

CONVERGENCE OF THE RATIO OF PERIMETER OF A REGULAR POLYGON TO THE LENGTH OF ITS LONGEST DIAGONAL AS THE NUMBER OF SIDES OF POLYGON INCREASES

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ABSTRACT

A regular polygon is a planar geometrical structure with all sides of equal length and all angles of equal magnitude. The ratio of perimeter of any regular polygon to the length of its longest diagonal is a constant term and the ratio converges to the value of π as the number of sides of the polygon increases. The result has been shown to be valid by actually calculating the ratio for each polygon by using corresponding formula and geometrical reasoning. A computational calculation of the ratio has also been presented to validate the convergence. The values have been calculated up to 30 significant digits.

Keywords: Regular Polygon, π , convergence

INTRODUCTION

A regular polygon is a planar geometrical structure with equal sides and equal angles. For a regular polygon of n sides there are $\frac{n(n-3)}{2}$ diagonals. Each angle of a regular polygon of n sides is given by $180^\circ \frac{n-2}{n}$ while the sum of interior angles is $180^\circ(n-2)$. The angle made at the center of any polygon by lines from any two consecutive vertices (center angle) of a polygon of sides n is given by $\frac{360^\circ}{n}$. It is evident that each exterior angle of a regular polygon is always equal to its center angle.

The ratio of the perimeter to the longest diagonal or diameter is characteristic feature of any regular polygon. For a regular polygon of even number of sides the ratio is given by $n \sin \left[\frac{180^\circ}{n} \right]$, while for a regular polygon of odd number of sides the ratio is given by $2n \sin \left[\frac{90^\circ}{n} \right]$. As the number of sides of a regular polygon becomes infinitesimally large, i.e. $n \rightarrow \infty$ the resulting polygon is called regular apeirogon which resembles a circle. The regular polygon at this state has countably infinite number of equal edges. The ratio of the perimeter to the longest diagonal reduces to $C \frac{\sin \theta}{p}$, where C is circumference of the resembling circle, θ is angle opposite to the perpendicular arm and p is perpendicular arm of the right angled triangle whose hypotenuse is diameter of the circle.

For inclusion of all regular polygons equilateral triangle poses a significant hurdle. It has no recognizable diagonal or diameter. The ratio of the perimeter of the equilateral triangle to the length of one of its sides is taken in this case which is equal to 3.

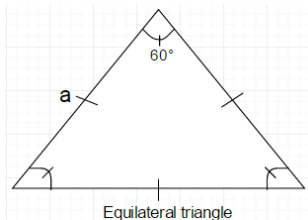
The formulas for the ratio of perimeter to the longest diagonal can be summarized as

- I. $BK = 3$, for $n=3$
- II. $BK = f(n) = n \sin\left[\frac{180^\circ}{n}\right]$, for n is an even number
- III. $BK = f(n) = 2n \sin\left[\frac{90^\circ}{n}\right]$, for n is an odd number
- IV. $BK = C \frac{\sin\theta}{p}$, for $n \rightarrow \infty$

The ratio for individual regular polygon has been represented by a single term, BK; BK is short form of Bishwakarma. BK ratio for convenience.

CALCULATION OF BK RATIOS FOR SOME REGULAR POLYGONS

1. For $n=3$, representing an equilateral triangle



$$BK = \frac{\text{total perimeter of equilateral triangle}}{\text{a side length}} = \frac{3a}{a} = 3.$$

2. For a regular polygon with even number of sides, i.e. n is an even number and $n \geq 4$
Square ($n = 4$), Hexagon ($n = 6$), Octagon ($n = 8$) are the basic examples of this types of polygons.

The length of the longest diagonal d of these regular polygons is given as by $\frac{a}{\sin\left(\frac{180^\circ}{n}\right)}$ [ref.

MM], where a is the length of the side of the polygon.

