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More about the inconsistency solution of the Maxwell equations

Annotation

A brief description of the non-contradictory solution of the Maxwell equations given in [1] and new additions are given.

Contents

1. Introduction

"To date, whatsoever effect that would request a modification of Maxwell's equations escaped detection" [4]. Nevertheless, recently criticism of validity of Maxwell equations is heard from all sides. Have a look at the Fig.1 that shows a wave being a known solution of Maxwell's equations. The confidence of critics is created first of all by the violation of the Law of energy conservation. And certainly "*the density of electromagnetic energy flow (the module of Umov-Pointing vector) pulsates harmonically. Doesn't it violate the Law of energy conservation?*" [3]. Certainly, it is violated, **if** the electromagnetic wave satisfies the **known solution** of Maxwell equations. But there is no other solution: "*The proof of solution's uniqueness in general is as follows. If there are two different solutions, then their difference due to the system's linearity, will also be a solution, but for zero charges and currents and for zero initial conditions. Hence, using the expression for electromagnetic field energy we must conclude that the difference between solutions is equal to zero, which means that the solutions are identical. Thus the uniqueness of Maxwell equations solution is proved*" [4]. So, <u>the uniqueness of solution is being proved on the base of using the law which is violated in this solution.</u>

Another result following from the existing solution of Maxwell equations is phase synchronism of electrical and magnetic components of intensities in an electromagnetic wave. This is contrary to the idea of constant transformation of electrical and magnetic components of energy in an electromagnetic wave. In [3[, for example, this fact is called "one of the vices of the classical electrodynamics".

Such results following from the known solution of Maxwell equations allow doubting the authenticity of Maxwell equations. However, we must stress that these results follow **only from the found solution**. But this solution, as has been stated above, can be different (in their partial derivatives, equations generally have several solutions).

In [1] **another solution** of Maxwell equation is derived, in which the density of electromagnetic energy flow remains constant in time, and electrical and magnetic components of intensities in the electromagnetic wave are shifted in phase.

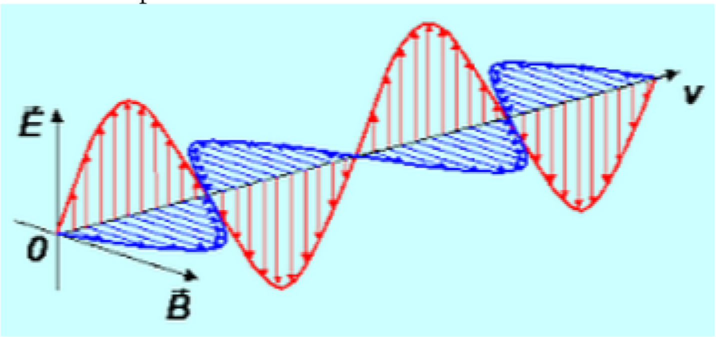

Fig. 1.

Below it is shown that from this solution one can also find the velocity of electromagnetic energy movement, which in general differs from the speed of light. Some other additions to [1] are given.

For the convenience of the reader, the solution proposed in [1] is first briefly considered.

2. Solution of Maxwell's Equations

The system of Maxwell's equations for vacuum has the form

$$\mathrm{rot}(E)+\frac{\mu}{c}\frac{\partial H}{\partial t}=0, \tag{1}$$

$$\mathrm{rot}(H)-\frac{\varepsilon}{c}\frac{\partial E}{\partial t}=0, \tag{2}$$

$$\mathrm{div}(E)=0, \tag{3}$$

$$\mathrm{div}(H)=0. \tag{4}$$

In cylindrical coordinates system r, φ, z these equations look as follows:

$$\frac{E_r}{r} + \frac{\partial E_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial E_\varphi}{\partial \varphi} + \frac{\partial E_z}{\partial z} = 0, \tag{5}$$

$$\frac{1}{r} \cdot \frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} = -\frac{\mu}{c} \frac{\partial H_r}{\partial t}, \tag{6}$$

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\mu}{c} \frac{\partial H_\varphi}{\partial t}, \tag{7}$$

$$\frac{E_\varphi}{r} + \frac{\partial E_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial E_r}{\partial \varphi} = -\frac{\mu}{c} \frac{\partial H_z}{\partial t}, \tag{8}$$

