

## Special Rule for Certain Prime Numbers

Abstract: A quadratic equation for prime numbers is assumed to be true that satisfy the following four rules. Some prime numbers violate these rules. Whereas some non prime numbers satisfy the four rules. They are not prime, therefore to make them violate the fourth rule we need to study how to choose the value of  $m$  and  $n$  so as make the quadratic equation as the primes generating formula.

The prime numbers of the form  $N=a^2+a+d$

Where  $a$  is any positive integer and  $d$  is a number needed to make  $N$  prime.

Where  $1 < d < a$

If  $N$  is prime, we find that the following four rules are satisfied.

- 1)  $a^2+a+1$  is coprime with  $d$
- 2)  $2a+1$  is coprime with  $d$
- 3)  $a^2-a-1$  is coprime with  $d$
- 4)  $(a-2), (a-1), a, (a+1), (a+2) \dots\dots (a+r)$  is coprime with  $d$

Here  $r$  varies from  $r=-m$  to  $r=n$  including 0.

We find that, some prime numbers violate these rules. The rule  $(a+r)$  is the exception that is violated by most prime numbers. Also the four rules is valid for numbers that are not prime and they follow the rule  $(a+r)$  is coprime with  $d$ .

There is no problem if certain prime numbers violate these rules. But for numbers that are not prime follow these rules, we need to make them violate by choosing  $m$  and  $n$ . We need to study how to choose  $m$  and  $n$ . Therefore if we are able to choose correct value of  $m$  and  $n$  for all numbers  $a$ , then this special rule becomes the prime numbers generating formula.

We clarify with examples.

- 1) For  $a=6, d=5$ , then  $N=47$

Choosing  $m=-1$  to  $n=2$

We find that first 3 rules are satisfied.

We find that  $(6-1)$  is not coprime with 5

Therefore primality test fails. But  $N=47$  is a prime number. Therefore we have to choose  $m=0$  to  $n=2$ .

2) For  $a=7$ ,  $d=3$ , then  $N=59$

Choosing  $m=-1$  to  $n=2$

The first rule yields 57 is not coprime with 3.

The second rule yields 15 is not coprime with 3.

The third rule yields 41 is coprime with 3.

The fourth rule  $(7-1)$  is not coprime with 3.

But  $N=59$  is a prime number. Therefore primality test fails.

3) For  $a=13$ ,  $d=5$ , then  $N=187$

Choosing  $m=-1$  to  $n=2$

We find that first 2 rules are satisfied.

The third rule yields 155 is not coprime with 5.

The fourth rule yields  $(13+2)$  is not coprime with 5.

Therefore  $N=187$  is not a prime number which is true.

The prime number density decreases as the number approaches infinity. Therefore for large number we need to know how to choose the value of  $m$  and  $n$  to make the non prime numbers violate the rule  $(a+r)$  is coprime with  $d$ . This technique is useful in factoring product of two prime numbers quickly.

References:

Wikipedia, .