If a regular polygon has even number of edges, then length of the longest diagonal of that polygon will be hypotenuse of a right angled triangle for which one of the sides of the polygon is the perpendicular side as illustrated in figure

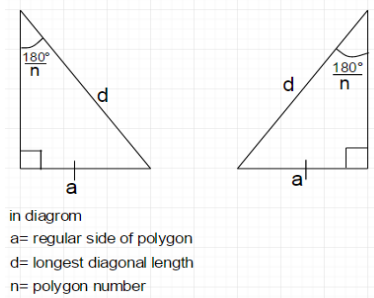


Figure 1: Illustration of the longest diagonal as the perpendicular of a right angled triangle

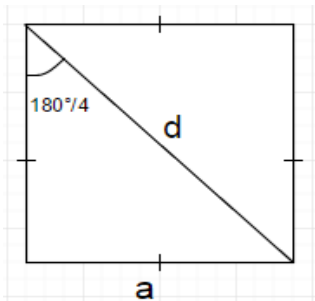
$$\sin\left(\frac{180^\circ}{n}\right) = \frac{a}{d}$$

$$\text{or, } d = \frac{a}{\sin\left(\frac{180^\circ}{n}\right)}$$

ILLUSTRATION I

The values of length of the longest diagonal for some regular polygons of have been geometrically presented below. The calculation of the ratio of the perimeter of the corresponding polygon to the length of the longest diagonal has been presented alongside.

I. Square (n=4)



$$\sin\left(\frac{180^\circ}{4}\right) = \frac{a}{d}$$

$$d = \frac{a}{\sin 45^\circ}$$

$$= (\sqrt{2})a$$

The ratio of the perimeter of the square to the length of its longest diagonal is

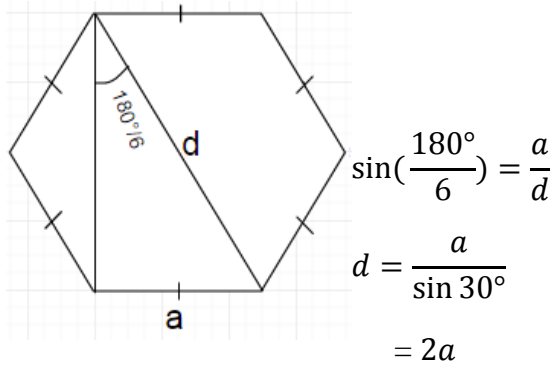
From geometry,

$$\begin{aligned} \text{BK} &= \frac{\text{perimeter of the square}}{\text{length of the longest diagonal}} \\ &= \frac{4a}{\sqrt{2}a} \\ &= 2\sqrt{2} \end{aligned}$$

From formula,

$$\begin{aligned} \text{BK} &= n \sin \left[\frac{180^\circ}{n} \right] \\ &= 4 \sin \left[\frac{180^\circ}{4} \right] \\ &= 4 \sin 45^\circ \\ &= 2\sqrt{2} \end{aligned}$$

II. Regular Hexagon (n=6)



The ratio of the perimeter of the square to the length of its longest diagonal is

From geometry,

$$\begin{aligned} \text{BK} &= \frac{\text{perimeter of the hexagon}}{\text{length of the longest diagonal}} \\ &= \frac{6a}{2a} \\ &= 3 \end{aligned}$$

From formula,

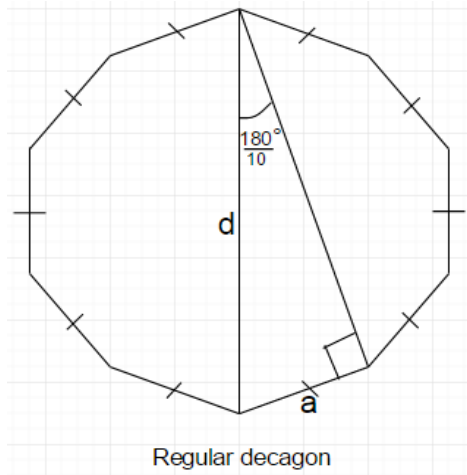
$$\text{BK} = n \sin \left[\frac{180^\circ}{n} \right]$$

$$= 6 \sin\left[\frac{180^\circ}{6}\right]$$

$$= 6 \sin 30^\circ$$

$$= 3$$

Regular Decagon (n=10)



$$\sin\left(\frac{180^\circ}{10}\right) = \frac{a}{d}$$

$$d = \frac{a}{\sin 18^\circ}$$

The ratio of the perimeter of the square to the length of its longest diagonal is

