy S_A and conservable radiations of a Schwarzshild black body. It is equivalent to associate it with *Bekenstein-Hawking* (16.2) radiation.

$$S_A(\omega_g, T) = 4\left(\frac{c_g^3}{4\hbar G}\right) = 4\frac{E_g^- E_g^+}{(\hbar c_g)^2} \quad \to \quad E_g^{\pm} = \mp \frac{i}{2}\sqrt{\hbar c_g^5 / G} \quad (14.9)$$

where the number 4 is factored or normalized for a dual-state system of triplet quarks similar to (8.23). Consequently, the gravitational energies $E_g^{\pm}$ contain not only a duality of the complex functions but also an irreducible unit: ***Graviton***, introduced at 21:30 November 25 of 2017, as a pair of graviton units:

$$E_g^{\pm} = \mp i \frac{1}{2} E_p \qquad : E_p = \sqrt{\hbar c_g^5 / G} \qquad \mathbf{(14.10)}$$

where E_p is *Plank* energy. It exhibits a coupling constant 1/2 to emit gravitational radiations, meaning a minimum of a pair of gravitons for a black hole emanations.

Artifacts 13.6: Energy–momentum of Graviton. The complex form of the graviton energy derives the gravitational momentum p_g in the following expression:

$$p_g = \frac{1}{2}\sqrt{\hbar c_g^3 / G} \qquad : \mathbf{P} = i c_g \mathbf{p}_g \qquad (14.11)$$

In the center of entanglement, the colliding duality has no net momentum, whereas gravitons always have the source temperature. Conservation of momentum, or equivalently transportation invariance, requires that at least two gravitons are created for entanglements transporting at their zero net momentum.

Artifact 13.7: Law of Conservation of Gravitation. The above equations (14.1)-(14.11) represents the characteristics of gravitation:

Law of Conservation of Gravitation

1) *Gravitation remains constant and conserves over time during its transportation.*
2) *Gravitation transports in wave formation virtually and acts on objects physically.*
3) *A gravitation energy of potential density neither can be created nor destroyed.*
4) *Gravitation consists of an energy duality as an irreducible complex graviton.*
5) *Gravitation has at least two gravitons for entanglement at zero net momentum.*
6) *Gravitation is operated by Torque transportations and the potential messages.*
7) *Without an energy supply, no gravitation can be delivered to its surroundings.*
8) *The net flexion across a region is sunk to or drawn from physical resources.*
9) *External to objects, gravity is inversely proportional to a square of the distance.*

XV. THERMODYNAMICS

During the formation of the horizons, movements of macro objects undergo interactions with and are propagated by the Y^+ fields, while events of motion objects are characterized by the Y^- dynamics. Under the formations of the ground horizons, the Y^-Y^+ dynamics of the symmetric system aggregates timestate objects to develop thermodynamics related to bulk energies, statistical works, and interactive forces towards the second horizon of macroscopic variables for processes and operations characterized as a macro system, associated with the rising temperature.

For a bulk system of N particles, each is in one of three possible states: Y^- |-⟩, Y^+ |+⟩, and neutral |o⟩ with the energy of these states given as E_n^-, E_n^+ and E_n^o, respectively. If the bulk system has $N_n^{\pm}$ particles at non-zero charges and $N^o = N - N_n^{\pm}$ neutrons at neutral charge, the interruptible internal energy of the system is $E_n = N_n^{\pm} E_n^{\pm}$. The number of states $\Omega(E_n)$ of the total system of energy E_n is the number of ways to pick $N_n^{\pm}$ particles from a total of N,

$$\Omega(E) = \prod \Omega(E_n) = \prod \frac{N!}{N_n^{\pm}!(N - N_n^{\pm})!} \qquad : N_n^{\pm} = \frac{E_n}{|E_n^{\pm}|} \qquad (15.1)$$

and the entropy, a measure of state probability, is given by

$$S(E) = \sum_n S(E_n) = -k_B \sum_n \log \frac{N!}{(N_n^{\pm})!(N - N_n^{\pm})!} \qquad (15.2)$$

