A Minor Theorem Related with the Fermat Conjecture

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It is obtained a minor theorem related with the Fermat conjecture.

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For non-zero positive integers numbers x, y, z and n, the Fermat conjecture says that the equation $x^n + y^n = z^n$ is false for $n > 2$.

Theorem: for non-zero positive integers numbers x, y, z and n, the equation

$$2^n + y^n = z^n \tag{1}$$

is false for $n \geq 2$.

Proof: from (1)

$$2^n = z^n - y^n = (z - y)(z^{n-1} + z^{n-2}y + z^{n-3}y^2 + ... + z^2y^{n-3} + zy^{n-2} + y^{n-1}) \tag{2}$$

Also from (1), $z^n > 2^n$ and $z^n > y^n$, then $z > 2$ and $z > y$, and for $n \geq 2$, from (1) and from the binomial formula, $(2 + y)^n = 2^n + y^n +$ *other non-zero positive integers values* $> 2^n + y^n = z^n$, then $2 + y > z$ and $2 > z - y = 1$, because $z > y$, then $0 < z - y < 2$ and $z - y$ is a non-zero positive integer number between 0 and 2, that is, $z - y = 1$. Also from (1) and for $n \geq 2$, it is for $y = 2$, $2 \cdot 2^n = z^n$, then $2^{1/n} \cdot 2 = z$, which is false for the non-zero positive integer number z, then $y \neq 2$. From $z > 2$, $z > y$, $2 + y > z$, $z - y = 1$ and $y \neq 2$, the minimum values for y and z are $y = 3$ and $z = y + 1 = 4$. And substituting these values in (2), it would be

$$2^n = 4^n - 3^n = 4^{n-1} + 4^{n-2} \cdot 3 + 4^{n-3} \cdot 3^2 + ... + 4^2 \cdot 3^{n-3} + 4 \cdot 3^{n-2} + 3^{n-1} = A \tag{3}$$

which is false because $A > 2^n$, since all the terms of A are non-zero positive integers numbers and its first term alone is already greater than 2^n: $4^{n-1}/2^n = (2^2)^{n-1}/2^n = 2^{2n-2}/2^n = 2^{2n-2-n} = 2^{n-2} > 1$ for $n > 2$. For $n = 2$, $2^n = 2^2 = 4$ and $A = 4^2 - 3^2 = 7$, and also $A > 2^n$ for $n = 2$. As for $n \geq 2$, it is $z - y = 1$, then for values of y and z greater than the minimum, $y = 3$ and $z = 4$, it would be from (2)

$$2^n = z^n - y^n = z^{n-1} + z^{n-2}y + z^{n-3}y^2 + ... + z^2y^{n-3} + zy^{n-2} + y^{n-1} = B \tag{4}$$

which would be false because $B > A > 2^n$ for $n \geq 2$. Therefore, (1) is false for $n \geq 2$.

Note: this theorem would correspond to the theorem 1.7 of the first version (v1) of [1].

[1] Haofeng Zhang, The Simplest Proving Method of Fermat's Last Theorem, viXra: 1709.0080 [Number Theory].
http://vixra.org/abs/1709.0080
http://vixra.org/pdf/1709.0080v1.pdf (first version, v1)