From geometry,

$$BK = \frac{\text{perimeter of the decagon}}{\text{length of the longest diagonal}}$$

$$= \frac{10a}{\frac{a}{\sin 18^\circ}}$$

$$= 3.090169$$

From formula,

$$BK = n \sin\left[\frac{180^\circ}{n}\right]$$

$$= 10 \sin\left[\frac{180^\circ}{10}\right]$$

$$= 10 \sin 18^\circ$$

$$= 3.0901$$

Similarly we can use $BK = n \sin \left[\frac{180^\circ}{n} \right]$ formula for all regular polygons having even number of edges. Therefore $BK = n \sin \left[\frac{180^\circ}{n} \right]$ is a valid formula for given domain.

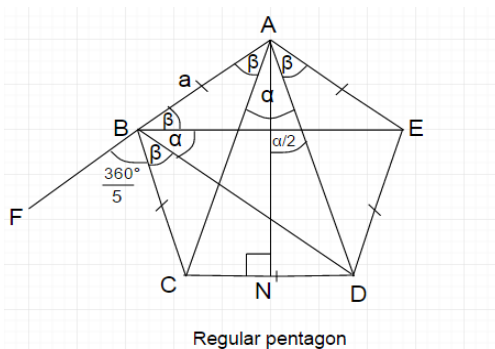
3. For a regular polygon with odd number of sides, i.e. n is an odd number and $n \geq 4$
 Pentagon ($n = 5$), Heptagon ($n = 7$), Nonagon ($n = 9$) are the basic examples of this types of polygons.

The length of the longest diagonal d of these regular polygons is given as by $\frac{a}{2 \sin(\frac{90^\circ}{n})}$ [ref. MM], where a is the length of the side of the polygon.

ILLUSTRATION 2

The value of length of the longest diagonal for some regular polygons of have been geometrically presented below. The calculation of the ratio of the perimeter of the corresponding polygon to the length of the longest diagonal has been presented alongside.

I. Regular Pentagon ($n = 5$)



Diagonals from D and E are drawn to B such that it subtends an angle α , and creates equal angles β on either side of the angle α . Similarly diagonals AC and AD subtend an angle α at A creating equal angles β on either side of the angle α .

Side AB is extended upto F and $AN \perp CD$ such that

- Triangle ACD and BDE are isosceles
- Triangle CAN and DAN are right angled triangles

From $\triangle ABD$,

$$(\alpha + \beta) + (\alpha + \beta) + \alpha = 180^\circ \text{ (sum of interior angles of a triangle)}$$

$$3\alpha + 2\beta = 180^\circ \dots\dots\dots(i)$$

On straight line ABF,

$$\angle ABE + \angle EBD + \angle DBC + \angle CBF = 180^\circ \text{ (angle add up on a straight line)}$$

$$B + \alpha + \beta + \frac{360^\circ}{n} = 180^\circ$$

$$2\beta = 180^\circ - (\alpha + \frac{360^\circ}{n}) \dots \dots \dots \text{(ii)}$$

From equation (i) and (ii)

$$180^\circ - (\alpha + \frac{360^\circ}{n}) = 180^\circ - 3\alpha$$

$$\alpha = \frac{360^\circ}{2n} \dots \dots \dots \text{(iii)}$$

In triangle ACD, line AN divides triangle ACD into equal two parts

$$\angle CAN = \angle DAN = \frac{\alpha}{2} \text{ \& } CN = ND = a/2$$

From right angled triangle CAN and DAN, AC=AD, both AC and AD are longest diagonal(d) for pentagon and hypotenuse for right angled triangles CAN and DAN respectively.

$$\sin(\frac{\alpha}{2}) = \frac{a/2}{AC}$$

$$AC = d = \frac{(a/2)}{\sin(\frac{\alpha}{2})}$$

$$AC = \frac{a}{2 \sin(\frac{\alpha}{2})} \dots \dots \dots \text{(iv)}$$

From equation (iii) and (iv)