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0, \tag{9}$$

$$\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} = \frac{\varepsilon}{c} \frac{\partial E_r}{\partial t}, \tag{10}$$

$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = \frac{\varepsilon}{c} \frac{\partial E_\varphi}{\partial t}, \tag{11}$$

$$\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial H_r}{\partial \varphi} = \frac{\varepsilon}{c} \frac{\partial E_z}{\partial t}, \tag{12}$$

For the sake of brevity further we shall use the following notations:

$$co = \cos(\alpha\varphi + \chi z + \omega t), \tag{13}$$

$$si = \sin(\alpha\varphi + \chi z + \omega t), \tag{14}$$

where α, χ, ω – are certain constants. Let us present the unknown functions in the following form:

$$H_r. = h_r(r)co, \tag{15}$$

$$H_\varphi. = h_\varphi(r)si, \tag{16}$$

$$H_z. = h_z(r)si, \tag{17}$$

$$E_r. = e_r(r)si, \tag{18}$$

$$E_\varphi. = e_\varphi(r)co, \tag{19}$$

$$E_z. = e_z(r)co, \tag{20}$$

where $h(r)$, $e(r)$ - certain function of the coordinate r.

In [1] it is shown that for such a system there **exists** a solution of the following form:

$$h_z(r) = 0,\ e_z(r) = 0, \tag{21}$$

$$e_r(r) = e_\varphi(r) = \frac{A}{2} r^{(\alpha-1)}, \tag{22}$$

$$h_\varphi(r) = \sqrt{\frac{\varepsilon}{\mu}} e_r(r), \quad . \tag{23}$$

$$h_r(r) = -\sqrt{\frac{\varepsilon}{\mu}} e_\varphi(r), \tag{24}$$

$$\chi = \pm\omega\sqrt{\mu\varepsilon}/c, \tag{25}$$

where $A, \varepsilon, \mu, c, \alpha, \chi, \omega$ – constants.

3. Intensities

Fig. 2 shows the vectors of intensities originating from the point $A(r, \varphi)$. In this case, the **vectors** E, H **are always orthogonal**.

In order to demonstrate phase shift between the wave components let's consider the functions (2.13, 2.14) and (2.15-2.20). It can be seen, that **at each point with coordinates** r, φ, z **intensities** H, E **are shifted in phase by a quarter-period**.

The energy density is

$$W = \left(\frac{\varepsilon}{2}E^2 + \frac{\mu}{2}H^2\right) = \frac{A^2}{4} \cdot \varepsilon \cdot r^{2(\alpha-1)}. \tag{1}$$

Thus, **electromagnetic wave energy density is constant in time and equal in all points of the cylinder of given radius**.

The solution exists also for changed signs of the functions (2.13, 2.14). This case is shown on Fig 3. Fig. 2 and Fig. 3 illustrate the fact that **there are two possible type of electromagnetic wave circular polarization.**

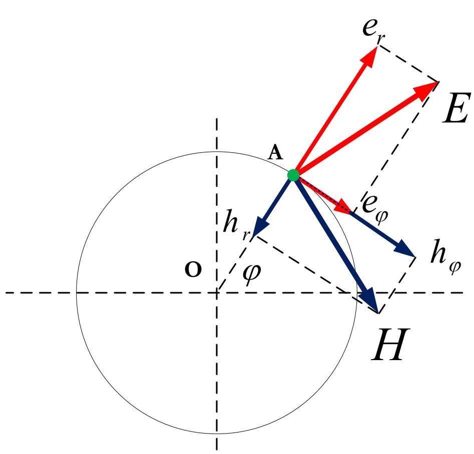

Fig. 2.

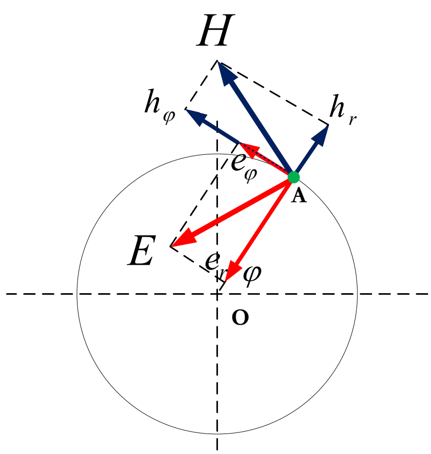

Fig. 3.

