《**Universal and Unified Field Theory**》

# 2. Quantum Field Generators of Horizon Infrastructure

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***Abstract***: Harnessed with the *Universal Topology, Mathematical Framework* and *Universal Field Equations*, the comprehensive discoveries and a broad range of applications to both classical and contemporary physics prevail throughout the following contexts but are not limited to,

1. Two sets of boost transform and spiral transport *Generators* construct a communication infrastructure and function as the event actors producing photons from the well-known grammar-matrices and gravitons from the exceptional ***chi-matrices***.
2. An ***evolutionary process*** reveals its superphase modulations and appears as the *Gauge* fields, giving rise to the horizons.
3. At the second horizon, the gravitation conserves invariance at its divergence with ***singularity-free*** on *World* planes.
4. ***Laws of Conservation of Creation and Reproduction*** illustrate the philosophical and mathematical derivations of classical quantum mechanics including but not limited to *Dirac*, *Hamiltonian*, *Pauli*, *Schrödinger*, *Lorentz*, *Klein–Gordon*, *Weyl* equations.
5. ***Embody Structure of Mass Enclave*** is discovered as a part of the horizon evolutionary processes. Particles at inauguration acquire their partial mass and evolve into the third horizons to become its full mass object, where the energy embodies its mass enclave and extends the extra freedom of the rotational coordinates into an integrity of the three-dimensional physical-space and one-dimensional virtual-time.
6. A set of the complex *matrices* formalizes ***Speed of Light and Gravitational Fields*** at a superphase modulation such that their r-directional amplitude of world line is at a constant *c*.
7. The virtual entangling functions are further decoherence or collapsed into the conventional physical interpretations that extend the fully virtual states to reformulate the classic equations, for example, *Lagrangians as well as Einstein* mass-energy equivalence.

Consequently, this unified theory testifies to, complies with and extends at precisely the empirical physics of *Pauli* matrices, *Lorentz* generators, quantum electrodynamics, spacetime evolution, horizon processes, and beyond.

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## INTRODUCTION

The main objective of this manuscript is to clearly demonstrate that, under *Universal Topology* $W = P \pm iV$ [1], a duality of the potential entanglements lies at the heart of all event operations as the natural foundation giving rise to and orchestrating relativistic transformations for photons and spiral transportations for gravitons. Besides, the superphase modulation conducts laws of evolutions and horizon of conservations, and maintains field entanglements of coupling weak and strong forces compliant to quantum electrodynamics of classic physics. As a result, it inaugurates a unified physics dawning at special remarks of the following:

Chapter VII: Discovers that a duality of the boost $S_i^{\pm}$ and spiral $R_i^{\pm}$ matrices constructs two sets of the *Generators*, establishes the quantum communication infrastructure, and institutes the well-known *Pauli* matrices or gamma-$\tilde{\gamma}^{\nu}$ transformations, and the exceptional chi-$\tilde{\chi}^{\nu}$ transportations, incepted at the second horizon on *World Planes*. Surprisingly, it reveals that the spiral torque is not only a source of graviton, but also has a hidden singularity such that its divergency of torque matrix is conserved to and yields the superphase modulations.

Chapter VIII: Demonstrates that a physical infrastructure is incepted by and given rise to the third horizon to have the extra freedom of the rotational coordinates as a three-dimensional space. As expected, the gravitation fields turn out the principle of the central-singularity.

Chapter IX: Derives all well-known quantum fields to include but not be limited to *Dirac Equation*, *Schrödinger Equation*, *Pauli Theory*, *Weyl Equation,* the enhanced *Klein–Gordon* equation, and the *Speed Matrices* of light and gravitational fields with the superphase modulations. In addition, it exposes mass formation during the quantum harmonic oscillations between the horizons. Remarkably, it further reveals *Embody Structure of Mass Enclave* such that one bids farewell to the hypothesized "*Big Bang*" model.

Consistently landing on classical and modern physics, this manuscript uncovers a series of the philosophical and mathematical groundbreakings accessible and testified by the countless artifacts.

## VII. COMMUNICATION INFRASTRUCTURES

As a part of the *Universal Topology*, a communication infrastructure formalizes the ontological processes in mathematical presentation driven by axiomatic creators and evolutions of the event operations that transform and transport informational messages and conveyable actions. Empowered with the speed of light, the *two-dimensional* $\{\mathbf{r} \mp i\,\mathbf{k}\}$ communication of the *World Planes* is naturally contracted or operated for tunneling between the $Y^-$ and $Y^+$ domains at both local residual and relativistic interaction among virtual dark and physical massive energies, which is mathematically describable by local invariances and relativistic commutations of entanglements cycling reciprocally and looping consistently among the four potential fields of the dual manifolds.

Remarkably, there are the environmental settings of originators and commutators that establish entanglements between the manifolds as a duality of the $Y^-Y^+$ infrastructures for the life transformation, transportation, or commutation simultaneously and complementarily. When the event $\lambda = t$ operates at constant speed *c*, the $Y^-Y^+$ dynamics incepts the matrices of (3.1-3.2) and (3.5,3.7) [1] at the second horizon of the world planes. Each world contracts a two-dimensional manifold, generates a pair of the boost and spiral transportations, and entangles an infinite loop between the manifolds:

$$\hat{\partial}^{\lambda} \circlearrowleft \hat{\partial}_{\lambda} \rightleftharpoons \check{\partial}^{\lambda} \circlearrowright \check{\partial}_{\lambda} \qquad : x_m \in \{ict, \tilde{r}\}, x^{\mu} \in \{-ict, \tilde{r}\} \tag{7.1}$$

This infrastructure has a set of constituents, named as *Generators* which are a group of the irreducible foundational matrices and constructs a variety of the applications in forms of horizon evolution, fields or forces. At the second horizon *SU*(2), the *Generators* institutes the infrastructure with a set of the metric signatures, *Local* originators, the *Horizon* commutators. For example, it features *Pauli* matrices, *Gamma* matrices, *Dirac* basis, *Weyl* spinors, *Majorana* basis, etc. At the third horizon *SU*(3) in the parallel fashion, another infrastructure institutes, but are not limited to, the symmetric and asymmetric transform or transport fields featuring electromagnetism, gravitation, weak, and strong forces, cosmological fields, etc.

***Artifact 7.1: Virtual and Physical Manifolds***. Both manifolds $\hat{x}\{\mathbf{r}-i\mathbf{k}\}$ and $\check{x}\{\mathbf{r}+i\mathbf{k}\}$ simultaneously govern and alternatively perform the event operations as one integral stream of any physical and virtual dynamics. Apparently, the virtual positions $\pm i\mathbf{k}$ naturally forms a duality of the conjugate manifolds: $x^{\nu}\in\hat{x}\{\mathbf{r}-i\mathbf{k}\}$ and $x_m\in\check{x}\{\mathbf{r}+i\mathbf{k}\}$. Each of the super two-dimensional coordinate system $G(\lambda)\in G\{\mathbf{r}\pm i\mathbf{k}\}$ constitutes its *World Plane* $W^{-}\in G(\lambda=t)$ or $W^{+}\in G(\lambda=t)$ distinctively, forms a duality of the universal topology $W^{\mp}=P\pm iV$ cohesively, and maintains its own sub-coordinate system $\mathbf{r}$ or $\mathbf{k}$ extendable, respectively. A sub-coordinate system has its own rotational freedom of either physical sub-dimensions $\mathbf{r}(\theta,\varphi)$ or virtual sub-dimensions $\mathbf{k}(x^0,\cdots)$. Together, they compose two rotational manifolds as a reciprocal or conjugate duality operating and balancing the world events.

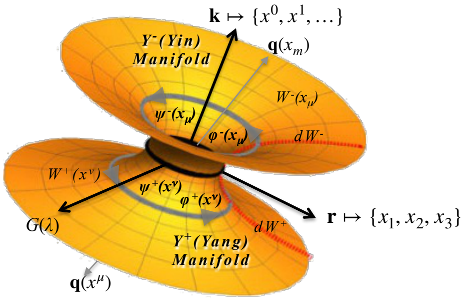


Figure 7a: Dual Manifolds of Communication Infrastructure

***Artifact 7.2: Boost Generators***. On the world planes at a constant speed $c$, this event flow naturally describes and concisely derives a set of the *Boost* matrix tables as the *Quadrant-State*:

$$S_2^+=\frac{\partial x^\nu}{\partial x^m}=\begin{pmatrix}1&-i\\ i&1\end{pmatrix}\equiv s_0+is_2 \qquad :\hat{\partial}^\lambda=\dot{x}^m S_2^+\partial^\nu \tag{7.2a}$$

$$S_1^+=\frac{\partial x^\nu}{\partial x_m}=\begin{pmatrix}-1&-i\\ -i&1\end{pmatrix}\equiv s_3-is_1 \qquad :\hat{\partial}_\lambda=\dot{x}_m S_1^+\partial^\nu \tag{7.2b}$$

$$S_1^-=\frac{\partial x_m}{\partial x^\nu}=\begin{pmatrix}-1&i\\ i&1\end{pmatrix}\equiv s_3+is_1 \qquad :\check{\partial}^\lambda=\dot{x}^\nu S_1^-\partial_m \tag{7.2c}$$

$$S_2^-=\frac{\partial x_m}{\partial x_\nu}=\begin{pmatrix}1&i\\ -i&1\end{pmatrix}\equiv s_0-is_2 \qquad :\check{\partial}_\lambda=\dot{x}_\nu S_2^-\partial_m \tag{7.2d}$$

The $S_1^\pm$ matrices are a duality of the horizon settings for transformation between the two-dimensional world planes without $r$-singularity. The $S_2^\pm$ matrices are the local or residual settings for $Y^-$ or $Y^+$ transportation within their own manifold, respectively. Defined as the *Infrastructural Boost Generators*, this $s_\kappa$ group consists of the distinct members, shown by the following:

$$s_\kappa=\left[\begin{pmatrix}1&0\\0&1\end{pmatrix}_0,\begin{pmatrix}0&1\\1&0\end{pmatrix}_1,\begin{pmatrix}0&-1\\1&0\end{pmatrix}_2,\begin{pmatrix}-1&0\\0&1\end{pmatrix}_3\right] \tag{7.3}$$

Intuitively simplified to a group of the 2x2 matrices, the infinite (7.1) loops of entanglements compose an integrity of the boost generators $s_n$ that represents law of conservation of life-cycle transform continuity of motion dynamics, shown by the following:

$$[s_a,s_b]=2\epsilon^{-}_{cba}s_c \qquad \langle s_a,s_b\rangle=0 \qquad :a,b,c\in\{1,2,3\} \tag{7.4}$$

where the *Levi-Civita* [5] connection $\epsilon^{-}_{cba}$ represents the right-hand chiral. In accordance with our philosophical anticipation, the non-zero commutation reveals the loop-processes of entanglements, reciprocally. The zero continuity illustrates the conservations of virtual supremacy that are either extensible from or degradable back to the global two-dimensions of the world planes.