$$AC = \frac{a}{2 \sin(\frac{360^\circ}{2n} \times \frac{1}{2})}$$

$$= \frac{a}{2 \sin(\frac{90^\circ}{n})}$$

For pentagon

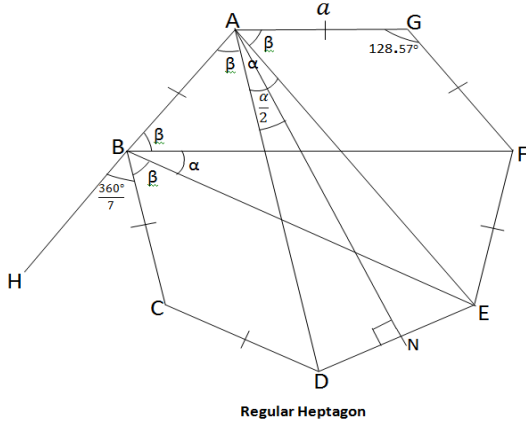
$$BK = \frac{\text{perimeter of regular pentagon}}{\text{longest diagonal length}}$$

$$= \frac{5a}{\frac{a}{2 \sin(\frac{90^\circ}{5})}}$$

$$= 3.090169\dots$$

From formula,
 $BK = 2n \sin\left[\frac{90^\circ}{n}\right]$
 $= 10 \sin\left[\frac{90^\circ}{5}\right]$
 $= 3.090169\dots$

II. Regular Heptagon(n=7)



From both vertex angle A and B of Heptagon an angle ‘ α ’ taken from the middle so that by symmetry all angles marked ‘ β ’ are equal in diagram.

Line AB extended upto H

Triangle ADE and BEF are isosceles

Triangle DAN and EAN are right angled triangle

$\angle AEB = \angle DAE = \angle EBF = \alpha$ (isosceles triangles)

From ABE triangle,

$$(\alpha + \beta) + (\alpha + \beta) + \alpha = 180^\circ \{ \because \text{sum of interior angles of a triangle} \}$$

$$3\alpha + 2\beta = 180^\circ \dots\dots\dots(i)$$

On straight line ABH,

$$\angle ABF + \angle FBE + \angle EBC + \angle CBH = 180^\circ$$

$$B + \alpha + \beta + \frac{360^\circ}{n} = 180^\circ$$

$$2\beta = 180^\circ - \left(\alpha + \frac{360^\circ}{n}\right) \dots\dots\dots(ii)$$

From equation (i) and (ii)

$$180^\circ - \left(\alpha + \frac{360^\circ}{n}\right) = 180^\circ - 3\alpha$$

$$\alpha = \frac{360^\circ}{2n} \dots\dots\dots\text{(iii)}$$

In triangle ADE, line AN divides triangle ADE into equal two parts

$$\angle DAN = \angle EAN = \frac{\alpha}{2} \text{ and } DN = NE = a/2$$

From right angled triangle DAN and EAN, AD=AE, both AD and AE are longest diagonal(d) for heptagon and hypotenuse for right angled triangles DAN and EAN respectively.

Here in right angled triangle DAN

$$\sin\left(\frac{\alpha}{2}\right) = \frac{a/2}{AD}$$

$$AD = d = \frac{a/2}{\sin\left(\frac{\alpha}{2}\right)}$$

$$AD = \frac{a}{2 \sin\left(\frac{\alpha}{2}\right)} \dots\dots\dots\text{(iv)}$$

From equation (iii) and (iv)

$$AD = \frac{a}{2 \sin\left(\frac{360^\circ}{2n} \times \frac{1}{2}\right)} = \frac{a}{2 \sin\left(\frac{360^\circ}{4n}\right)} = \frac{a}{2 \sin\left(\frac{90^\circ}{n}\right)}$$

This beautiful rule holds all the regular polygons having odd number of sides

Since n=7 for heptagon

$$BK = \frac{\text{perimeter of regular pentagon}}{\text{longest diagonal length}} = \frac{7a}{\frac{a}{2 \sin\left(\frac{90^\circ}{7}\right)}} = 7 \times 2 \sin\left(\frac{90^\circ}{7}\right) = 14 \sin\left(\frac{90^\circ}{7}\right) = 3.115293\dots$$

From formula,

$$BK = 2n \sin\left[\frac{90^\circ}{n}\right]$$

$$BK = (2 \times 7) \sin\left[\frac{90^\circ}{7}\right]$$

$$= 14 \sin\left[\frac{90^\circ}{7}\right] = 3.115293\dots$$

Similarly this formula gives an accurate ratio of perimeter to the diagonal of all polygons having odd number of sides.