where k_B is ***Boltzmann*** constant [7]. For large N, there is an accurate approximation to the factorials as the following:

$$log\,(N!) = N\,\log(N) - N + \frac{1}{2}\log(2\pi N) + \Re(1/N) \qquad (15.3)$$

known as the *Stirling's* formula, introduced 1730s [6]. Therefore, the entropy is simplified to:

$$S(N_n^{\pm}) = -k_B N \left[\left(1 - \frac{N_n^{\pm}}{N}\right)\log\left(1 - \frac{N_n^{\pm}}{N}\right) + \frac{N_n^{\pm}}{N}\log\left(\frac{N_n^{\pm}}{N}\right)\right] \qquad (15.4)$$

Generally, one of the characteristics of a bulk system can be presented and measured completely by the thermal statistics of energy $k_B T$ such as a scalar function of the formless entropy shown by Eq. (15.2). In a bulk system with intractable energy $E_n^{\pm}$, its temperature can be risen by its entropy as the following:

$$\frac{1}{T} = \sum_n \frac{\partial S_n}{\partial E_n} = \sum_n \frac{\mp i k_B}{E_n^{\pm}} \log\left(\frac{N E_n^{\pm}}{E_n} - 1\right) \qquad : k_B T \in (0, \pm i E_n^{\pm}) \quad (15.5)$$

With a bulk system of n particles, both energies $E_n^{\pm}(T)$ and horizon factor $h_n(T)$ are temperature-dependent.

$$E_n = N E_n^{\pm} h_n = \frac{N E_n^{\pm}}{e^{\pm i E_n^{\pm}/k_B T} + 1} = k_B T N_n^{\pm} \log\left(\frac{N}{N_n^{\pm}} - 1\right) \qquad (15.6)$$

Apparently, the horizon factor gives rise to and emerges as the temperature T of a bulk system.

Artifacts 14.1: Horizon Factor. During processes that give rise to the bulk horizon, the temperature emerges in forms of energy between zero and $k_B T \simeq E_n^{\pm}$, reproducing the particle n balanced at its population $N_n^{\pm}$. Remarkably, the horizon factor is simplified to:

$$h_n^{\pm} = \frac{N_n^{\pm}}{N} = \frac{1}{e^{\pm \beta E_n^{\pm}} + 1} \qquad : \beta = \frac{i}{k_B T} \qquad (15.7)$$

where i presents that the temperature $k_B T$ is a virtual character similar to the virtual time dimension ict.

Artifacts 14.2: State Probability. Fundamental to the statistical mechanics, we recall that all accessible energy states are equally likely. This means the probability that the system sits in state |n⟩ is just the ratio of this number of states to the total number of states, emerged and reflected in the above equations at the state probabilities, $p_n^{\pm} = p_n(h_n^{\pm})$, to form the macroscopic density and to support the equations of (11.1)-(11.6) by the following expression:

$$p_n^{\pm} = \frac{h_n^{\pm}}{\sum h_\nu} = \frac{e^{\pm \beta E_n^{\pm}}}{Z}, \qquad Z \equiv \sum_\nu e^{\pm \beta E_\nu^{\pm}} = \frac{e^{\pm \beta E_\nu^{\pm}/2}}{1 - e^{\pm \beta E_\nu^{\pm}}} \qquad (15.8)$$

known as the *Boltzmann* distribution [7], or the canonical ensemble, introduced in 1877. The average energy in a mode can be expressed by the partition function:

$$\tilde{E}^{\pm} = -i\frac{d\,\log(Z)}{d\beta} = \pm\frac{iE_n^{\pm}}{2} \pm \frac{iE_n^{\pm}}{e^{\pm \beta E_n^{\pm}} - 1} \qquad : E_n^{\pm} = \mp imc^2 \quad (15.9)$$

As $T \to 0$, the distribution forces the system into its ground state at the lowest energy before transforming to the virtual world. All higher energy states have vanishing probability at zero temperature, the mirroring effects of infinite temperature.