***Artifact 7.3: Torque Generators***. Simultaneously on the world planes at a constant speed, the loop event naturally describes and concisely elaborates another set of the *Spiral* matrix tables. The world planes are supernatural or intrinsic at the two-dimensional coordinates presentable as a vector calculus in polar coordinates. Because of the superphase modulation, in *Cartesian* coordinates all *Christoffel* symbols vanish, which implies the superphase modulation becomes hidden. Therefore, we consider the polar manifold $\{\tilde{r},\pm i\tilde{\vartheta}\}\in\mathscr{R}^2$ that a physical world has its superposition $\tilde{r}$ superposed with the virtual world through the superphase $\vartheta$ coordinate:

$$ds^2=(d\tilde{r}+i\tilde{r}d\tilde{\vartheta})(d\tilde{r}-i\tilde{r}d\tilde{\vartheta})=d\tilde{r}^2+\tilde{r}^2d\tilde{\vartheta}^2 \tag{7.6}$$

$$x^m\in\check{x}\{\tilde{r},+i\tilde{\vartheta}\},\,x^\nu\in\hat{x}\{\tilde{r},-i\tilde{\vartheta}\} \tag{7.7}$$

The relationship of the metric tensor and inverse metric components is given straightforwardly by the following

$$\check{g}_{\nu\mu}=\hat{g}^{\nu\mu}=\begin{pmatrix}1&0\\0&\tilde{r}^2\end{pmatrix},\qquad \check{g}^{\nu\mu}=\hat{g}_{\nu\mu}=\begin{pmatrix}1&0\\0&\tilde{r}^{-2}\end{pmatrix} \tag{7.8}$$

where $\check{g}_{\nu\mu}\in Y^-$, and $\hat{g}^{\nu\mu}\in Y^+$. Normally, the coordinate basis vectors $\mathbf{b}_{\tilde{r}}$ and $\mathbf{b}_{\tilde{\vartheta}}$ are not orthonormal. Since the only nonzero derivative of a covariant metric component is $\check{g}_{\tilde{\vartheta}\tilde{\vartheta},\tilde{r}}=2\tilde{r}$, the toques in *Christoffel* symbols for polar coordinates are simplified to and become as a set of *Quadrant-State* matrices,

$$R_2^+=x^\mu\Gamma^{+}_{\nu\mu a}=x^\mu\begin{pmatrix}0&\tilde{r}\\ \tilde{r}&-\tilde{r}\end{pmatrix}\equiv\tilde{r}^2\epsilon_0+i\epsilon_2\tilde{r}\tilde{\vartheta} \qquad :\hat{\partial}^\lambda=\dot{x}^m R_2^+\partial^\nu \tag{7.9a}$$

$$R_1^+=x^\mu\Gamma^{+\nu}_{\mu a}=x^\mu\begin{pmatrix}0&1/\tilde{r}\\ 1/\tilde{r}&-\tilde{r}\end{pmatrix}\equiv\tilde{r}^2\epsilon_3-i\epsilon_1\tilde{r}\tilde{\vartheta} \qquad :\hat{\partial}_\lambda=\dot{x}_m R_1^+\partial^\nu \tag{7.9b}$$

$$R_1^-=x_s\Gamma^{-m}_{s\alpha}=x_s\begin{pmatrix}0&1/\tilde{r}\\ 1/\tilde{r}&-\tilde{r}\end{pmatrix}\equiv\tilde{r}^2\epsilon_3+i\epsilon_1\tilde{r}\tilde{\vartheta} \qquad :\check{\partial}^\lambda=\dot{x}^\nu R_1^-\partial_m \tag{7.9c}$$

$$R_2^-=x_m\Gamma^{-}_{nma}=x_m\begin{pmatrix}0&\tilde{r}\\ \tilde{r}&-\tilde{r}\end{pmatrix}\equiv\tilde{r}^2\epsilon_0-i\epsilon_2\tilde{r}\tilde{\vartheta} \qquad :\check{\partial}_\lambda=\dot{x}_\nu R_2^-\partial_m \tag{7.9d}$$

The $R_1^\pm$ matrices are a duality of the interactive settings for transportation between the two-dimensional world planes. The $R_2^\pm$ matrices are the residual settings for $Y^-$ and $Y^+$ transportation or within their own manifold, respectively. Defined as a set of the *Infrastructural Torque Generators*, this $\epsilon_\kappa$ group consists of the distinct members, featured as the following:

$$\epsilon_\kappa=\left[\begin{pmatrix}0&1\\1&0\end{pmatrix}_0,\begin{pmatrix}0&0\\0&-1\end{pmatrix}_1,\begin{pmatrix}0&0\\0&1\end{pmatrix}_2,\frac{1}{\tilde{r}^2}\begin{pmatrix}0&1\\1&0\end{pmatrix}_3\right] \tag{7.10}$$

As a group of the 2x2 matrices, the infinite (7.9) loops of entanglements institute an integrity of the spiral generators $\epsilon_n$ sourced by the transport generators $\epsilon_0$, shown by the following:

$$[\varepsilon_2,\varepsilon_1]=0=[\varepsilon_1,\varepsilon_0] \qquad :\textit{Independent Freedom} \tag{7.11a}$$

$$[\varepsilon_2,\varepsilon_3]=\frac{1}{\tilde{r}^2}s_2=[\varepsilon_3,\varepsilon_1] \qquad :\textit{Force Exposions} \tag{7.11b}$$

$$[\varepsilon_2,\varepsilon_0]=s_2=[\varepsilon_0,\varepsilon_1] \qquad :\textit{Commutation Invariance} \tag{7.11c}$$

In accordance with our philosophical anticipation, the above commutations between manifolds reveals that

a. Double loop entanglements are invariant and yield local independency, respectively.

b. Conservations of transportations are operated at the superposed world planes.

c. Spiral commutations generate the $s_2$ spinor to maintain its torsion conservation.

d. Commutative generators exert its physical contortion at inverse r-dependent.

Besides, the continuity of life-cycle transportations has the characteristics of

$$\langle\varepsilon_3,\varepsilon_0\rangle=\frac{2}{\tilde{r}^2}s_0 \qquad \langle\varepsilon_2,\varepsilon_1\rangle=2\varepsilon_1 \tag{7.12a}$$

$$\langle\varepsilon_2,\varepsilon_3\rangle=\varepsilon_3=-\langle\varepsilon_3,\varepsilon_1\rangle \qquad \langle\varepsilon_2,\varepsilon_0\rangle=\varepsilon_0=-\langle\varepsilon_0,\varepsilon_1\rangle \tag{7.12b}$$

It demonstrates the commutative principles among the torque generators:

a. The entire torque is sourced from the inception of the transformation $s_0$ and the physical contorsion $\varepsilon_3$; and

b. Each of the physical or virtual torsion is driven by the real force $\varepsilon_3$ or superposing torsion $\varepsilon_0$, respectively.

Similar to the boost generators, the double streaming torques orchestrate a set of the four-status.

***Artifact 7.4: Conservation of Superposed Torsion***. At the constant speed, the divergence of the torsion tensors are illustrated by the following:

$$\nabla \cdot R_2^- = \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\epsilon_0 \tilde{r}^2\right) - \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{\vartheta}}\left(\epsilon_2 \tilde{r}\,\tilde{\vartheta}\right) = 2\epsilon_0 - \epsilon_2 \tag{7.13a}$$

$$\nabla \cdot R_1^- = \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\epsilon_3 \tilde{r}^2\right) + \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{\vartheta}}\left(\epsilon_1 \tilde{r}\,\tilde{\vartheta}\right) = \epsilon_1 \tag{7.13b}$$

Because of the $Y^-Y^+$ reciprocity, each superphase $\tilde{\vartheta}$ is paired at its mirroring spiral opponent. Remarkably, on the world planes at $\tilde{r} = 0$, the total of each $Y^-Y^+$ torsion derivatives is entangling without singularity and yields invariant, introduced at 8:17 July 17 of 2018.

$$Y^- : \nabla \cdot (R_1^- + R_2^-) = 2\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \tag{7.14a}$$

$$Y^+ : \nabla \cdot (R_1^+ + R_2^+) = 2\begin{pmatrix} 0 & 1 \\ 1 & +1 \end{pmatrix} \tag{7.14b}$$

As the *Conservation of Superposed Torsion* under the superposed global manifolds, it implies that the transportations of the spiral torques between the virtual and physical worlds are

a. Modulated by the superphase $2\tilde{\vartheta}$-chirality, bi-directionally,

b. Operated at independence of spatial $\tilde{r}$-coordinate, respectively,

c. Streaming with its residual and opponent, commutatively, and

d. Entangling a duality of the reciprocal spirals, simultaneously.

This virtual-supremacy nature features the world planes a principle of *Superphase Ontology*, which, for examples, operates a macroscopic galaxy or blackhole system, or generates a microscopic spinor of particle system.

***Artifact 7.5: Manifold Signature***. The scaling $s_0$ and transform $s_3$ generators operate as the evolutionary processes giving rise to the infrastructure of the second horizon $SU(2)$ at two-dimensions and the third horizon $SU(3)\times SU(2)\times U(1)$ at four-dimensions. Each constitutes a pair of the bilinear forms

$$S_0^{\pm} = s_0 \pm i s_3 \equiv \eta^0 + i\eta^{\pm} \qquad : \textit{Manifold Signatures,} \tag{7.15}$$

$$s_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{I}_3 \end{pmatrix}, \qquad s_3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \mapsto \begin{pmatrix} -1 & 0 \\ 0 & \mathbf{I}_3 \end{pmatrix} \tag{7.16}$$

$$\eta^{\pm} = \pm \begin{pmatrix} -1 & 0 \\ 0 & \mathbf{I}_3 \end{pmatrix} \qquad : \textit{Metric Signature} \tag{7.17}$$

where $\eta^0 = s_0$ and $\eta^{\mp} = \pm s_3$ are known as the metric signature of the manifolds, and $S_0^-$ or $S_0^+$ is defined as the $Y^-$ or $Y^+$ *Manifold Signature*, respectively. When the metric signatures are a diagonal matrix, *Lie* algebra $O(1,3)$ consists of 2x2 or 4x4 matrices $M$ such that it has the transform with the metric signatures $\eta^{\mp} \rightleftharpoons \eta^{\pm}$ between the manifolds:

$$\eta^{\mp} M \eta^{\pm} = -M \qquad : \eta^{\mp} \rightleftharpoons \eta^{\pm} \tag{7.18}$$

Consequently, with one dimension $\tilde{r}$ in the world planes, the global manifolds are operated to extend the extra freedom of the two dimensions to its spatial coordinates or ***r****-vector*, where the group $SU(2)$ is locally isomorphic to $SU(3)$, and the physical $\boldsymbol{r}$ generators follow the same *Lie* algebra [2].

***Artifact 7.6: Infrastructural Signatures***. Upon the foundations of originator $s_0$ and commutator $s_3$, the infrastructures $S_n^+$ and $S_n^-$ contract the $s_1$ matrix as an *evolutionary producer* between manifolds, and the $s_2$ matrix as a *transformer* within each of the manifolds.

$$S_1^{\pm} = s_3 \mp i s_1 \qquad : \textit{Horizon Signature} \tag{7.19a}$$

$$S_2^{\pm} = s_0 \pm i s_2 \qquad : \textit{Transform Signature} \tag{7.19b}$$

Meanwhile, the spiral torques operate at signatures of the rotational infrastructure:

$$R_1^{\pm} = \tilde{r}(\tilde{r}\epsilon_3 \mp i\epsilon_1 \tilde{\vartheta}) \qquad : \textit{Interactive Signature} \tag{7.20a}$$

$$R_2^{\pm} = \tilde{r}(\tilde{r}\epsilon_0 \pm i\epsilon_2 \tilde{\vartheta}) \qquad : \textit{Transport Signature} \tag{7.20b}$$

Instinctively, the (7.1) flow institutes naturally the loop signature of the infrastructures:

$$S_2^+ \hookrightarrow S_1^+ \rightleftharpoons S_1^- \hookleftarrow S_2^- \qquad : \textit{Loop Infrastructure} \tag{7.21}$$

$$R_2^+ \rightleftharpoons R_1^+, \;\; R_1^- \rightleftharpoons R_2^- \qquad : \textit{Torque Infrastructure} \tag{7.22}$$

$$S_1^- = (S_1^+)^*, S_2^+ = (S_2^-)^* \qquad R_1^- = (R_1^+)^*, R_2^+ = (R_2^-)^* \tag{7.23}$$

Incredibly, the loop infrastructure orchestrates a life-cycle of the double streaming entanglements giving rise to the horizon and force fields.