4. For $n \rightarrow \infty$ representing a apeirogon

For a given perimeter, if n become very large (or countably infinite) then a regular polygon becomes regular apeirogon [ref]. Regular apeirogon is ultimate form of regular polygon and it has countably infinite number of equal edges.

The ratio of the perimeter of this polygon to the length of its largest diagonal is calculated by the limiting value as $n \rightarrow \infty$. The limiting value is calculated to be π .

Proof

For polygons of even number of sides

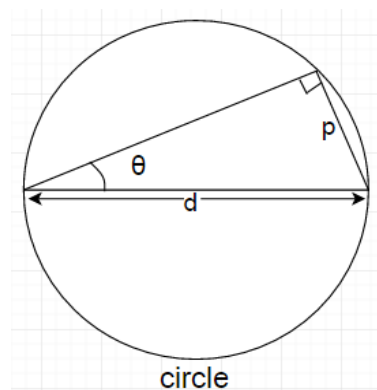
$$BK_{n \rightarrow \infty} = \lim_{n \rightarrow \infty} n \sin \left[\frac{180^\circ}{n} \right] \\ = \pi$$

For polygons of odd number of sides

$$BK_{n \rightarrow \infty} = \lim_{n \rightarrow \infty} 2n \sin \left[\frac{90^\circ}{n} \right] \\ = \pi$$

ANALOGY TO CIRCLE

By analogy to regular polygons with large number of edges, regular apeirogon resembles a circle. In the figure below we can see that C is circumference of circle, θ is angle opposite to the perpendicular arm and p is length of the perpendicular arm of the right angled triangle whose hypotenuse is diameter of the circle.



If two lines from diametrically opposite points of the circle intersect at any point on the circumference then angle formed on the circumference by these two lines is always 90° .

ILLUSTRATION

From $\triangle ABC$

$$\sin \theta = \frac{p}{d}$$

$$\text{or, } d = \frac{p}{\sin \theta}$$

The BK ratio is then given by

$$\begin{aligned} BK &= \frac{C}{d} \\ &= \frac{C}{\frac{p}{\sin \theta}} \\ &= C \frac{\sin \theta}{p} \end{aligned}$$

$$\therefore BK = C \frac{\sin \theta}{p}$$

COMPUTATIONAL ILLUSTRATION

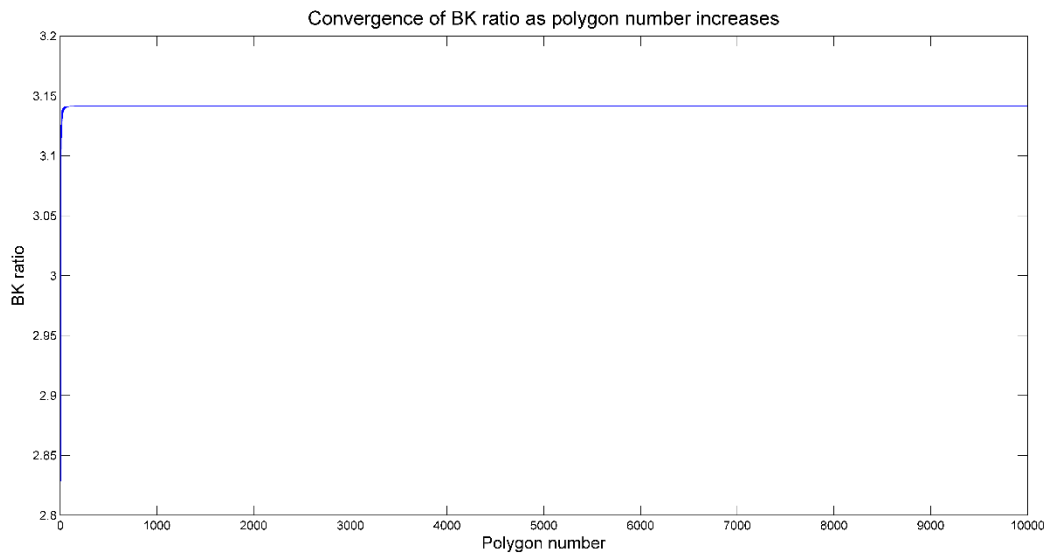


Figure 2: Convergence of the BK ratio as the number of sides of a regular polygon increases