Artifacts 14.3: Chemical Potential. For a bulk system with the internal energy shown as above and the intractable energy of E_n, the chemical potential $\mu = -\sum \mu_n$, rises from the numbers of particles:

$$\mu = -\sum_n \left(\frac{\partial E_n}{\partial N_n^{\pm}}\right)_{S,V} = k_B T \sum_n \frac{1 - \left(1 - N_n^{\pm}/N\right)\log\left(N/N_n^{\pm} - 1\right)}{\left(1 - N_n^{\pm}/N\right)}$$

$$= -\sum_n \left[E_n^\pm - k_B T\left(1 + e^{\pm\beta E_n^\pm}\right)\right] \tag{15.10}$$

Its heat capacity can be given by the following definition:

$$C_V \equiv \sum_n \left(\frac{\partial E_n}{\partial T}\right)_{V, N_n^\pm} = k_B \sum_n \frac{N\left(E_n^\pm\right)^2 e^{\pm\beta E_n^\pm}}{\left[k_B T\left(e^{\pm\beta E_n^\pm}+1\right)\right]^2} \tag{15.11}$$

The maximum heat capacity is around $k_B T \to |E^\pm|$. As $T \to 0$, the specific heat exponentially drops to zero, whereas $T \to \infty$ drops off at a much slower pace defined by a power-law.

Artifacts 14.4: Thermodynamics. Consider a system with entropy $S(E, V, N_n)$ that undergoes a small change in energy, volume, and number $N_n^\pm$, the change in entropy is

$$dS = \frac{\partial S}{\partial E} dE + \frac{\partial S}{\partial E}\frac{\partial E}{\partial V} dV + \frac{\partial S}{\partial E}\sum_n \left(\frac{\partial E}{\partial N_n^\pm} dN_n^\pm\right)$$

$$= \frac{1}{T}\left(dE + PdV - \sum_n \mu_n dN_n^\pm\right) \quad : \frac{1}{T} = \frac{\partial S}{\partial E}, P = \left(\frac{\partial E}{\partial V}\right)_T \tag{15.12}$$

known as fundamental laws of thermodynamics of common conjugate variable pairs. The principles of thermodynamics were established and developed by *Rudolf Clausius*, *William Thomson*, and *Josiah Willard Gibbs*, introduced during the period from 1850 to 1879.

Artifacts 14.5: $Y^\mp$ Thermal Density Equations. Furthermore, convert all parameters to their respective densities as internal energy density $\rho_E = E/V$, thermal entropy density $\rho_s = S/V$, mole number density $\rho_{n_i} = N_i/V$, state density of $\rho_\Phi \sim 1/V$; The above equation has the entropy relationship among their density as the following:

$$S_\rho = -k_s \int \rho_\psi dV = -k_s \int \frac{d\rho_E - Td\rho_s - \sum_i \mu_i d\rho_{n_i}}{T\rho_s + \sum_i \mu_i \rho_{n_i} - (P + \rho_E)} dV \tag{15.13}$$

Satisfying entropy equilibrium at extrema results in the general density equations of the thermodynamic fields:

$$d\rho_E^- = Td\rho_s^- + \sum\nolimits_i \mu_i d\rho_{n_i}^- \tag{15.14}$$

$$P + \rho_E^+ = T\rho_s^+ + \sum\nolimits_i \mu_i \rho_{n_i}^+ \tag{15.15}$$

The first equation indicates that entropy increases towards Y^- maximum in physical disorder, so that the dynamics of the internal energy are the interactive fields of thermal and chemical reactions as they influence substance molarity. The second equation indicates that entropy decreases towards Y^+ minimum in physical order, so that both external forces and internal energy hold balanced macroscopic fields in one bulk system.