***Artifact 7.7: Commutation of Infrastructure***. As a loop sequence of the matrices, the infrastructure consists of the self-circular commutations and constructs miraculously a pair of the tangent manifold spaces to facilitate the generalization of horizon entanglements from world planes to affine spaces, shown as the following:

$$[S_2^+, S_1^+] = +S_1^+ = [S_1^+, S_2^-], \qquad [S_2^+, S_1^-] = -S_1^- = [S_1^-, S_2^-] \tag{7.24}$$

$$[R_2^+, R_1^+] = i s_2 (1 + \tilde{r}^2)\vartheta = [R_1^-, R_2^-], \quad \lim_{\tilde{r}\to 0}[R_1^{\pm}, R_2^{\pm}] = \mp i\vartheta s_2 \tag{7.25}$$

$$[S_1^+, S_1^-] = i s_2 \tag{7.26}$$

where the "–" sign implies the reverse or mirroring loop charity between the $Y^-Y^+$ manifolds. Apparently, the horizon signatures $S_1^{\pm}$ and the interactive signatures $R_1^{\pm}$ lie at a center of the core infrastructure dynamically bridging the two district activities or residual dynamics $S_2^{\pm}$ or $R_2^{\pm}$ distinctively. Phenomenally, the transform *Generator* $s_2$ plays an essential role as the natural resource tie bonding and streaming the double entanglements.

***Artifact 7.8: Entangling Independence***. At the local environment, the relationship of their commutations and continuities can be derived as the following, respectively:

$$[S_2^+, S_2^-] = 0 \qquad \langle S_2^+, S_2^- \rangle = 0 \tag{7.27}$$

$$[R_2^+, R_2^-] = [R_1^+, R_1^-] = 0 \qquad \lim_{\tilde{r}\to 0}\langle R_2^+, R_2^- \rangle = \tilde{r}^2(s_0 + \epsilon_2 \tilde{\vartheta}^2) \to 0 \tag{7.28}$$

When any two objects are commutative at zero, it implies and reveals amazingly the independence between the manifold opponents:

1) The residual dynamics is independent to its opponent $[S_2^+, S_2^-] = 0$ and $[R_2^+, R_2^-] = [R_1^+, R_1^-] = 0$ while they jointly conserve the density flow $\langle S_2^+, S_2^- \rangle = 0$ and torque flow $\lim_{\tilde{r}\to 0}\langle R_2^+, R_2^- \rangle \to 0$, harmonizing an integrity of their environment and conserving a duality of the dynamic invariant.

2) The super force interaction between objects is independent to their torque commutation $[\varepsilon_0, \varepsilon_3] = 0$ while, under invariance of the torque transportations, they are jointly modulated by the superposing phases to maintain or preserve the transformational $s_0$ generator.

$$\lim_{\tilde{r}\to 0}\langle R_1^+, R_1^- \rangle = s_0 + \epsilon_2 \tilde{r}^2 \tilde{\vartheta}^2 \to s_0 \tag{7.29}$$

Inconceivably, the infrastructural invariant orchestrates the life-cycle generators of the double entanglements, giving rise to the horizons and force fields.

***Artifact 7.9: Chiral Entanglement***. When an axis passes through the center of an object, the object is said to rotate upon itself locally, or spin. Furthermore, when there are two axes passing through the center of an object, the object is said to be under the entanglements of the *YinYang* duality. Remarkably, the infrastructure consists of a pair of the double rotation fields such that each of the entangling matrix $S_1^{\pm}$ or $S_2^{\pm}$ has its corresponding normalized *Eigenvectors* ($|S_n^{\pm}\psi - \lambda I| = 0$, $S_n^{\pm}\psi = \lambda\psi$), respectively:

$$S_2^+, S_1^- \mapsto \psi^L = \begin{pmatrix} 1 \\ -i \end{pmatrix}^{\circlearrowleft} \qquad : \textit{Left-hand chirality} \tag{7.30}$$

$$S_1^+, S_2^- \mapsto \psi^R = \begin{pmatrix} 1 \\ +i \end{pmatrix}^{\circlearrowleft} \qquad : \textit{Right-hand chirality} \tag{7.31}$$

Together, the $S_1^+ + S_2^-$ matrix of virtual supremacy produces the left-hand eigenvector or (7.14) while the $S_1^- + S_2^+$ matrix of physical supremacy severs the right-hand eigenvector or (7.15). With respect to the whole-cycle of the spin-up and spin-down potentials, the "$-i$" sign represents the left-hand chiral in the $Y^+$ manifold, and the "$+i$" sign depicts the right-hand chiral in the $Y^-$ manifold. Therefore, spin chirality is a type of the virtual $Y^+$ and physical $Y^-$ transformation that object entanglements on the world lines $(\hat{\partial}_\lambda, \check{\partial}^\lambda)$ consist of the residual transportations $(\hat{\partial}^\lambda, \check{\partial}_\lambda)$ of the $Y^-Y^+$ spinors, reciprocally, such that the nature materializes the spinors characterized by the left-handed and the right-handed chirality sourced from or driven by each of the manifolds of the virtual $Y^+$ and physical $Y^-$ dynamics. Following the trajectory (7.1), it takes in total two full rotations 720° from the $W^+$ to $W^-$ and then back to $W^+$ world plane, and vice versa, for an object to return to its original state. With its opponent companionship, the infrastructure (7.10) of a whole system yields the parity conservation by maintaining and entangling the double duality reciprocally and simultaneously.

***Artifact 7.10: Entangle Invariance***. As the dynamic streaming, the entanglements of commutative densities are operated and balanced by the dark energies providing the common resources between the physical and virtual existences in order to maintain conservation of entanglements, shown by the chart. It represents *Law of Conservation of Entanglements*, or simply *Entangle Invariance*. Conservation of momentum applies only to an isolated system of entangle objects as a whole. Under this condition, an isolated system is one that is not acted on by objects external to the system, and that both entanglers are closed in a virtual space of the world regardless of their physical distance.

Artifact 7.10: ***Law of Conservation of Entanglements***

1) *At least two types of densities are required in order to entangle fluxions.*
2) *Flux transports and performs as a duality of virtual fields and real forces.*
3) *Total fluxion of an entangle streaming must be sustainable and invariant.*
4) *Flux remains constant and conserves over time during its transportation.*
5) *A fluxion density can neither be created nor destroyed by entanglement.*
6) *Transportation of entangle momentum is conserved at its zero net value.*
7) *Momentum is exchangeable through cross interactions among participants.*

***Artifact 7.11: Gamma Matrices***. Considering the mirroring effects $-f^*(z^*)$ between manifolds, the (7.2) matrices institutes an infrastructure,

$$\tilde{\gamma}^\nu \equiv \begin{pmatrix} \{S_1^-, S_2^+\} \\ -\{S_1^+, S_2^-\}^* \end{pmatrix} \qquad \tilde{\gamma}_\nu \equiv \begin{pmatrix} \{S_1^+, S_2^-\} \\ -\{S_1^-, S_2^+\}^* \end{pmatrix} \tag{7.32}$$

$$\tilde{\gamma}^\nu = \left[ \begin{pmatrix} s_0 & 0 \\ 0 & -s_0 \end{pmatrix}_0, -i \begin{pmatrix} 0 & s_1 \\ s_1 & 0 \end{pmatrix}_1, i \begin{pmatrix} 0 & s_2 \\ s_2 & 0 \end{pmatrix}_2, \begin{pmatrix} 0 & s_3 \\ -s_3 & 0 \end{pmatrix}_3 \right] \tag{7.33}$$

$$\tilde{\gamma}_\nu^- = -\eta^- \tilde{\gamma}^\nu \qquad \{S_1^-, S_2^+\} = \{S_1^+, S_2^-\}^* \tag{7.34}$$

Simply extended by the mirroring chirality $-(S_n^\pm)^*$, the tilde-gamma matrices $\tilde{\gamma}^\nu$ represent the upper-row of one manifold dynamic stream $\{\hat{\partial}^\lambda, \check{\partial}^\lambda\}$ and the lower-row for its opponent $\{\hat{\partial}_\lambda, \check{\partial}_\lambda\}$.

***Artifact 7.12: Chi Matrices***. In parallel to the tilde-gamma matrices, one can contract another superposed tilde-chi matrices $\tilde{\chi}^\nu$ representing (7.9) a set of the mirroring spiral torque tensors.

$$\tilde{\chi}^\nu = \tilde{r} \left[ \tilde{r} \begin{pmatrix} \epsilon_0 & 0 \\ 0 & -\epsilon_0 \end{pmatrix}_0, -i\tilde{\vartheta} \begin{pmatrix} 0 & \epsilon_1 \\ \epsilon_1 & 0 \end{pmatrix}_1, i\tilde{\vartheta} \begin{pmatrix} 0 & \epsilon_2 \\ \epsilon_2 & 0 \end{pmatrix}_2, \tilde{r} \begin{pmatrix} 0 & \epsilon_3 \\ -\epsilon_3 & 0 \end{pmatrix}_3 \right] \tag{7.35}$$

$$\tilde{\chi}_\nu = -\eta^- \tilde{\chi}^\nu \qquad \{R_1^-, R_2^+\} = \{R_1^+, R_2^-\}^* \tag{7.36}$$

Each of the $\tilde{\chi}_\nu^\pm$ matrices is a set of the vector matrices with the upper-row for one infrastructural stream and the lower-row for its opponent manifold. Together, they further descend into its higher dimensional manifold. The $\chi_\mu^\pm$ fields are a pair of the torque-graviton potentials.

***Artifact 7.13: Superphase Fields at Second Horizon***. At the loop entanglements $\phi^+(\hat{x}) \rightleftharpoons \varphi^-(\check{x})$, the processes operate the particle fields in forms of transformations $S_i^\pm$, torque representations $R^\mu{}_\nu$ and $R_\mu{}^\nu$, and *Gauge* potentials $A_\nu \mapsto eA_\nu/\hbar$ for electrons and $A^\nu \mapsto eA^\nu/\hbar$ for positrons. Consequently, we have the total effective fields in each of the respective manifolds:

$$\check{\partial}_\lambda \phi^- + \hat{\partial}_\lambda \varphi^+ = \dot{x}_\nu \tilde{\zeta}_\nu \left[ \begin{pmatrix} \partial_\nu \\ \partial^\nu \end{pmatrix}' \pm i \frac{e}{\hbar} \begin{pmatrix} A_\nu \\ A^\nu \end{pmatrix}' \right] \psi^- \quad : \psi^- = \begin{pmatrix} \phi^- \\ \varphi^+ \end{pmatrix} \tag{7.37}$$

$$\check{\partial}_\lambda = \dot{x}_\nu \left( S_2^- + R_2^- \right) \left( \partial_m + i \frac{e}{\hbar} A_\nu \right), \hat{\partial}_\lambda = \dot{x}_\nu \left( S_1^+ + R_1^+ \right) \left( \partial^\mu - i \frac{e}{\hbar} A^\mu \right)$$

$$\hat{\partial}^\lambda \phi^+ + \check{\partial}^\lambda \varphi^- = \dot{x}^\nu \tilde{\zeta}^\nu \left[ \begin{pmatrix} \partial^\nu \\ \partial_\nu \end{pmatrix}' \mp i \frac{e}{\hbar} \begin{pmatrix} A^\nu \\ A_\nu \end{pmatrix}' \right] \psi^+ \quad : \psi^+ = \begin{pmatrix} \phi^+ \\ \varphi^- \end{pmatrix} \tag{7.38}$$

$$\hat{\partial}^\lambda = \dot{x}^\nu \left( S_2^+ + R_2^+ \right) \left( \partial^m - i \frac{e}{\hbar} A^\nu \right), \check{\partial}^\lambda = \dot{x}^\nu \left( S_1^- + R_1^- \right) \left( \partial_\mu + i \frac{e}{\hbar} A_\mu \right)$$

$$\tilde{\zeta}^\nu = \tilde{\gamma}^\nu + \tilde{\chi}^\nu \qquad \tilde{\zeta}_\nu = \tilde{\gamma}_\nu + \tilde{\chi}_\nu \tag{7.39}$$