Let's consider the functions (2.13, 2.14) and (2.25). Then, we can find

$$co = \cos(\alpha\varphi + \sqrt{\varepsilon\mu}\frac{\omega}{c}z + \omega t), \tag{2}$$

$$si = \sin(\alpha\varphi + \sqrt{\varepsilon\mu}\frac{\omega}{c}z + \omega t). \tag{3}$$

Let's consider a point moving along a cylinder of constant radius r, where the value of intensity depends on time as follows:

$$H_{r}. = h_r(r)\cos(\omega t) \tag{4}$$

Comparing this equation with (2.15) and taking (2) into account, we can notice that equations (4) is the same as (2.15), if at any moment of time

$$\alpha\varphi + \sqrt{\varepsilon\mu}\frac{\omega}{c}z = 0 \tag{5}$$

or

$$\varphi = -\frac{\omega\sqrt{\varepsilon\mu}}{\alpha \cdot c}z. \tag{6}$$

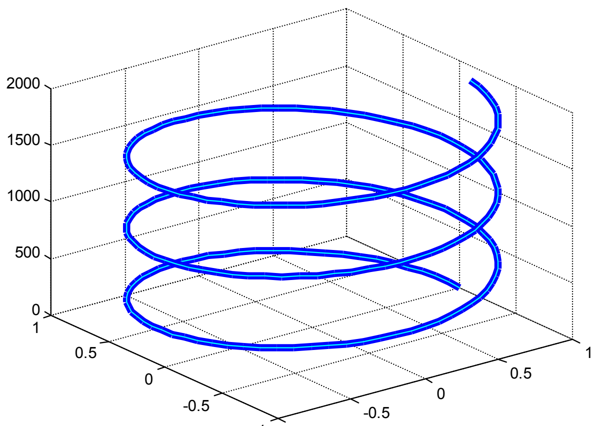

Fig. 4. (TokPotok33.m)

Thus, at the cylinder of constant radius r a path of this point exists, which is described by equations (2, 6), where all the intensities vary harmonically. On the other hand, this path is a helix. Thus, the line, along which the point moves in such a way, that its intensity $H_r.$ varies in a sinusoidal manner, is a helix. The same conclusion can be repeated for other intensities (2.16-2.20). Thus,

the path of the point, which moves along a cylinder of given radius in such a manner, that each intensity varies harmonically with time, is described by a <u>helix</u>	**(A)**

For example, Fig. 4 shows a helix, for which $r=1,\ c=300000,\ \omega=3000,\ \alpha=-3,\ \varphi=[0 \div 2\pi]$. Fig. 4a shows helices in the same conditions, but for different radii, where $r=[0.5,\ 0.6,\ ...1.0,\ 1.1]$. Straight lines indicate the geometric loci of points with equal φ.

We denote $e_r(r)=e_\varphi(r)=e_{r\varphi}(r)$. Then it follows from (2.21-2.25) that in each point there are only vectors

$$H_r. = -\sqrt{\frac{\varepsilon}{\mu}} e_{r\varphi}(r) cos(\omega t), \qquad H_\varphi. = \sqrt{\frac{\varepsilon}{\mu}} e_{r\varphi}(r) sin(\omega t), \tag{7}$$

$$E_r. = e_{r\varphi}(r) \sin(\omega t), \qquad E_\varphi. = e_{r\varphi}(r) \cos(\omega t). \tag{8}$$

In this case resultant vectors $\mathrm{H}_{r\varphi} = \mathrm{H}_r + \mathrm{H}_\varphi$ and $\mathrm{E}_{r\varphi} = \mathrm{E}_r + \mathrm{E}_\varphi$ lay in plane $r,\ \varphi$, and their modules are $|H_{r\varphi}| = \sqrt{\frac{\varepsilon}{\mu}} e_{r\varphi}(r)$ and $|E_{r\varphi}| = e_{r\varphi}(r)$. Fig. 4b shows all these vectors. It can be seen, that when the point T moves along the helix, resultant vectors $\mathrm{H}_{r\varphi}$ and $\mathrm{E}_{r\varphi}$ rotate in plane $r,\ \varphi$. Their moduli are constant and equal one to the other. These vectors $\mathrm{H}_{r\varphi}$ and $\mathrm{E}_{r\varphi}$ are always orthogonal.