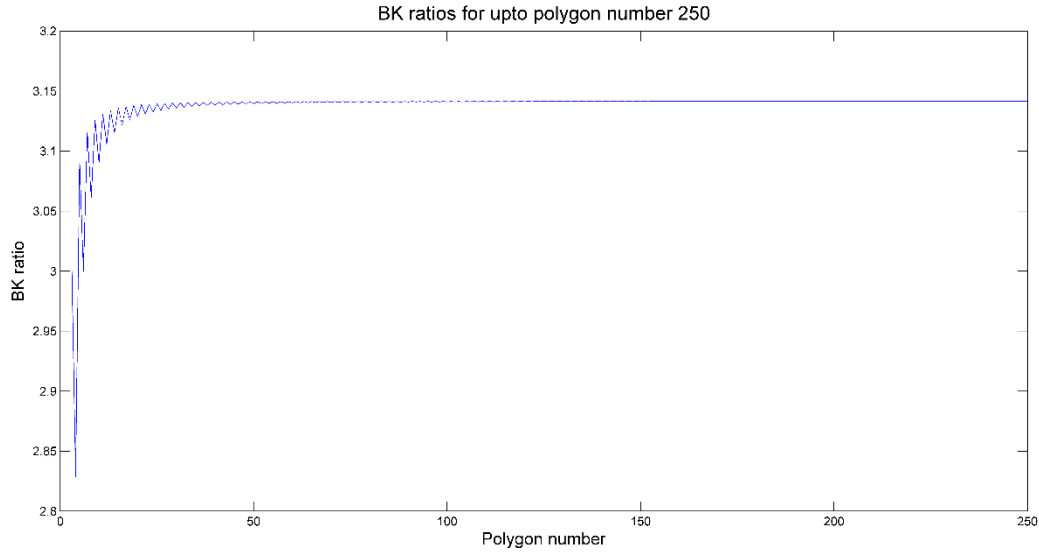


Figure 3: Fluctuations of the BK ratio for the number of sides of a regular polygon from 3 to 250

Figure 2 shows the convergence of the BK ratios as the polygon number increases. However, at lower polygon number there are significant fluctuations in the value of the ratio. As the polygon number exceeds 250 the fluctuation is approximately negligible and approaches the value of π .

APPENDIX

The numerical value of the BK Ratios have been presented below:

Serial number of regular polygon	Name of regular polygon	BK Ratio	BK ratio with precision up to 30 decimal digits
1	Equilateral triangle	$BK = 3$	3.00000000000000000000000000000000
2	Square	$BK = n \sin\left[\frac{180^\circ}{n}\right] = 4 \sin\left[\frac{180^\circ}{4}\right] = 4\sin 45^\circ$	2.82842712474619009760337744842
3	Pentagon	$BK = 2n \sin\left[\frac{90^\circ}{n}\right] = 2 \times 5 \sin\left[\frac{90^\circ}{5}\right] = 10\sin 18^\circ$	3.09016994374947424102293417183
4	Hexagon	$BK = 6 \sin 30^\circ$	3.00000000000000000000000000000000
5	Heptagon	$BK = 14\sin(90^\circ/7)$	3.11529307538840166004463590295
6	Octagon	$BK = 8\sin(180^\circ/8)$	3.06146745892071817382767987224
7	Nonagon	$BK = 18\sin 10^\circ$	3.12566719800474627933089928185