The potential $\psi^-$ or $\psi^+$ implies each of the loop entanglements is under its $Y^-$ or $Y^+$ manifold, respectively. The first equation represents the horizon potentials at the local $\check{\partial}_\lambda \varphi^-$ of the $Y^-$ manifold and the transformation $\hat{\partial}_\lambda \phi^+$ from its $Y^+$ opponent. Likewise, the second equation corresponds to the horizon potentials at the local $\hat{\partial}^\lambda \phi^+$ of the $Y^+$ manifold and the transformation $\check{\partial}^\lambda \varphi^-$ from its $Y^-$ opponent. To collapse the above equations together, we have a duality of the states expressed by or degenerated to the classical formulae:

$$\check{\partial}\psi^- \equiv \check{\partial}_\lambda \phi^- + \hat{\partial}_\lambda \varphi^+ = \dot{x}_\nu \tilde{\zeta}_\nu D_\nu \psi^- \quad : D_\nu = \partial_m + i \frac{e}{\hbar} A_m \tag{7.41}$$

$$\hat{\partial}\psi^+ \equiv \hat{\partial}^\lambda \phi^+ + \check{\partial}^\lambda \varphi^- = \dot{x}^\nu \tilde{\zeta}^\nu D^\nu \psi^+ \quad : D^\nu = \partial^\nu - i \frac{e}{\hbar} A^\nu \tag{7.42}$$

To our expectation, the $A_\nu$ and $A^\nu$ fields are a pair of the graviton-photon potentials. Intuitively, both photons and gravitons are the outcomes or products of a duality of the double entanglements.

## VIII. PHYSICAL HORIZON INFRASTRUCTURES

At motion dynamics of the second horizon, the tangent of the scalar density fields constructs the vector fields. As an astonishing consequence, under the two-dimensions of the world planes, the horizon generator $s_1$ incepts a freedom of the extra dimensions into the physical or virtual world, respectively giving rise to the third horizon:

$$s_1(2) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & 1 \\ 1 & s_1(3) \end{pmatrix}, \quad s_1(3) = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \tag{8.1}$$

By means of transformation between the manifolds, the matrix $s_1$ functions as *Generators* giving rise to the three-dimensional space. Simultaneously, by means of the transportation, the residual freedom of the $s_2$ matrix rotating itself into three-dimensions of the spatial manifolds: $SO(3)$.

$$s_2(2) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 & -1 \\ 1 & s_2(3) \end{pmatrix}, \quad s_2(3) = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \tag{8.2}$$

Together, a pair of the matrices, $s_1$ and $s_2$, institutes the third horizon and constructs an infrastructure in four-dimensions: $SU(2) \times SO(3)$.

***Artifact 8.1: Pauli Matrices***. Apparently, the *Infrastructural Generators* can contract alternative matrices that might extend to the physical topology. Among them, one popular set is shown as the following:

$$\sigma_\kappa = \left[ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_0, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}_1, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}_2, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_3 \right] \tag{8.3}$$

$$\sigma_0 = s_0 \qquad \sigma_1 = s_1 \qquad \sigma_2 = i s_2 \qquad \sigma_3 = -s_3 \qquad \sigma_n^2 = I \tag{8.3a}$$

$$[\sigma_a, \sigma_b]^- = 2i\varepsilon_{cba}^- \sigma_c \qquad [\sigma_a, \sigma_b]^+ = 0 \quad : a, b, c \in (1,2,3) \tag{8.3b}$$

known as *Pauli* spin matrices, introduced in 1925 [11]. In this definition, the residual spinors $S_2^\pm$ are extended into the physical states toward the interpretations for the decoherence into a manifold of the four-dimensional spacetime-coordinates of physical reality.

***Artifact 8.2: Physical Gamma and Chi Matrices***. Aligning to the topological comprehension, we extend the gamma-matrix $\gamma^\nu$, introduced by *W. K. Clifford* in the 1870s [8], and chi-matrix $\chi^\nu$ for physical coordinates.

$$\zeta^\nu = \gamma^\nu + \chi^\nu \qquad \zeta_\nu = \gamma_\nu + \chi_\nu \tag{8.4}$$

$$\gamma^\nu = \left[ \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix}_0, \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}_1, \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}_2, \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}_3 \right] \tag{8.4a}$$

$$\chi^\nu = \left[ \begin{pmatrix} \varsigma_0 & 0 \\ 0 & -\varepsilon_0 \end{pmatrix}_0, \begin{pmatrix} 0 & \varsigma_1 \\ -\varsigma_1 & 0 \end{pmatrix}_1, \begin{pmatrix} 0 & \varsigma_2 \\ -\varsigma_2 & 0 \end{pmatrix}_2, \begin{pmatrix} 0 & \varsigma_3 \\ -\varsigma_3 & 0 \end{pmatrix}_3 \right] \tag{8.4b}$$

$$\varsigma_0 = \tilde{r}^2 \epsilon_0 \qquad \varsigma_1 = \tilde{r}\,\vartheta\,\epsilon_1 \qquad \varsigma_2 = i\tilde{r}\,\vartheta\,\epsilon_2 \qquad \varsigma_3 = \tilde{r}^2 \epsilon_3 \tag{8.4c}$$

The superphase $\mathrm{d}\vartheta$ of polar coordinates extends into the circumference-freedom $\mathrm{d}\vartheta \mapsto \mathrm{d}\theta \pm i \sin\theta\, \mathrm{d}\phi$ of sphere coordinates.

$$\mathrm{d}\vartheta^2 \mapsto (\mathrm{d}\theta + i\sin\theta\,\mathrm{d}\phi)(\mathrm{d}\theta - i\sin\theta\,\mathrm{d}\phi) = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \tag{8.5}$$

Similar to *Pauli* matrices, the gamma $\gamma^\nu$ and chi $\chi^\nu$ matrices are further degenerated into a spacetime manifold of the physical reality. To collapse the (7.41, 7.42) equations together, we have a duality of the states expressed by or degenerated to the formulae of event operations:

$$\check{\partial} = \check{\partial}_\lambda + \hat{\partial}_\lambda = \dot{x}_\nu \zeta_\nu D_\nu = \dot{x}_\nu\, \zeta_\nu \Big( \partial_\nu + i\frac{e}{\hbar}A_\nu + \tilde{\kappa}_2^- \partial_\nu A_\mu + \cdots \Big) \tag{8.6a}$$

$$\hat{\partial} = \hat{\partial}^\lambda + \check{\partial}^\lambda = \dot{x}^\mu \zeta^\mu D^\mu = \dot{x}^\mu \zeta^\mu \Big( \partial^\mu - i\frac{e}{\hbar}A^\mu - \tilde{\kappa}_2^+ \partial^\mu A^\nu - \cdots \Big) \tag{8.6b}$$

Accordingly, all terms have a pair of the irreducible and complex quantities that preserves the full invariant and streams a duality of the $Y^-$ and $Y^+$ loop $\hat{\partial}^\lambda \hookrightarrow \hat{\partial}_\lambda \rightleftharpoons \check{\partial}^\lambda \hookleftarrow \check{\partial}_\lambda$ entanglements.

***Artifact 8.3: Superphase Fields at Second Horizon***. As the superphase function from the second to third horizon, the vector field $A^\nu$ bonds and projects its potentials superseding with its conjugator, arisen by or acting on its opponent $A_\nu$ through a duality of reciprocal interactions dominated by boost $\tilde{\gamma}$ and twist $\tilde{\chi}$ fields, evolution into the second ($\tilde{\zeta} \mapsto \zeta$) horizon. Under the $Y^-$ or $Y^+$ primary, the event operates the third terms of (8.6) in a pair of the relativistic entangling fields:

$$F^{-n}_{\nu\mu} = \left( \zeta_\nu \partial_\nu A_\mu - \zeta^\mu \partial^\mu A^\nu \right)_n = -F^{+n}_{\mu\nu} \qquad : \tilde{F}^{\pm n}_{\nu\mu}(\tilde{\zeta}) \mapsto F^{\pm n}_{\mu\nu}(\zeta) \tag{8.7}$$

The tensor $F^{\pm n}_{\nu\mu}$ is the transform and torque fields at second horizon. The transform and transport tensors naturally consist of the antisymmetric field components and construct a pair of the superphase potentials in world planes, giving rise to the third horizon fields, emerging the four-dimensional spacetime, and producing the electromagnetism and gravitation fields.

***Artifact 8.4: Lorentz Generators***. Giving rise to the third horizon, the (8.1, 8.2) generators contract with the $\zeta$ infrastructure and evolve into the four-dimensional matrices $SU(2)_{s_1} \times SO(3)_{s_2}$, shown by the following:

$$J_1 = \begin{pmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&-1\\0&0&1&0 \end{pmatrix}, J_2 = \begin{pmatrix} 0&0&0&0\\0&0&0&1\\0&0&0&0\\0&-1&0&0 \end{pmatrix}, J_3 = \begin{pmatrix} 0&0&0&0\\0&0&-1&0\\0&1&0&0\\0&0&0&0 \end{pmatrix} \tag{8.10}$$

$$K_1 = \begin{pmatrix} 0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0 \end{pmatrix}, K_2 = \begin{pmatrix} 0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0 \end{pmatrix}, K_3 = \begin{pmatrix} 0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0 \end{pmatrix} \tag{8.11}$$

$$L^-_\nu = K_\nu + iJ_\nu \qquad L^+_\nu = K_\nu - iJ_\nu \tag{8.12a}$$

$$[J_1, J_2]^- = J_3 \qquad [K_1, K_2]^- = -J_3 \qquad [J_1, K_2]^- = K_3 \tag{8.12b}$$

known as *Generator* of the *Lorentz* group, discovered since 1892 [3] or similar to *Gell-Mann* matrices [17]. Conceivably, the $K_\nu$ or $J_\nu$ matrices are residual $\{\hat{\partial}^\lambda, \hat{\partial}_\lambda\}$ or rotational $\{\check{\partial}_\lambda, \check{\partial}^\lambda\}$ components, respectively. During the transitions between the horizons, the redundant degrees of freedom is developed and extended from superphase $\vartheta$ of world-planes into the extra physical coordinates (such as $\theta$ and $\phi$ in).

***Artifact 8.3: Fields at Third or Higher Horizons***. For the field structure at the third or higher horizons, a duality of reciprocal interactions dominated by boost $\gamma$ and twist $\chi$ fields is developed into the third ($\zeta \mapsto L$) horizon.

$$T^{-n}_{\nu\mu}(L) = \left( L^-_{\nu\mu} \partial_\nu A_\mu - L^+_{\mu\nu} \partial^\mu A^\nu \right)_n \qquad : F^{\pm n}_{\nu\mu}(\gamma) \mapsto T^{\pm n}_{\mu\nu}(L) \tag{8.13}$$

$$\Upsilon^{-n}_{\nu\mu}(L) = \left( L^-_{\nu\mu} \partial_\nu V_\mu - L^+_{\mu\nu} \partial^\mu V^\nu \right)_n \qquad : F^{\pm n}_{\nu\mu}(\chi) \mapsto \Upsilon^{\pm n}_{\mu\nu}(L) \tag{8.14}$$

where $T^{\pm n}_{\mu\nu}(L^\pm_\nu)$ is electromagnetic fields and $\Upsilon^{\pm n}_{\mu\nu}(L^\pm_\nu)$ is gravity fields. Under the $Y^-$ or $Y^+$ primary, the event operates the third terms of (8.6) in a pair of the relativistic entangling fields.

