Theorems on Pythagorean triples and Prime numbers

Faisal Amin Yassein Abdelmohssin[1]

Sudan Institute for Natural Sciences, P.O.BOX 3045, Khartoum, Sudan

September 07, 2017

Abstract
Relationships among natural numbers constituting a Pythagorean triple (PT) and between these natural numbers constituting the Pythagorean triples (PTs) and Prime Numbers (PNs) have been found. These relationships are formulated as theorems; first theorem is that the natural numbers constituting a Pythagorean triple (PT) satisfy a certain equation related to sum of their differences; second theorem is that differences of sum of the natural numbers constituting a Pythagorean triple (PT) are prime numbers.

MSC numbers: 11Axx, 11Dxx, 11D04, 11D09, 11A41, 11N80[2]
Keywords: Elementary number theory, Diophantine equations, linear equation, generalized primes and integers.

Introduction

Pythagorean triples and prime numbers have been the subject of much recreational material as well as the basis of some of the most important and fundamental topics in number theory. In my search for a relation among prime numbers I came across a relation between the natural numbers of Pythagorean triples and between the natural numbers of Pythagoreans triples and prime numbers.

Discussion: Theory

I formulated theorems of relationships among natural numbers constituting a Pythagorean triple (PT) and between these natural numbers constituting the Pythagorean triples (PTs) and Prime Numbers (PNs) as follows:

---

[1] f.a.y.abdelmohssin@gmail.com

[2] https://cran.r-project.org/web/classifications/MSC.html
http://www.ams.org/msc/msc2010.html

Theorem I
For every Pythagorean triple (a,b,c) of natural numbers a,b and c, the following equations hold:
$(c-a)=(c-b)+(b-a)$, if $(a<b<c)$,
and
$(c-b)=(c-a)+(a-b)$, if $(b<a<c)$.

Brackets should be calculated first.

Theorem II
For every Pythagorean triple (a,b,c) of natural numbers a,b and c, then, $(b+c)-(a+c)$ and $(a+c)-(a+b)$ are prime numbers, if $(a<b<c)$, and, $(a+c)-(b+c)$ and $(b+c)-(b+a)$ are prime numbers, if $(b<a<c)$.

Brackets should be calculated first.

Figure (1) and (2) below show the relationships of the natural numbers of a Pythagorean triple.

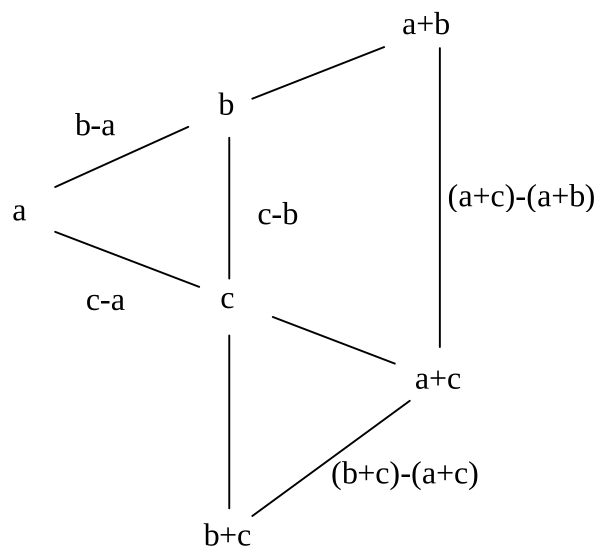

Fig (1): Natural numbers of a Pythagorean triple (a,b,c), in case $(a<b<c)$, differences $(b-a)$, $(c-b)$ and $(c-a)$, sums $(a+b)$, $(b+c)$ and $(c+a)$ and differences of sums $(a+c)-(a+b)$ and $(b+c)-(a+c)$.

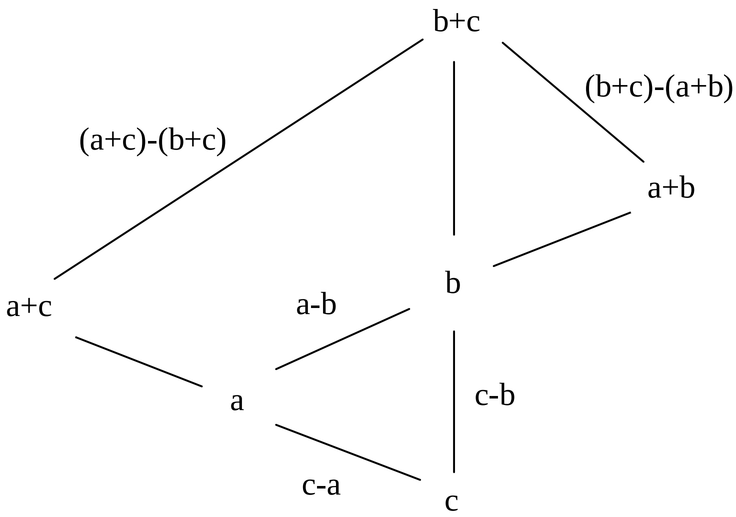

Fig (2): Natural numbers of a Pythagorean triple (a,b,c), in case $(b<a<c)$, differences $(a-b)$, $(c-a)$ and $(c-b)$, sums $(a+b)$, $(b+c)$ and $(c+a)$ and differences of sums $(a+c)-(b+c)$ and $(b+c)-(a+b)$.

Conclusion

Search for a pattern in prime numbers should be sought with relation to other theorem relating natural numbers. I found a relation among prime numbers with relation to Pythagorean Theorem and Pythagorean triples.

References:

[1] Stillwell J (2003). *Elements of Number Theory*. New York: Springer-Verlag. pp. 110–112. ISBN 0-387-95587-9. Retrieved 2016-03-17.

[2] Barlow P (1811). *An Elementary Investigation of Theory of Numbers.* St. Paul's Church-Yard, London: J. Johnson.

[3] Richinick, Jennifer (July 2008). *"The upside-down Pythagorean Theorem".* Mathematical Gazette. **92**: 313–317.

[4] Koshy T (2001). *Elementary number theory with applications.* New York: Academic Press. ISBN 978-0-12-421171-1.

[5] *Stark H* (1978). *An Introduction to Number Theory.* MIT Press. ISBN 0-262-69060-8.

[6] G.H. Hardy*; E. M. Wright* (2008). *An Introduction to the Theory of Numbers* (6th edition.). Oxford University Press. *p. 6.* ISBN 978-0-19-921986-5.

[7] F. A.Y. Abdelmohssin viXra: **1708.0158** *On the Pythagorean Triples* (12, y, z), submitted on *2017-08-14 08:00:35.*Category: Number Theory