8	Decagon	$BK = 10\sin 18^\circ$	3.090169943749474241 02293417183
9	Hendecagon/Un decagon	$BK = 22\sin(90^\circ/11)$	3.130926442012273089 76343870956
10	Dodecagon	$BK = 12\sin 15^\circ$	3.105828541230249148 18678605149
11	Tridecagon	$BK = 26\sin(90^\circ/13)$	3.133953686638399387 07575987377
12	Tetradecagon	$BK = 14\sin \sin(180^\circ/14)$	3.115293075388401660 04463590295
13	Pentadecagon	$BK = 30\sin 6^\circ$	3.135853898029604141 99502464407
14	Hexadecagon	$BK = 16\sin(180^\circ/16)$	3.121445152258052285 57255789563
15	Heptadecagon	$BK = 34\sin(90^\circ/17)$	3.137124221752267838 14813764325
16	Octadecagon	$BK = 18\sin 10^\circ$	3.125667198004746279 33089928185
17	Enneadecagon	$BK = 38\sin(90^\circ/19)$	3.138015127948628334 81306950102
18	Icosagon	$BK = 20\sin(180^\circ/20)$	3.128689300804617380 20210638934
19	Icosihenagon	$BK = 42\sin(90^\circ/21)$	3.138663930629818680 21946932086
20	Icosikaidigon	$BK = 22\sin(180^\circ/22)$	3.130926442012273089 76343870956
21	Icosikaitrigon	$BK = 46\sin(90^\circ/23)$	3.139151014774864892 37467003503
22	Icosikaitetragon	$BK = 24\sin(180^\circ/24)$	3.132628613281238197 16174946949
23	Icosikaipentago n	$BK = 50\sin(90^\circ/25)$	3.139525976465668803 80891122828
24	Icosikaihexagon	$BK = 26\sin(180^\circ/26)$	3.133953686638399387 07575987377
25	Icosikaiheptago n	$BK = 54\sin(90^\circ/27)$	3.139820761165694741 09239290974
26	Icosikaioctagon	$BK = 28\sin(180^\circ/28)$	3.135005330892620037 12376618762
27	Icosikaienneago n	$BK = 58 \sin(90^\circ/29)$	3.140056697954216516 69468290652
28	Tricontagon	$BK = 30\sin 6^\circ$	3.135853898029604141 99502464407
29	Tricontakaihena gon	$BK = 62\sin(90^\circ/31)$	3.140248468000188161 28261516408
30	Tricontakaidigo n	$BK = 32\sin(180^\circ/32)$	3.136548490545939263 81425804444
31	Tricontakaitrigo n	$BK = 66\sin(90^\circ/33)$	3.140406444366991631 68595810608
32	Tricontakaitetra gon	$BK = 34\sin(180^\circ/34)$	3.137124221752267838 14813764325

33	Tricontakaipentagon	$BK = 70\sin(90^\circ/70)$	3.140538124536044782 06632357613
34	Tricontakaihexagon	$BK = 36\sin 5^\circ$	3.137606738915694248 09031375015
35	Tricontakaiheptagon	$BK = 74\sin(90^\circ/37)$	3.140649036514974634 97711987755
36	Tricontakaiioctagon	$BK = 38\sin(180^\circ/38)$	3.138015127948628334 81306950102
37	Tricontakaienneagon	$BK = 78\sin(90^\circ/39)$	3.140743328534381182 23244911686
38	Tetracontagon	$BK = 40\sin(180^\circ/40)$	3.138363829113797801 31840983974
39	Tetracontakaihe-nagon	$BK = 82\sin(90^\circ/41)$	3.140824162582898601 85044758480
40	Tetracontakaidigon	$BK = 42\sin(180^\circ/42)$	3.138663930629818680 21946932086
41	Tetracontakaitri-gon	$BK = 86\sin(90^\circ/43)$	3.140893982958659807 29056484846
42	Tetracontakaitet-ragon	$BK = 44\sin(180^\circ/44)$	3.138924060766222974 40286111695
43	Tetracontakaipen-tagon	$BK = 90\sin 2^\circ$	3.140954703225087448 13956634628
44	Tetracontakaihexagon	$BK = 46\sin(180^\circ/46)$	3.139151014774864892 37467003503
45	Tetracontakaiheptagon	$BK = 94\sin(90^\circ/47)$	3.141007838721409662 27484961792
46	Tetracontakaiioctagon	$BK = 48\sin(180^\circ/48)$	3.139350203046867207 13514682121
47	Tetracontakaienneagon	$BK = 98\sin(90^\circ/49)$	3.141054602022207074 88795148387
48	Pentacontagon	$BK = 50\sin(180^\circ/50)$	3.139525976465668803 80891122828
49	Pentacontakaihe-nagon	$BK = 102\sin(90^\circ/51)$	3.141095972729376095 20561928758
50	Pentacontakaidigon	$BK = 52\sin(180^\circ/52)$	3.139681865958874778 77704164220
.	.	.	.
99	hectohenagan	$BK = 202\sin(90^\circ/101)$	3.141466007910876550 05140024123
100	Hectokaidigon	$BK = 102\sin(180^\circ/102)$	3.141095972729376095 20561928757
.	.	.	.
.	.	.	.
198	Diectogon	$BK = 200\sin(180^\circ/200)$	3.141463462364135150 65907066198
199	Diectohenagon	$BK = 402\sin(90^\circ/201)$	3.141560676058298068 47912624424
200	Diectokaidigom	$BK = 202\sin(180^\circ/202)$	3.141466007910876550 05140024123