***Artifact 8.5: Physical Torque Singularity***. Descendent from the world planes with the convention coordinates $\{r, \theta, \varphi\}$, a physical coordinate system is further extended its metric elements of $ds^2 = dr^2 + r^2(d^2\theta + sin^2 d\varphi^2)$ in a physical $\mathscr{R}^3$ space. The redundant degrees has its freedom of $\{\theta, \varphi\}$ coordinates with the metric and its inverse elements of:

$$\breve{g}_{\nu\mu} = \begin{pmatrix} 1&0&0\\0&r^2&0\\0&0&r^2 sin^2\theta \end{pmatrix}, \qquad \breve{g}^{\nu\mu} = \begin{pmatrix} 1&0&0\\0&r^{-2}&0\\0&0&r^{-2} sin^{-2}\theta \end{pmatrix} \tag{8.15}$$

The *Christoffel* symbols of the sphere coordinates become the matrices:

$$\Gamma^-_{r\nu\mu} = \Gamma^{-r}_{\nu\mu} = \begin{pmatrix} 0&0&0\\0&-r&0\\0&0&-r sin^2\theta \end{pmatrix} \tag{8.16a}$$

$$\Gamma^{-\theta}_{\nu\mu} = \begin{pmatrix} 0&\frac{1}{r}&0\\ \frac{1}{r}&0&0\\0&0&-sin\theta cos\theta \end{pmatrix}, \quad \Gamma^{-\varphi}_{\nu\mu} = \begin{pmatrix} 0&0&\frac{1}{r}\\0&0&cot\theta\\ \frac{1}{r}&cot\theta&0 \end{pmatrix} \tag{8.16b}$$

$$\Gamma^-_{\theta\nu\mu} = r^2 \Gamma^{-\theta}_{\nu\mu}, \qquad \Gamma^-_{\varphi\nu\mu} = r^2 sin^2\theta\, \Gamma^{-\varphi}_{\nu\mu} \tag{8.16c}$$

Apparently, the divergence of the spiral torque fields has the $r$-dependency, expressed by the divergence in spherical coordinates:

$$\nabla \cdot R^-_1 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \Gamma^{-r}_{\nu\mu}) + \frac{1}{r sin\theta}\Big[ \frac{\partial}{\partial\theta}(sin\theta\, \Gamma^{-\theta}_{\nu\mu}) + \frac{\partial}{\partial\varphi}(\Gamma^{-\varphi}_{\nu\mu}) \Big] \tag{8.17}$$

When the $r$-coordinate aligns to the superposition $\tilde{r}$, the three-dimensions of a physical space has its redundant degrees of freedom $\{\theta, \varphi\}$ such that the torque transportation becomes $r$-dependent inversely proportional to the square of distance or appears as the gravitational singularity. Therefore, one spatial dimension on the world planes evolves its physical world into the extra two-coordinates with a rotational *Central-Singularity*. This nature of physical-supremacy characterizes forces between objects and limits their interactive distances. As an associative affinity, this principle of the central-singularity, for examples, operates the gravitational attractions between the mass bodies, or gives weight to physical objects in residence.

***Artifact 8.6: Friedmann-Lemaître Model***. Introduced in the 1920s, the *Friedmann–Lemaître–Robertson–Walker* (FLRW) [25] metric attempts a solution of *Einstein's* field equations of general relativity. Aimed to the gravitational inverse-square law, the research discovered that the desired outcome leads to the polar coordinates on a world plane:

$$\mathrm{d}\Sigma^2 = \mathrm{d}r^2 + S_k(r)^2\,\mathrm{d}\vartheta^2 \qquad : \mathrm{d}\vartheta^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 \tag{8.18}$$

$$S_k(r) = sinc(r\sqrt{k}) = \begin{cases} \sin\left(r\sqrt{k}\right)/\sqrt{k}, & k > 0 \\ r, & k = 0 \\ \sinh\left(r\sqrt{|k|}\right)/\sqrt{k}, & k < 0. \end{cases} \tag{8.19}$$

Apparently, it represents the virtual (k<0) and physical (k>0) of the "hyperspherical coordinates" bridged by the polar coordinate system (k=0), which emerges into the third horizon to gain the extra two-coordinates. Therefore, it evidently supports a proof to our full description of the evolutionary process coupling the horizons between the two-dimensional *World Planes* and the three-dimensional physical spacetime manifold.

***Artifact 8.7: Spacetime Evolution***. Generally, a spacetime of the third horizon is manifested and given rise from the second horizon to gain the extra freedom and evolution into three-dimensions of a physical space. The event operation of evolution is mathematically describable through transitioning functions from the tilde-zeta-matrices of the first horizon to the zeta-matrices $\tilde{\zeta} \mapsto \zeta$ of the second horizon, to the *Lorentz*-matrices $\zeta \mapsto L_\nu^\pm$ of the third horizon. Dependent on their $Y^-Y^+$ commutations or continuities through the tangent curvatures of potentials, the entangling processes develop the dark fluxions of fields, forces and entanglements to evolve the physical spacetime, prolific ontology, and eventful cosmology.

***Artifact 8.8: Inauguration of Gravity***. At the second horizon, conservation of *light* is sustained by its potential fields $F_{\nu\mu}^{\pm n}(\gamma)$ and transported by its companion partner: torque $F_{\nu\mu}^{\mp n}(\chi)$ fields. At the third horizon, given rise to, $\zeta^\nu \mapsto L_\nu^\pm$, the freedom of the extra rotations, the world planes are further evolved into *Spacetime* manifolds, where the torque $F_{\nu\mu}^{\pm n}(\chi)$ fields are transited to gravitational $\Upsilon_{\mu\nu}^{\pm n}(L_\nu^\pm)$ forces with a central-singularity. Therefore, at the inauguration of mass enclave at the third horizon, appearing as if there were from nothing at the second horizon, the fluxion of the superphase entanglement exerts gravity fields in a spacetime manifold.

## IX. QUANTUM FIELD EQUATIONS

At the first horizon, the individual behaviors of objects or particles are characterized by their timestate functions of $\phi_n^+$ or $\phi_n^-$ in the $W_a$ equations. Due to the nature superphase modulation of virtual and physical coexistences, particle fields appear as quantization in mathematics.

Under a steady environment of the energy fluxions $W_n^\pm$, the equations (6.7) and (6.13) [1] can be reformulated into the compact forms for the $Y^+$ supremacy of the entanglements: *the $Y^+$ Quantum Field Equations*

$$\frac{-\hbar^2}{2E_n^+}\hat{\partial}_\lambda\hat{\partial}_\lambda\phi_n^+ - \frac{\hbar}{2}\left(\hat{\partial}_\lambda - \check{\partial}^\lambda\right)\phi_n^+ + \frac{\hbar^2}{2E_n^+}\check{\partial}_\lambda\left(\hat{\partial}_\lambda - \check{\partial}^\lambda\right)\phi_n^+ = \frac{W_n^+}{c^2}\phi_n^+ \quad \mathbf{(9.1)}$$

$$\frac{\hbar^2}{2E_n^-}\check{\partial}^\lambda\check{\partial}^\lambda\varphi_n^- - \frac{\hbar}{2}\left(\check{\partial}^\lambda - \hat{\partial}_\lambda\right)\varphi_n^- + \frac{\hbar^2}{2E_n^-}\left(\check{\partial}_\lambda - \hat{\partial}_\lambda\right)\check{\partial}^\lambda\varphi_n^- = \frac{W_n^-}{c^2}\varphi_n^- \quad \mathbf{(9.2)}$$

$$\kappa_1 = \hbar c^2/2 \qquad \kappa_2 = \pm(\hbar c)^2/(2E_n^\mp) \qquad W_n^\pm = c^2 E_n^\pm \qquad (9.3)$$

where $E_n^\pm$ is an energy state of a virtual object or a physical particle. It emerges that the bi-directional transformation has two rotations one with left-handed $\phi_n^+ \mapsto \phi_n^L$ acting from the $Y^+$ source to the $Y^-$ manifold, and the other right-handed $\varphi_n^- \mapsto \phi_n^R$ reacting from the $Y^-$ back to the $Y^+$ manifold. Both fields are alternating into one another under a parity operation with relativistic preservation.

The entanglement of $Y^+$-supremacy represents one of the important principles of natural governances - ***Law of Conservation of Virtual Creation and Annihilation***:

1. The operational action $\hat{\partial}^\lambda$ of virtual supremacy results in the physical effects as the parallel and reciprocal reactions or emanations $\check{\partial}_\lambda$ in the physical world;
2. The virtual world transports the effects $\hat{\partial}_\lambda\hat{\partial}_\lambda$ emerging into or appearing as the creations of the physical world, even though the bi-directional transformations seem balanced between the commutative operations of $\hat{\partial}_\lambda$ and $\check{\partial}^\lambda$; and
3. As a part of the reciprocal processes, the physical world transports the reactive effects $\check{\partial}^\lambda\check{\partial}_\lambda$ concealing back or disappearing as annihilation processes of virtual world.

As a set of the universal laws, the events incepted in the virtual world not only generate its opponent reactions but also create the real-life objects in the physical world. The obvious examples are the formations of the elementary particles that a) the antiparticles in a virtual world generate the physical particles through their opponent duality of the event operations; b) by carrying and transitioning the informational massages, particles and antiparticles grow into real-life objects vividly in a physical world and maintain their living entanglement; c) recycling objects of a physical world as one of continuity processes for virtual-life streaming.

As a reciprocal process, another pair of the equations (6.12) and (6.8) [1] simultaneously formulates the following components for the $Y^-$ supremacy of entanglements: *the $Y^-$ Quantum Field Equations*

$$\frac{\hbar^2}{2E_n^-}\check{\partial}^\lambda\check{\partial}_\lambda\phi_n^- - \frac{\hbar}{2}\left(1 + \frac{\hbar}{E_n^-}\hat{\partial}^\lambda\right)\left(\check{\partial}_\lambda - \hat{\partial}^\lambda\right)\phi_n^- = \frac{W_n^-}{c^2}\phi_n^- \quad \mathbf{(9.4)}$$

$$\frac{-\hbar^2}{2E_n^+}\hat{\partial}^\lambda\hat{\partial}^\lambda\varphi_n^+ - \frac{\hbar}{2}\left(1 - \frac{\hbar}{E_n^+}\check{\partial}^\lambda\right)\left(\hat{\partial}^\lambda - \check{\partial}_\lambda\right)\varphi_n^+ = \frac{W_n^+}{c^2}\varphi_n^+ \quad \mathbf{(9.5)}$$

The $Y^-$ parallel entanglement represents another essential principle of $Y^-$ natural behaviors - ***Law of Conservation of Physical Animation and Reproduction***:

1. The operational action $\check{\partial}_\lambda$ of physical supremacy results in their conjugate or imaginary effects of animations because of the parallel reaction $\hat{\partial}^\lambda$ in the virtual world;
2. Neither the actions nor reactions impose their final consequences $\check{\partial}^\lambda\check{\partial}^\lambda$ on their opponents because of the parallel mirroring residuals for the horizon phenomena of reproductions $\hat{\partial}^\lambda\hat{\partial}^\lambda$ during the symmetric fluxions;
3. There are one-way commutations of $\check{\partial}^\lambda\check{\partial}_\lambda$ in transporting the events of the physical world into the virtual world asymmetrically. As a part of the reciprocal processes, the virtual world replicates $\hat{\partial}^\lambda$ the physical events during the mirroring $\hat{\partial}^\lambda\check{\partial}_\lambda$ processes in the virtual world.