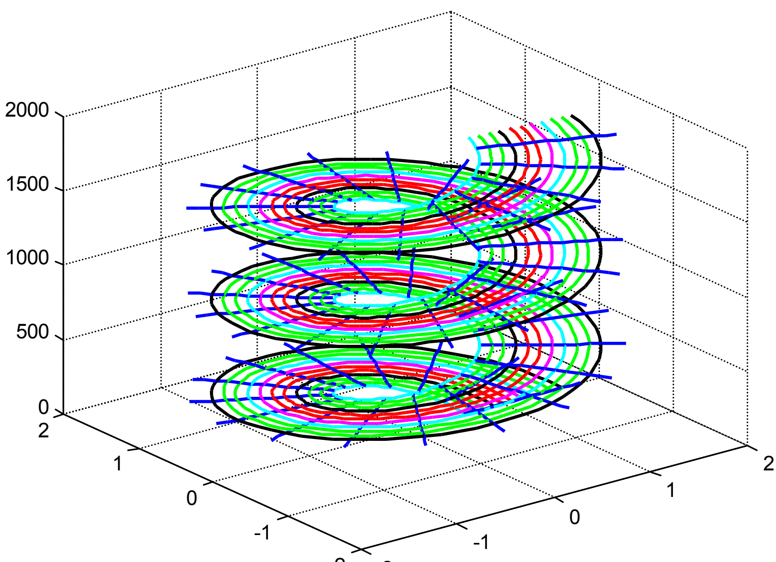

Fig. 4a. (TokPotok33.m)

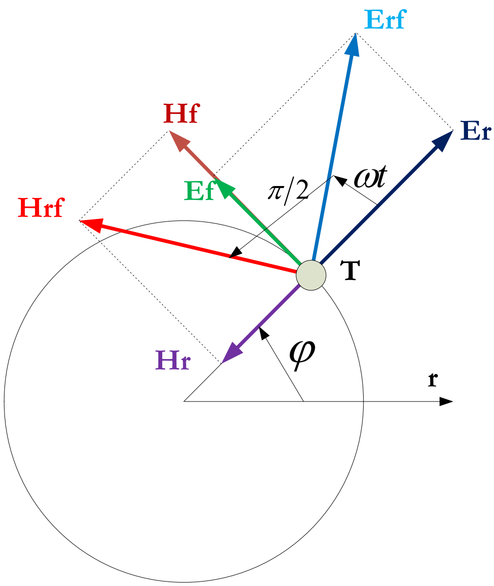

Рис. 4б.

So, **harmonic wave is propagating along the helix**, and in this case at each point T, which moves along this helix, projections of vectors of magnetic and electric intensities:

- exist only in the plane which is perpendicular to the helix axis, i.e. there only two projections of these vectors exist,
- vary in a sinusoidal manner,
- are shifted in phase by a quarter-period.

Resultant vectors:

- rotate in these plane,
- have constant moduli,
- are orthogonal to each other.

4. Energy Flows

The density of electromagnetic flow is Umov-Pointing vector

$$S = \eta E \times H \,, \tag{1}$$

where

$$\eta = c/4\pi \,. \tag{2}$$

Отсюда и из предыдущих формул следует, что the energy flow extends only along the axis oz and is equal

$$\overline{S} = \overline{S_z} = \eta \iint_{r,\varphi} \left[s_z \cdot si \cdot co \right] dr \cdot d\varphi \,, \tag{3}$$

where the energy flux density

$$s_z = \sqrt{\frac{\varepsilon}{\mu}}\left(e_r^2 + e_\varphi^2\right). \qquad (4)$$

Lack of radial energy flux indicates that area of wave existence is **<u>NOT</u>** <u>growing</u>. Existence of laser provides evidence of this fact.

As shown in [1], it follows from the preceding formulas that

$$s_z = \frac{A^2 c}{64\alpha\pi}\sqrt{\frac{\varepsilon}{\mu}}\left(1-\cos(4\alpha\pi)\right)r^{2(\alpha-1)}. \qquad (5)$$

It follows that

- flux density is unevenly distributed over the flow cross section – there is a <u>picture of the distribution of flow density</u> by the cross section of the wave
- this picture is rotated while moving on the axis oz;
- the flow of energy, passing through the cross-sectional area, not depend on $t,\ \varphi,\ z$; the main thing is that the value does not change with time, and, следовательно, <u>поток энергии электромагнитной волны является постоянным во времени</u>; this <u>complies with the</u> **<u>Law of energy conservation</u>**.