Artifacts 14.6: Bloch Density Equations. At the arisen horizon, a macroscopic state consists of pairs of $Y^-\{\rho^-, \varrho^+ = \rho^{*-}\}$ and $Y^+\{\rho^+, \varrho^- = \rho^{*+}\}$ thermal density fields. By mapping $\phi_n^\pm \mapsto \rho^\pm$, $\varphi_n^\pm \mapsto \varrho^\pm$ and $x_0 \mapsto \beta$, the same mathematical framework in deriving (8.16) and (8.21) can be reapplied to formulate a duality of the thermal densities, shown by the following:

$$-i\frac{\partial \rho^-}{\partial \beta} = \hat{H}\rho^-, \quad -h_\beta \frac{\partial^2 \rho}{\partial \beta^2} = \hat{H}\rho \quad : \hat{H} \equiv -h_\beta \nabla^2 + \hat{U}(\mathbf{r}, \beta_0) \tag{15.16}$$

where $\rho = \rho^+\rho^-$ and h_β is a horizon constant of thermodynamics. These equations are known as *Bloch* equations introduced in 1932 [8].

XVI. BLACK BODY AND BLACK HOLE

Every physical body spontaneously and continuously emits electromagnetic radiation. At near thermodynamic equilibrium, the emitted radiation is closely described by either Planck's law for black bodies or *Bekenstein-Hawking* radiation for black holes, or both. Because of its dependence on temperature and area, *Planck* and *Schwarzshild* radiation are said to be thermal radiation obeying area entropies. The higher the temperature or area of a body the more radiation it emits at every wavelength of light and gravitation. Since a black hole acts like an ideal black body as it reflects no light, their entropies of area law are equivalent to (8.23) and (14.19) for light and gravitational radiation.

Artifacts A.1: Electromagnetic Radiation. With the horizon factor (15.7), *Planck* derived in 1900 [9] that an area entropy S_A of radiance of a body is given by frequency at absolute temperature *T*.

$$S_A(\omega_c, T) = \frac{\hbar\omega_c^3}{4\pi^3 c^2 k_B T}\left(e^{\hbar\omega_c/k_B T} - 1\right)^{-1} \simeq \frac{\omega_c^2}{4\pi^3 c^2} \tag{16.1}$$

Expressed as an energy distribution of entropy, it is the unique stable radiation in quantum electromagnetism, applicable to (8.23).

Artifacts A.2: Black Hole Radiation. By associating spacetime horizon factor with *Schwarzshild* radius $r_s = 2GM/c^2$ of black hole, derived in 1915, an area entropy S_A of the quantum-gravitational radiance of a black hole is given by frequency at absolute temperature T and constant c_g as the following:

$$S_A(\omega_g, T) = \frac{c_g^3}{4\hbar G} \tag{16.2}$$

known as *Bekenstein-Hawking* radiation [10,11], introduced in 1974.

CONCLUSION

From *First Universal Field Equations* (6.7-8) and (6.12-13) [1], the Y^-Y^+ fluxions are operated to give rise to the horizons where a set of continuities is instituted symmetrically to function as the horizon of *Second Universal Dynamic Equations* (12.2) and (12.4), unifying the fields of electromagnetism, gravitation and thermodynamics.

Remarkably, *Law of Conservation of Gravitation* demonstrates that graviton is conserved to invariance of the *Torque Transportations*, given by Gravitational fields of Eq (14.1)-(14.8) that cohesively unify with light, electromagnetic and black hole fields at the horizons factored by statistical thermodynamics.

In the center of a black hole, a system of partial differential equations forms the entanglements of gravitational and electromagnetic fields and emerges the associated forces from massive objects for internal communications. Essentially, the natural context of *inertial boost* $J_{\mu a}^+$ and spiral torque $K_{ma}^\pm$ tensors constitutes and acts as the sources of "photon" and "graviton" fields being operated at the heart of energy formulations of stress strengths and twist torsions, driven by the events descending from the two-dimensional world planes of the dual manifolds and the affine connections aligning to each of the curvatures.

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