As another set of laws, the events initiated in the physical world must leave a life copy of its mirrored images in the virtual world without the intrusive effects in the virtual world. In other words, the virtual world is aware of and immune to the physical world. In this perspective, continuity for a virtual-life streaming might become possible as a part of recycling or reciprocating a real-life in the physical world.

***Artifact 9.1: Mass-energy Equivalence***. In mathematical formulations of entanglements, we redefine the energy-mass conversion in the forms of virtual complexes as the following:

$$E_n^\mp = \pm i m c^2 \qquad : \hbar\omega \rightleftharpoons mc^2 \qquad (9.6)$$

where $m$ is the rest mass. Compliant with a duality of *Universal Topology* $W = P \pm iV$, it extends *Einstein* mass-energy equivalence, introduced in 1905 [10], into the virtual energy states as one of the essential formulae of the topological framework.

***Artifact 9.2: Dirac Equation***. Intrinsically heterogeneous, one of the characteristics of spin is that the events in the $Y^+$ or $Y^-$manifold transform into their opponent manifold in forms of bispinors of special relativity, reciprocally. Considering the first order $\dot{\partial}$ only and applying the transformational characteristics (8.6), we add (9.1)-(9.5) together to formulate the simple compartment:

$$\frac{\hbar}{2}\left(\dot{x}_\nu\zeta_\mu D_\nu - \dot{x}^\mu\zeta^\mu D^\mu\right)\psi_n^\pm \mp E_n^\pm\psi_n^\pm = 0 \qquad (9.7)$$

$$\psi_n^+ = \begin{pmatrix}\phi_n^+ \\ \varphi_n^-\end{pmatrix}, \ \overline{\psi}_n^- = \bar{\kappa}\begin{pmatrix}\varphi_n^- \\ \phi_n^+\end{pmatrix}, \ \psi_n^- = \begin{pmatrix}\phi_n^- \\ \varphi_n^+\end{pmatrix}, \ \overline{\psi}_n^+ = \bar{\kappa}\begin{pmatrix}\varphi_n^+ \\ \phi_n^-\end{pmatrix} \qquad (9.8)$$

where $\overline{\psi}_n^\pm$ is the adjoint potential and $\bar{\kappa}$ is a constant subject to renormalization. Ignoring the torsion fields $\chi^\mu$ and $\chi_\mu$, we have the above compact equations reformulated into the formulae:

$$\tilde{\mathscr{L}}_D^+ = \overline{\psi}_n^-\gamma^\mu\left(i\hbar c\,\partial^\mu + eA^\mu\right)\psi_n^+ + mc^2\overline{\psi}_n^-\psi_n^+ \to 0 \qquad (9.9a)$$

$$\tilde{\mathscr{L}}_D^- = \overline{\psi}_n^+\gamma_\nu\left(i\hbar c\,\partial_\nu - eA_\nu\right)\psi_n^- - mc^2\overline{\psi}_n^+\psi_n^- \to 0 \qquad (9.9b)$$

where $\tilde{\mathscr{L}}_D^\pm$ is defined as the classic *Lagrangians*. As a pair of entanglements, they philosophically extend to and are known as *Dirac Equation*, introduced in 1925 [7]. For elementary (unit charge, massless) fermions satisfying the *Dirac* equation, it suffices to note their field entanglements [23]:

$$\left(\gamma^\mu D^\mu\right)\left(\gamma_\nu D_\nu\right) = D^\mu D_\nu + \frac{i}{4}[\gamma^\mu, \gamma^\nu]F_{\mu\nu}^{-n} \tag{9.10}$$

Historically, the *Dirac* equation was a major achievement and gave physicists great faith in its overall correctness.

***Artifact 9.3: Spinor Fields***. As the function quantity from the first to second horizon, a scalar field $\phi^-$ bonds and projects its potentials superseding its surrounding space, arisen by or acting on its opponent $\varphi^+$ through a duality of reciprocal interactions dominated by *Lorentz Generators*. From the *gamma matrix* (8.4a) to *Lorentz generators* (8.12), the respective transformations of spinors are given straightforwardly by the matrixes of spinor $\sigma_n$ quantities [24].

$$\phi_n^L = S(\Lambda^+)\phi_n^+(\hat{x}) \qquad : (\phi_n^L)^{-1}\gamma^\mu\phi_n^L = \Lambda^+\gamma^\nu, \check{x} = \Lambda^+\hat{x} \tag{9.11a}$$

$$\phi_n^R = S(\Lambda^-)\varphi_n^-(\check{x}) \qquad : (\phi_n^R)^{-1}\gamma_\mu\phi_n^R = \Lambda^-\gamma_\nu, \hat{x} = \Lambda^-\check{x} \tag{9.11b}$$

$$S(\Lambda^\pm) = exp\left\{\frac{1}{2}\left(i\sigma_k\hat{\theta}_k \pm \sigma_m\hat{\phi}_m\right)\right\} \quad , \ \Lambda^\pm = exp\left(\frac{\omega_k}{2}L_\kappa^\pm\right) \tag{9.11c}$$

Each of the first terms of $S(\Lambda^\pm)$ is the transformation matrix of the two dimensional world planes, respectively. Each of the second terms of $S(\Lambda^\pm)$ is an extension to the additional dimensions for the physical freedoms. The quantities are irreducible, preserve full parity invariant with respect to the physical change $\check{\theta}_i \to -\hat{\theta}_i$ for spin-up and spin-down positrons, which has the extra freedoms and extends the two degrees from a pair of each physical dimension of the world planes.

***Artifact 9.4: Weyl Equation***. In the limit as m → 0, the above *Dirac* equation is reduced to the massless particles:

$$\sigma_\mu\partial_\mu\psi = 0, \quad or \quad I_2\frac{1}{c}\frac{\partial\psi}{\partial t} + \sigma_1\frac{\partial\psi}{\partial x} + \sigma_2\frac{\partial\psi}{\partial y} + \sigma_3\frac{\partial\psi}{\partial z} = 0 \tag{9.12}$$

known as *Weyl* equation introduced in 1918 [12].

***Artifact 9.5: Schrödinger Equation***. For observations under an environment of $W_n^- = -ic^2V^-$ at the constant transport speed $c$, the homogeneous fields are in a trace of diagonalized tensors. From the first to the second horizon, it is dominated by the virtual time entanglement with the equation of

$$\check{\partial}_\lambda - \hat{\partial}^\lambda = \dot{x}_\nu S_2^-\partial_m - \dot{x}^m S_2^+\partial^\nu = 2ic\begin{pmatrix}\partial_\kappa \\ -\partial^\kappa\end{pmatrix} \tag{9.13}$$

Referencing the (3.14-3.15) equations, we decode the quantum fields of (9.4, 9.5) into the following formulae:

$$-i\hbar\frac{\partial}{\partial t}\phi_n^- - \frac{i\hbar^2}{2E_n^-}\frac{\partial^2\phi_n^-}{\partial t^2} = -i\frac{(\hbar c)^2}{2E_n^-}\nabla^2\phi_n^- + V^-\phi_n^- \equiv \hat{H}\phi_n^- \tag{9.14a}$$

$$-i\hbar\frac{\partial}{\partial t}\varphi_m^+ + \frac{i\hbar^2}{2E_m^+}\frac{\partial^2\varphi_m^+}{\partial t^2} = -i\frac{(\hbar c)^2}{2E_m^+}\nabla^2\varphi_m^+ + V^-\varphi_m^+ \equiv \hat{H}\varphi_m^+ \tag{9.14b}$$

where $\hat{H}$ is known as the classical *Hamiltonian* operator, introduced in 1834 [4]. For the first order of time evolution, it emerges as the *Schrödinger* equation, introduced in 1926 [6].

$$-i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi \qquad \hat{H} \equiv -i\frac{(\hbar c)^2}{2E_n^-}\nabla^2 + V^- \tag{9.15}$$

The $Y^-Y^+$ entanglement of the (9.14) equations can be integrated into the following formulae:

$$-i\hbar\left\langle\frac{\partial}{\partial t}\right\rangle_{mn}^- - \frac{\hbar^2}{2mc^2}\left[\frac{\partial^2}{\partial t^2}\right]_{mn}^- = \tilde{H}_{mn}^- \quad \to \quad -i\hbar\frac{\partial}{\partial t} = \hat{H} \tag{9.16a}$$

$$\tilde{H}_{mn}^- \equiv -\frac{\hbar^2}{2m}\left\langle\nabla^2\right\rangle_{mn}^- + 2V^-\left(\phi_n^-\varphi_m^+\right) \tag{9.16b}$$

where the bracket $\langle\ \rangle_{mn}^\pm$ and $[\ ]_{mn}^\pm$ are given by (3.17-3.21) of fluxion entanglements. Remarkably, it reveals that the entanglement lies at the second order of the virtual time commutation $[\partial^2/\partial t^2]_{mn}^-$ of the event operations.

***Artifact 9.6: Pauli Theory***. In the gauge fields, a particle of mass $m$ and charge $e$ can be extended by the vector potential $\mathbf{A}$ and scalar electric potential $\phi$ in the form of $A^\nu = \{\phi, \mathbf{A}\}$ such that the (9.15) equation is conceivable by (8.6) as the following gauge invariant:

$$-i\hbar\zeta^0 D^\kappa\varphi^+ = -\frac{\hbar^2}{2m}(\zeta^r D^r)(\zeta^r D^r)\varphi^+ + \hat{V}\varphi^+ \quad : D^\nu = D^\kappa + D^r \tag{9.17a}$$

$$D^\kappa = \partial^t - i\frac{e}{\hbar}\phi, \quad D^r = \partial^r - i\frac{e}{\hbar}\mathbf{A} \qquad : A^\nu = \{\phi, \mathbf{A}\} \tag{9.17b}$$

Since $\gamma^r = (\sigma_x, \sigma_y, \sigma_z) \equiv \boldsymbol{\sigma}$, the *Schrödinger* Equation (9.15) becomes the general form of Pauli Equation, formulated by *Wolfgang Pauli* in 1927 [11]:

$$i\hbar\frac{\partial}{\partial t}|\varphi^+\rangle = \left\{\frac{1}{2m}\left[\boldsymbol{\sigma}\cdot\left(\mathbf{p} - e\mathbf{A}\right)\right]^2 + e\phi + \hat{V}\right\}|\psi\rangle \equiv \check{H}|\varphi^+\rangle \tag{9.18}$$

$$\mathbf{p} = -i\hbar\partial^r, \qquad \boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \qquad : \chi^\nu \mapsto 0, \partial^t = -\partial_t \tag{9.19}$$

where $\mathbf{p}$ is the kinetic momentum. The *Pauli* matrices can be removed from the kinetic energy term, using the *Pauli* vector identity:

$$(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b}) \qquad : \gamma^r = (\sigma_x, \sigma_y, \sigma_z) \equiv \boldsymbol{\sigma} \tag{9.20}$$

to obtain the standard form of *Pauli Equation* [20],

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \left\{\frac{1}{2m}\left(\mathbf{p} - e\mathbf{A}\right)^2 - \frac{e\hbar}{2m}\boldsymbol{\sigma}\cdot\mathbf{B} + \tilde{V}\right\}|\psi\rangle \equiv \check{H}|\psi\rangle \tag{9.21}$$

where $\mathbf{B} = \nabla\times\mathbf{A}$ is the magnetic field and $\tilde{V} = \hat{V} + e\phi$ is the total potential including the horizon potential $e\phi$. The *Stern–Gerlach* term, $e\hbar\boldsymbol{\sigma}\cdot\mathbf{B}/(2m)$, acquires the spin orientation of atoms with the valence electrons flowing through an inhomogeneous magnetic field [18]. As a result, the above equation is implicitly observable under the $Y^+$ characteristics. The experiment was first conducted by the *German* physicists *Otto Stern* and *Walter Gerlach*, in 1922. Analogously, the term is responsible for the splitting of quantum spectral lines in a magnetic field anomalous to *Zeeman* effect, named after *Dutch* physicist *Pieter Zeeman* [26] in 1898.