5. Velocity of energy movement

First of all, we find the propagation velocity of a monochromatic electromagnetic wave. Obviously, this velocity is equal to the derivative $\frac{dz}{dt}$ of the function given implicitly in the form (2.15-2.20). Having determined the derivative of these functions $z(t)$, we find the propagation velocity of a monochromatic electromagnetic wave

$$v_m = \frac{dz}{dt} = -\frac{\omega}{\chi} \qquad (1)$$

or, taking into account (2.25),

$$v_m = \mp\frac{c}{\sqrt{\mu\varepsilon}}. \qquad (2)$$

Consequently, the propagation velocity of a monochromatic electromagnetic wave is equal to the velocity of light.

Umov's concept [5] is generally accepted, according to which the energy flux density s is a product of the energy density w and the velosity of energy movement v_e:

$$s = w \cdot v_e . \tag{3}$$

From (4.12, 3.1, 18) we obtain:

$$K_{vc} = \frac{v_e}{c} = \frac{(1 - \cos(4\alpha\pi))}{16\alpha\pi\sqrt{\varepsilon\mu}} . \tag{4}$$

When small α, equation (4) is transformed to the form:

$$K_{vc} \approx \frac{\pi\alpha}{2\sqrt{\varepsilon\mu}} . \tag{5}$$

Thus, the velocity of the electromagnetic energy movement and the magnitude are proportional. In particular, the velocity of the electromagnetic energy movement is equal to the propagation velocity of monochromatic electromagnetic wave at $K_{vc} = 1$, from which it follows that

$$\alpha \approx \frac{2}{\pi}\sqrt{\varepsilon\mu} \approx 2 \cdot 10^{-9} . \tag{6}$$

Under this condition, the energy flux and energy are related by the relation $s = w \cdot c$. The velocity of energy movement is constant for all points of the wave section (does not depend on *r*).

The velocity of movement of electromagnetic energy movement is not always equal to the velocity of light. For example, in a standing wave $v_e = 0$, and generally in a wave that is the sum of two monochromatic electromagnetic waves of the same frequency, propagating in opposite directions, the energy transfer is weakened and $v_e < c$.

Note that, based on the known solution and formula (3), we can not find the velocity v_e. Indeed, in the SI system we find:

$$v_e = \frac{S}{W} = \mathrm{EH}\Big/\left(\frac{\varepsilon E^2}{2} + \frac{H^2}{2\mu}\right) = 2\mu\Big/\left(\varepsilon\mu\frac{E}{H} + \frac{H}{E}\right).$$

If $\frac{\varepsilon E^2}{2} = \frac{H^2}{2\mu}$, then $\frac{H}{E} = \sqrt{\mu\varepsilon}$. Then for a vacuum

$$v_e = 2\mu\Big/\left(\varepsilon\mu\frac{1}{\sqrt{\varepsilon\mu}} + \sqrt{\varepsilon\mu}\right) = \sqrt{\frac{\mu}{\varepsilon}} \approx 376 ,$$

which is **not** true. In general, the solution obtained here can not be found in vector form.

6. Momentum and moment of momentum

It is known that

$$p = S/c^2, \tag{1}$$

$$m = p \cdot r, \tag{2}$$

where p - momentum density, m - density momentum at this point about an axis spaced from the given point by a distance r. It follows from the above that in the electromagnetic wave there exist energy flows, which directed along a radius, along a circle, along a axis. Consequently, in the electromagnetic wave there exist momentum, which directed along a radius, along a circle, along a axis. Also there exist momentum, which directed along a radius, along a circle, along a axis.

Let's consider the angular momentum about the axis z. According to (1) we can find this momentum as follows:

$$L_z = p_z r = s_z r / c^2. \tag{3}$$

This is orbital angular momentum, which can be detected in so called twisted light [6].