.	.	.	.
498	Pentahectogon	$BK = 500\sin(180^\circ/500)$	3.141571982779475624 86765507897
499	Pentahectohenagon	$BK = 1002\sin(90^\circ/501)$	3.141587506488546573 44911867315
500	Pentahectokaidigon	$BK = 502\sin(180^\circ/502)$	3.141572147158702773 77552989356
.	.	.	.
998	Chilliagon	$BK = 1000\sin(180^\circ/1000)$	3.141587485879563351 93322703549
999	Chilliahenagon	$BK = 2002\sin(90^\circ/1001)$	3.141591364241742741 96381045095
1000	Chilliaidigon	$BK = 1002\sin(180^\circ/1002)$	3.141587506488546573 44911867315
.	.	.	.
4998	Pentakischilliagon	$BK = 5000\sin(180^\circ/5000)$	3.141592446881286116 72626766426
4999	Pentakischilliahenagon	$BK = 10002\sin(90^\circ/5001)$	3.141592601933330344 29355391453
5000	Pentakischilliadigon	$BK = 5002\sin(180^\circ/5002)$	3.141592447046553751 97152596898
.	.	.	.
9998	Myriagon	$BK = 10000\sin(180^\circ/10000)$	3.141592601912665692 97934647928
9999	Myriahenagon	$BK = 20002\sin(90^\circ/10001)$	3.141592640673094773 13310688367
10000	Myriakaidigon	$BK = 10002\sin(180^\circ/10002)$	3.141592601933330344 29355391453
.	.	.	.
99998	Centachilliagon	$BK = 100000\sin(180^\circ/100000)$	3.141592653073021960 48314802075
99999	Centachilliahenagon	$BK = 200002\sin(90^\circ/100001)$	3.141592653460603002 78062061722
100000	Centachilliakaidigon	$BK = 100002\sin(180^\circ/100002)$	3.141592653073042630 71415718326
.	.	.	.
999998	Megagon	$BK = 1000000\sin(180^\circ/1000000)$	3.141592653584625525 68259596341
999999	Megahenagon	$BK = 2000002\sin(90^\circ/1000001)$	3.141592653588501312 85148356440
1000000	Megakaidigon	$BK = 1000002\sin(180^\circ/1000002)$	3.141592653584625546 35338507120
.	.	.	.
1012-2	Teragon (1012 sides)	$BK = 1012 \sin(180^\circ/10^{12})$	3.141592653589793238 46263821556
1012 -1	(1012+1)-gon	$BK = 2(1012+1)\sin(90^\circ/(1012+1))$	3.141592653589793238 46264209135

1012	(1012+2)-gon	$BK = (1012+2)\sin(180^\circ/(1012+2))$	3.141592653589793238 46263821556 6722854897991273463
.	.	.	.
1015-2	Petagon (1015 sides)	$BK = 1015 \sin(180^\circ/10^{15})$	3.141592653589793238 46264338327 43351714171194293458
1015-1	(1015+1)-gon	$BK = 2(1015+1)\sin(90^\circ/(1015+1))$	3.141592653589793238 46264338327 82109560021569094516
1015	(1015+2)-gon	$BK = (1012+2)\sin(180^\circ/(1012+2))$	3.141592653589793238 46264338327 43351714171194500168
.	.	.	.
∞	Regular Apeirogon (Precise Circle) It is ultimate regular polygon	$BK = C \frac{\sin \theta}{p} = \pi$	3.141592653589793238 46264338327

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