***Artifact 9.7: Current Density***. For the particle fluxion, the electromagnetic current associates it with flow of its probability. With the equation (8.6b), the (5.9) becomes the following:

$$\mathbf{j}_s^+ = \frac{\hbar c^2}{2E^+}\left[Tr(\zeta^\nu)\left(\varphi^-\partial\phi^+ - \phi^+\partial\varphi^-\right) - 2e\varphi^-\zeta^\nu\mathbf{A}\phi^+\right] \tag{9.22a}$$

$$\approx \frac{1}{2m}\left([\hat{p}]^+ - 2e\mathbf{A}\varphi^-\phi^+\right) + \frac{\mu_S}{s}\nabla\times\left(\varphi^-\mathbf{S}\phi^+\right) \tag{9.22b}$$

where the momentum operator is $\hat{p} = -i\hbar\nabla$. The spin vector $\mathbf{S}$ of the particles might be correspond with the spin magnetic moment $\mu_s$ and quantum number $s$ [21].

***Artifact 9.8: Continuity Equation***. Considering a pair of the wave function observed externally at a constant speed, the diagonal elements of (9.1, 9.2) has the potential density $\Phi_c^+ = \varphi^-\phi^+$of light transporting massless waves, conserving to a constant, and maintaining its continuity states of current density.

$$\partial_\mu J_c^\mu \mapsto \frac{\partial\rho_c^+}{\partial t} + \mathbf{u}^+\nabla\cdot\mathbf{j}_c^+ = 0 \qquad : J_c^\mu = \left(c\rho^+, \mathbf{j}^+\right) \tag{9.23}$$

$$\rho_c^+ = \frac{\hbar}{2E^+}\partial_t\Phi_c^+, \qquad \mathbf{j}_c^+ = \frac{\hbar c}{2E^+}\mathbf{u}^+\nabla\Phi_c^+ \qquad : \Phi_c^+ = \varphi^-\phi^+ \tag{9.24}$$

This continuity equation is an empirical law expressing charge neutral conservation. It implies that a pair of photons is transformable or convertible into a pair of the electron and positron or vice versa.

***Artifact 9.9: Mass Acquisition and Annihilation***. As a duality of evolution, consider $N$ harmonic oscillators of quantum objects. The energy spectra operates between the virtual wave and physical mass oscillating from one physical dimension on world planes into three dimensional *Hamiltonian* of *Schrödinger Equation* in spacetime dimensions, shown by the following:

$$\tilde{H} = \sum_{n=1}^{N}\frac{\hat{p}_n^2}{2m} + \frac{1}{2}m\omega_n^2 r_n^2 \qquad : \hat{p}_n = -i\hbar\frac{\partial}{\partial r_n} \tag{9.25}$$

Developed by *Paul Dirac* [13], the "ladder operator" method introduces the following operators:

$$\tilde{H} = \sum_{n=1}^{N} \hbar\omega_n \left(\tilde{a}_n^{\pm}\tilde{a}_n^{\mp} \mp \frac{1}{2}\right) \quad : \tilde{a}_n^{\mp} = \sqrt{\frac{m\,\omega_n}{2\hbar}}\left(r_n \pm \frac{i}{m\,\omega_n}\hat{p}_n\right) \tag{9.26}$$

Under the $Y^-$ supremacy, $\tilde{a}_n^+$ is the creation operation for the wave-to-mass of physical animation, while $\tilde{a}_n^-$ is the reproduction operation for mass-to-wave of virtual annihilation. Intriguingly, the solution to the above equation can be either one-dimension $SU(2)$ for ontological evolution or three-dimension for spacetime at the $SU(3)$ horizon.

$$\varphi_n^+(r_n) = \frac{1}{\sqrt{2^n\,n!}}\left(\frac{m\,\omega_n}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega_n r_n^2}{2\hbar}} H_n\!\left(\sqrt{\frac{m\,\omega_n}{\hbar}}r_n\right) \tag{9.27}$$

$$\phi_{nlm}^-(r_n,\theta,\phi) = N_{nl}r^l e^{-\frac{m\omega_n}{2\hbar}r_n^2} L_n^{(l+1/2)}\!\left(\frac{m\,\omega_n}{\hbar}r_n^2\right) Y_{lm}(\theta,\phi) \tag{9.28a}$$

$$N_{nl} = \left[\left(\frac{2\nu_n^3}{\pi}\right)^{1/2} \frac{2^{n+2l+3}\,n!\,\nu_n^l}{(2n+2l+1)!}\right]^{1/2} \quad : \nu_n \equiv \frac{m\,\omega_n}{2\hbar} \tag{9.28b}$$

The $H_n(x)$ is the *Hermite* polynomials, detail by *Pafnuty Chebyshev* in 1859 [14]. The $N_{nl}$ is a normalization function for the enclaved mass at the third horizon. Named after *Edmond Laguerre* (1834-1886), the $L_k^{\nu}(x)$ are generalized *Laguerre* polynomials [22] for the energy embody dynamically. Introduced by *Pierre Simon de Laplace* in 1782, the $Y_{lm}(\theta,\phi)$ is a spherical harmonic function for the freedom of the extra rotations or the basis functions for $SO(3)$. Apparently, the classic normalizations are at the second horizon for $\varphi_n^+$ and the third horizon for $\phi_{nlm}^-$.

***Artifact 9.10: Embody Structure of Mass Enclave***. Based on the above artifacts at the n=0 ground level $H_0 = L_0 = Y_{00} = 1$, the energy potentials embody the full mass enclave $\phi_n^-\varphi_n^+ \propto m$ that splits between the potential $\varphi_n^+ \propto m^{1/4}$ in the second horizon and $\phi_n^- \propto m^{3/4}$ in the third horizon. The density emerges from the second to third horizon for the full-mass acquisition:

$$\rho^- \approx \phi_0^-\varphi_0^+ = 2\frac{m\,\omega}{\pi\hbar}exp\left[-\frac{m\,\omega}{2\hbar}(r_s^2 + r_w^2)\right] \tag{9.29a}$$

$$\phi_0^- = 2\left(\frac{m\,\omega}{\pi\hbar}\right)^{3/4} e^{-\frac{m\omega}{2\hbar}r_s^2}, \qquad \varphi_0^+ = \left(\frac{m\,\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega r_w^2}{2\hbar}} \tag{9.29b}$$

where the radius $r_s$ or $r_w$ is the interactive range of the strong or weak forces, respectively. Therefore, the energy embodies its mass enclave in a process from its ¼ to ¾ core during its evolution of the second to third horizon, progressively. Vice versa for the annihilation.

Remarkably, the operations represent not only a duality of the creation and annihilation, but also the seamless transitions between the virtual world planes and the real spacetime manifold. For example, The Sun is the star at the center of the solar system between the virtual and physical worlds. The Sun rotates in the quantum layers with the innermost 1/4 (or higher to include the excited levels at n>0) of the core radius at the second and lower horizons. Between this core radius and 3/4 of the radius, it forms a "radiative zone" for energy embodied at the full mass enclave by means of photon radiation. The rest of the physical zone is known as the “convective zone” for massive outward heat transfer.

***Artifact 9.11: Speed of Light***. At an event $\lambda = t$, the observable light speed in a free space or vacuum has the relativistic effects of transformations. A summation of the right-side of the four (7.2) equations represents the motion fluxions:

$$\mathbf{f}_c^+ = \psi_c^-\begin{pmatrix}\hat{\partial}^{\nu}\\ \check{\partial}^{\nu}\end{pmatrix}'\psi_c^+ = \psi_c^-\dot{x}^{\nu}\tilde{\gamma}^{\nu}\begin{pmatrix}\partial^{\nu}\\ \partial_{\nu}\end{pmatrix}'\psi_c^+ \mapsto C_{\nu\mu}^+\psi_c^-\nabla\psi_c^+ \tag{9.30a}$$

$$\mathbf{f}_c^- = \psi_c^+\begin{pmatrix}\check{\partial}_{\nu}\\ \hat{\partial}_{\nu}\end{pmatrix}'\psi_c^- = \psi_c^+\dot{x}_{\nu}\tilde{\gamma}_{\nu}\begin{pmatrix}\partial_{\mu}\\ \partial^{\mu}\end{pmatrix}'\psi_c^- \mapsto C_{\nu\mu}^-\psi_c^+\nabla\psi_c^- \tag{9.30b}$$

where the equations are mapped to the three-dimensions of a physical space at the second horizon ($\tilde{\gamma} \mapsto \gamma$). For the potential fields $\psi_c^{\pm} = \psi_c^{\pm}(r)exp(i\vartheta^{\pm})$ at massless in the second horizon, we derive the $C$-matrices for the speed of light:

$$C_{\nu\mu}^+ = \dot{x}^{\nu}\gamma^{\nu}e^{-i\vartheta}, \qquad C_{\nu\mu}^- = \dot{x}_{\nu}\gamma_{\nu}e^{i\vartheta} \qquad : \vartheta = \vartheta^- - \vartheta^+ \tag{9.31}$$

where the quanta $\vartheta$ is the superphase, and $\nu \in (1,2,3)$. Remarkably, the speed of light is characterized by a pair of the above $Y^-Y^+$ matrices, revealing the intrinsic entanglements of light that constitutes of transforming γ-fields and superphase modulations. Philosophically, no light can propagate without the internal dynamics, which is described by the off-diagonal elements of the $C$-matrices. Applying to an external object, the quantities can be further characterized by the diagonal elements of the $C$-matrices at the r-direction of world lines, shown by the following:

$$C_{rr}^{\pm} = c\,e^{\mp i\vartheta} \qquad : \textit{Speed of Light } = |C_{rr}^{\pm}| = c \tag{9.32}$$

As expected, the speed of light is generally a non-constant matrix, representing its traveling dynamics sustained and modulated by the $Y^-Y^+$ superphase entanglements. Because the constituent elements of the γ-matrices are constants, the amplitude of the $C$-matrices at a constant $c$ is compliant to and widely known as a universal physical constant. The speed $C$-matrix applies to all massless particles and changes of the associated fields travelling in vacuum or free-space, regardless of the motion of the source or the inertial or rotational reference frame of the observer.

