

# A Discrete Bass-Riccati Diffusion Model for Forecasting The Adoption of New Carsharing Services

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## ABSTRACT

In recent years, there is rapid growth in the adoption of sharing economy as conceptual framework of digital innovations. The so-called sharing economy is likely to impact many areas of social life, not only hotel, travelling, and logistics, but also new transportation services including carsharing and ridesharing. Nonetheless, current travel demand models are unable to predict long-range trends in travel behavior as they do not entail a mechanism that projects membership and market share of new modes of transport (Uber, Lyft, Grab etc.) Following El Zarwi, Vij & Walker, in the present paper we will discuss the use of technology adoption model based on Bass-Riccati equation to forecast the future of carsharing as new transportation services. More researches are needed to verify our proposition.

## 1. Introduction: the basics of technology diffusion model

There is little doubt that the sharing economy has become a new business buzzword, and even it has become one of mantra for digital economy investors.[5][6][7]. Nonetheless, we shall admit that the existing forecasting tools do not include diffusion models to predict adoption of such a sharing economy in providing new transportation services. In the meantime, since its introduction to marketing in the 1960s, the diffusion theory perspective has been of interest to scholars of consumer behavior, marketing management, and management and marketing science. The main impetus underlying the work done in this area is a new product-growth model developed by Bass. The Bass

model has been investigated in mainly three aspects: adopter categorization, the communication structure between the two assumed groups of adopters of innovators and imitators, the development of diffusion models by specifying adoption decisions at the individual level. The Bass model and its revised forms have been successfully demonstrated for forecasting innovation diffusion in many products and services.[4] Following El Zarwi, Vij & Walker [1], in the present paper we will discuss the use of technology adoption model based on Bass-Riccati equation to forecast the future of carsharing as new transportation services. Nonetheless, while their analysis is fresh and insightful, they do not consider the discrete Riccati version as analogy of Bass equation, as will be discussed briefly in this paper.

## 2. Recent development: discrete Riccati equation analog of Bass model

Since the Bass model was first reported, diffusion theory has often been used to model the first-purchase sales growth of a new product over time. In his 1969 article, Bass suggested that the following differential equation can be used to represent the diffusion process:[2]

$$\frac{dN(t)}{dt} = \left( p + \frac{q}{m} N(t) \right) (m - N(t)), \quad (1)$$

Where  $N(t)$  is the cumulative number of adopters at time  $t$ ,  $m$  is the cumulative number of adopters,  $p$  is the coefficient of innovation, and  $q$  is the coefficient of imitation. It can be shown, that by defining:  $a=mp$ ,  $b=(q-p)/2$ ,  $c=-(q/m)$ , then equation (1) can be regarded as Riccati equation as follows:[2]

$$\frac{du}{dt} = a(t) + 2b(t)u + c(t)u^2, \quad (2)$$

Where  $a(t)$ ,  $b(t)$  and  $c(t)$  are given functions of  $t$ . In this model, the Riccati equation is considered when  $a, b, c$  are constants.

Moreover, a discrete model conserves the properties of the continuous model, and so the parameter estimation would likely to be simpler and more accurate. A discrete Bass model can be obtained by using a discrete Riccati equation. This model is described by a difference equation. The difference equation has an exact solution, although an ordinary forward difference equation does not. The discrete Bass model enables to forecast innovation diffusion without a continuous-time Bass model.[2][4]

Hirota used a discrete Riccati equation that has an exact solution. His discrete Riccati equation is described as:[2]

$$\frac{N_{(t+\delta)} - N_{(t-\delta)}}{2\delta} = a + b(N_{(t+\delta)} + N_{(t-\delta)}) + cN_{(t+\delta)}N_{(t-\delta)}, \quad (3)$$

Where  $\delta$  is the constant time-difference length. The exact solution of equation (3) has been found in [2].

Moreover, by using the discrete Riccati equation, Satoh obtained the discrete Bass model:[2][4]

$$\frac{N_{(n+1)} - N_{(n-1)}}{2\delta} = p \left( m - \frac{N_{(n+1)} + N_{(n-1)}}{2} \right) + \frac{q}{m} \left( m \left( \frac{N_{(n+1)} + N_{(n-1)}}{2} \right) - N_{(n+1)}N_{(n-1)} \right) \quad (4)$$

Again the exact solution to equation (4) was obtained and reported in [2][4].

We are convinced that the discrete Bass equation (4) can be used as better forecasting model for adoption of carsharing as new transportation services. And a better forecasting tool may be used for developing better fiscal policies to promote more innovation-based economic growth.[7]

### **3. Concluding Remarks**

Considering the ever-increasing demand for better transportation and travelling services, it can be expected that carsharing and ridesharing as technology innovations will grow rapidly as new transportation solution both in developed countries and also in developing countries, such as China, India, Indonesia etc. For forecasting purpose, it is suggested that Bass-Riccati diffusion model including its discrete version may be used to predict such a carsharing future growth. We are convinced that the discrete Bass equation (4) can be used as better forecasting model for adoption of carsharing as new transportation services. And a better forecasting tool may be used for developing better fiscal policies to promote more innovation-based economic growth.

Nonetheless, this is only an early investigation. More researches and observations are recommended to verify our propositions.

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