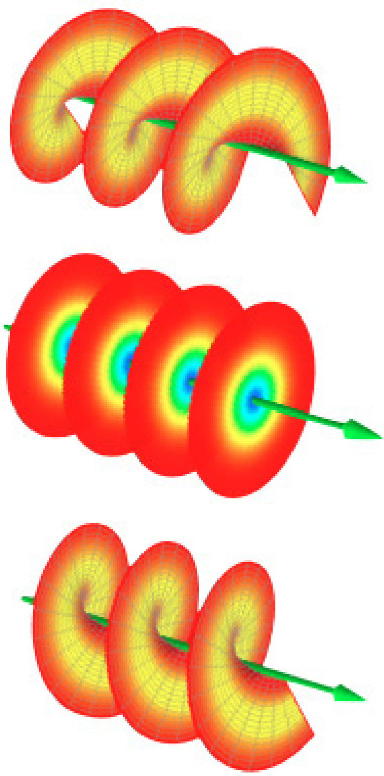

Fig. 7a.

However, it should be noted that existence of the twisted light does **not** follow from the existing solution of Maxwell's equations. But it naturally follows from the proposed solution — see (6). In Fig. 7a (taken from [64]) "*the picture with the twisted light doesn't show the electric field, but the wavefront* (the middle picture shows non-twisted light, and the upper and lower ones — the light twisted to one or another side). *It is not flat; in this case the wave phase changes not only along the beam, but also with shifting in cross-sectional plane… As the energy flow of the light wave is usually directed perpendicular to the wavefront, it occurs, that in the twisted light energy and momentum not only fly ahead, but also spin around the axis of movement.*" This particular fact was confirmed above — see Fig. 3.4a for comparison.

7. Discussion

The Fig. 8 shows the intensities in Cartesian coordinates. The resulting solution describes a wave. The main distinctions from the known solution are as follows:

1. Instantaneous (and not average by certain period) energy flow **does not** change with time, which complies with the Law of energy conservation.
2. The energy flow has a positive value
3. The energy flow extends along the wave.
4. Magnetic and electrical intensities on one of the coordinate axes r, φ, z phase-shifted by a quarter of period.
5. The solution for magnetic and electrical intensities is a real value.
6. The solution exists at constant velosity of wave propagation.
7. The existence region of the wave **does not** expand, as evidenced by the existence of laser.
8. The vectors of electrical and magnetic intensities are orthogonal.
9. There are two possible types of electromagnetic wave circular polarization.
10. The wave and its energy are determined if the parameters A, ω, R, α are specified.
11. The parameter α determines the velocity of energy movement in the electromagnetic wave.
12. The path of the point, which moves along a cylinder of given radius in such a manner, that each intensity value varies harmonically with time, is a helix.

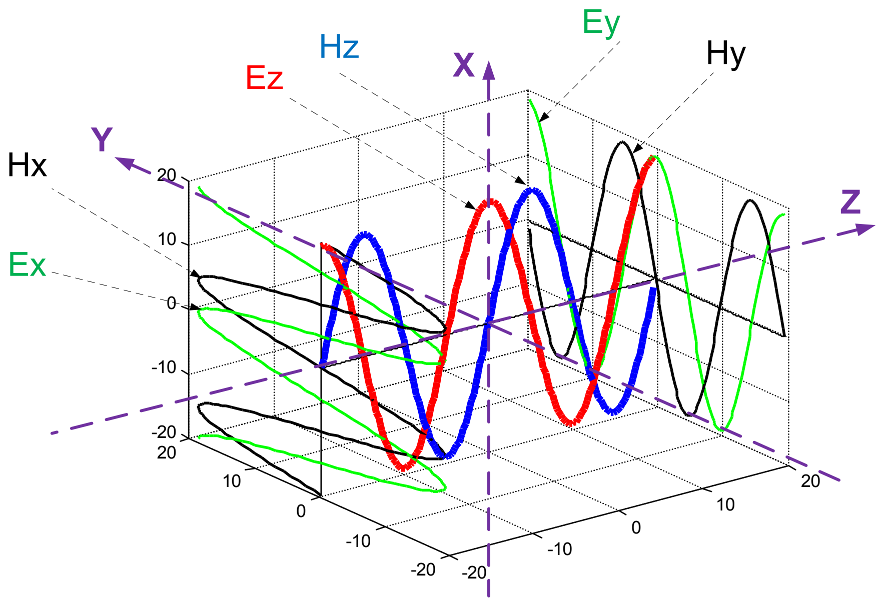

Fig. 8.

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