***Artifact 9.12: Speed of Gravitation***. Similar to the motion fluxions of light, one has the fluxions of gravitational fields in a free space or vacuum:

$$\mathbf{f}_g^+ = \psi_g^-\begin{pmatrix}\hat{\partial}^{\nu}\\ \check{\partial}^{\nu}\end{pmatrix}'\psi_g^+ = \psi_g^-\dot{x}^{\nu}\tilde{\chi}^{\nu}\begin{pmatrix}\partial^{\nu}\\ \partial_{\nu}\end{pmatrix}'\psi_g^+ \mapsto G_{\nu\mu}^+\psi_g^-\nabla\psi_g^+ \tag{9.33}$$

$$\mathbf{f}_g^- = \psi_g^+\begin{pmatrix}\check{\partial}_{\nu}\\ \hat{\partial}_{\nu}\end{pmatrix}'\psi_g^- = \psi_c^+\dot{x}_{\nu}\tilde{\chi}_{\nu}\begin{pmatrix}\partial_{\mu}\\ \partial^{\mu}\end{pmatrix}'\psi_g^- \mapsto G_{\nu\mu}^-\psi_g^+\nabla\psi_g^- \tag{9.34}$$

Unlike the light transformation seamlessly at massless, the uniqueness of gravitation is at its massless transportation of the $\chi$-matrices from the second horizon potential $\psi_g^+ = \psi_g(r)exp(i\vartheta)$ of world planes into the third horizon potential $\psi_g^- = \psi_{nlm}(r_n,\theta,\phi)$ of the $L$-matrices of spacetime manifolds for its massive gravitational attraction. At inception of the mass enclave in the second horizon, the $G$-matrices are free of its central-singularity $r \to 0$, and result in

$$G_{\nu\mu}^+ = \lim_{r\to 0}(x^{\nu}\dot{x}^{\nu}\chi^{\nu}e^{-i\vartheta}) = x^{\nu}\dot{x}^{\nu}\epsilon_3 e^{-i\vartheta} = c_g s_1 e^{-i\vartheta} \tag{9.35}$$

$$G_{\nu\mu}^- = \lim_{r\to 0}(x_{\nu}\dot{x}_{\nu}\chi_{\nu}e^{i\vartheta}) = x_{\nu}\dot{x}_{\nu}\epsilon_3 e^{i\vartheta} = c_g s_1 e^{i\vartheta} \tag{9.36}$$

$$\textit{Speed of Gravitation } = |G_{\mu\nu}^{\pm}| = c_g \qquad : \mu \neq \nu \tag{9.37}$$

Remarkably, the gravitational speed $c_g$ is a constant similar to the speed of light, but propagating orthogonally in the off-diagonal elements. Interrupting with mass objects at the third horizon, the gravitation becomes gravity that exerts a force inversely proportional to a square of the distance. Apparently, gravity has the same characteristics of the quantum entanglement.

***Artifact 9.13: Invariance of Flux Continuity***. At both of the boost and twist transformations at a constant speed, the (9.1, 9.2) equations obey the time-invariance, transform between virtual and physical instances, and transport into the third horizon $SU(3)$. For the external observation, the diagonal elements can be converted into a pair of dynamic fluxions of the $Y^-Y^+$ energy flows:

$$\hbar^2\check{\partial}_{\lambda}\check{\partial}^{\lambda}\phi_n^+ = 2E_n^+E_n^+\phi_n^+ \to \frac{1}{c^2}\frac{\partial^2\phi_n^+}{\partial t^2} - \nabla^2\phi_n^+ = 2\frac{E_n^+E_n^+}{(\hbar c)^2}\phi_n^+ \tag{9.39}$$

$$\hbar^2\hat{\partial}_{\lambda}\hat{\partial}_{\lambda}\varphi_n^- = 2E_n^-E_n^-\varphi_n^- \to \frac{1}{c^2}\frac{\partial^2\varphi_n^-}{\partial t^2} + \nabla^2\varphi_n^- = 2\frac{E_n^-E_n^-}{(\hbar c)^2}\varphi_n^- \tag{9.40}$$

where the (3.14-3.15) equations are applied. It extends and amends the *Klein–Gordon* equation, introduced in 1926 [15], by a factor of 2. Adding $\varphi_n^-$ times the first equation and $\phi_n^+$ times the second equation, one has an observable flux-continuity of the $Y^+$-primacy entanglement.

$$\Diamond_n^+ = 2\frac{E_n^+E_n^+}{(\hbar c)^2}\varphi_n^-\phi_n^+ \qquad : \Diamond_n^+ \equiv \left\langle\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\rangle_n^+ - \left[\nabla^2\right]_n^+ \tag{9.41}$$

Correspondingly, the diagonal elements of the (9.4, 9.5) equations can be similarly reformulated to the similar $Y^-$ flux-continuity.

$$\lozenge_n^- = 2\frac{E_n^- E_n^-}{(\hbar c)^2}\varphi_n^+\phi_n^- \qquad : \lozenge_n^- \equiv \left\langle \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\rangle_n^- + \left[\nabla^2\right]_n^- \tag{9.42}$$

Together, they represent a flux propagation of the $Y^-Y^+$ entanglements:

$$\lozenge_n \equiv \lozenge_n^+ + \lozenge_n^- = 4\frac{E_n^\pm E_n^\pm}{(\hbar c)^2}\Phi_n \quad : \Phi_n = \frac{1}{2}\left(\varphi_n^-\phi_n^+ + \varphi_n^+\phi_n^-\right) \tag{9.43}$$

Amazingly, it reveals that an integrity of entanglements lies at the continuity of virtual time and the commutators of physical space.

***Artifact 9.14: Lagrangian of Fluxions***. In reality, the above flux-continuities are a pair of virtual and physical energies in each of the asymmetric entanglements to give rise to the strong forces at higher horizons of $SU(2)$ and $SU(3)$. Therefore, under a trace of the diagonalized tensors, we can represent a pair of the *Lagrangians* as a duality of the area flux-continuities:

$$\mathscr{L}_{Flux}^{\pm SU1} \equiv \lozenge_n^\pm = 2\frac{E_n^\pm E_n^\pm}{(\hbar c)^2}\Phi_n^\pm \qquad : \Phi_n^\pm = \varphi_n^\mp\phi_n^\pm \tag{9.44}$$

$$\mathscr{L}_{Flux}^{SU1} = \lozenge_n^+ + \lozenge_n^- = -4\frac{E_n^\mp E_n^\pm}{(\hbar c)^2}\Phi_n \qquad : \Phi_n = \frac{1}{2}\left(\Phi_n^+ + \Phi_n^-\right) \tag{9.45}$$

The area flow of energy, $4E_n^+E_n^-/(\hbar c)^2$, represents a pair of the irreducible density units $E_n^-E_n^+$ that exists alternatively between the physical-particle $E_n^-$ and virtual-wave $E_n^+$ states.

***Artifact 9.15: Wave-Particle Duality***. Since light exhibits wave–particle duality, its properties must acquire characteristics of both waves and particles. A duality of the energy formations of light has both of its convertible form to physical $mc^2$ and its transportable form at virtual $\hbar\omega$. It is conservation of energy $\hat{E}^2 = \hat{\mathbf{P}}^2 + 4m^2c^4$ and invariance of momentum $\mathbf{P} = ic\hat{\mathbf{p}} \mapsto \mp i\hbar c\mathbf{k}$ that maintain the light transformable between a duality of virtual and physical states. Together, it derives a pair of irreducible virtual unit $\pm\hbar\omega$, known as *Planck–Einstein* relation as well as a pair of physical unit $mc^2$. The property of light becomes a complex form of virtual and physical duality as the following:

$$\tilde{E}_c^\mp = \hbar\omega \pm imc^2 \qquad : \hbar\omega \rightleftharpoons mc^2 \tag{9.46}$$

named as ***Photon Energy*** - a fundamental property of light. As a constant, a photon defines an irreducible unit of energy state either at virtual $\hbar\omega$ or at physical $mc^2$, but not at both. The photon's wave and quanta qualities are two observable aspects of a single phenomenon, which obey law of conservation of energy as the following:

$$\left(\hat{\mathbf{P}} + i\hat{E}\right)\left(\hat{\mathbf{P}} - i\hat{E}\right) = 4E_n^+E_n^- \tag{9.47}$$

$$\hat{E} = -i\hbar\partial_t, \qquad \mathbf{P} = ic\hat{\mathbf{p}}, \qquad \hat{\mathbf{p}} = -i\hbar\nabla \tag{9.48}$$

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = 4\frac{E_n^-E_n^+}{(\hbar c)^2} \qquad : \nabla^2 = \left[\nabla^2\right]_n^+ - \left[\nabla^2\right]_n^- \tag{9.49}$$

where $\mathbf{p}$ is the momentum vector. In the center of entanglement, the colliding duality has no net momentum. Whereas a single photon always has momentum, conservation of momentum (equivalently, transformation invariance) requires that at least two photons be created for entanglement, with zero net momentum.

For example, in a free space, a light traveling the 2-dimensional manifolds $\{r, \pm ict\}$ of the world planes has the two wave functions, respectively and simultaneously $\Phi_c \propto exp\left(\mp\frac{i}{\hbar c}(m_c c^2 t \pm \hbar\omega r)\right)$. They carry quantities that might be simulated by spin angular momentum. From the conservation of energy $E_c \mapsto \hbar\omega$, it appears that the magnitude of its spin were $\hbar$ at the component measured along its direction of motion. This is because the total $E_c$ energy includes both photon energies for the dual manifolds. There are two possible helicities $\pm\hbar$, called right-handed and left-handed, correspond to the two possible circular polarization states of the photons.

***Artifact 9.16: Big Bang Theory***. In the *Big Bang* theory, "the universe began from a singularity," introduced in 1927 by *Lemaître* [25], and the expansion of the observable universe began with the explosion of a single particle at a definite point in time. According to this horizon infrastructure, obviously, the universe is amazingly a chain of the seamlessly processes at the *conservation of superphase evolutions* for the progressive mass acquisitions from virtual non-singularity to physical spacetime singularity. The gravitational singularity exists only at the third horizon where the energy embodies its enclave as a mass object, which gains the rotational coordinates. Applicable to prevail in the earliest states of physical objects, *Big Bang Theory* would have been a cosmological model for the universe, if the ordinary matter in the universe were dominant or created virtual energy. Therefore, the model of "*Big Bang* theory" might be limited to a process of the mass inauguration in physical only. Besides, in reality, acceleration of a physical object is simply embarrassed by a common phenomenon or a result of the generation process of light radiations. A property of the entire universe is orchestrated as a whole rather than a phenomenon that applies just to one part of the universe or from the physical observation only.

## CONCLUSION

Complying with classical and contemporary physics, this universal and unified theory demonstrates its holistic foundations applicable to the well-known natural intrinsics of the following remarks:

1) At the two-dimensions of the world planes, a pair of transform and transport ***Entangling Generators***, $\tilde{\gamma}^\nu$- and $\tilde{\chi}^\nu$-matrices, incepts, acquires, and extends the empirical formulae of, but not limited to, *Lorentz* generators, *Pauli* spin $\gamma^\mu$-matrices, torque $\chi^\mu$-matrices, and transformation and transportation structures of quantum fields.

2) *Stateful Einstein mass-energy* is refined philosophically as the entanglements $\hbar\omega \rightleftharpoons mc^2$ of complex states with virtual imaginary interpretations $E_n^\mp = \pm imc^2$.

3) *Lagrangian* $\mathscr{L}$ is concisely redefined philosophically as the entanglements of continuity (10.10) dynamically transported and balanced between the manifolds and horizons.

4) *Quantum Physics* is derived as compliance to contemporary physics and particle physics, testified by the empirical theories of *Schrödinger*, *Dirac*, *Klein–Gordon* and *Pauli* equations, *Quantum Electrodynamics, etc*.

5) *Embody Structure of Mass Enclave* is an evolutionary process from the second horizon of world line giving rise to the third horizon of the physical spacetime manifold.

6) Besides a constant, the speed of light is entangled or operated by the *C*-matrices of the superphase modulations.

7) Likewise, the speed of gravitation has the superphase modulations operated by the *G*-matrices.

Consequently, this manuscript of the *Universal Topology* has testified and extended to the numerous theoretical foundations, mathematical framework, event operations, and world equations [1] for the quantum physics.

***Natural Secret of Scalar Fields***. Since the evolutionary processes of the mass inauguration is between the second and third horizons, the scalar fields are massless instances under the virtual supremacy dominant at the first and second horizons. In addition, the scalar potentials are the gauge fields, operated by the superphase modulation and subjected to the event actions. Conceivably and strikingly, the scalar fields behaves or known as *Dark Energy*.

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