Morphogenesis of Gravitational Structures through a Non-differentiable Approach

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Abstract. The present paper analyzes the morphogenesis of gravitational structures, assuming that dynamics of a test particle in a gravitational field takes place on continuous but non-differentiable curves. The dynamics of such gravitational system is described by an equation for a complex speed that characterizes its rheological behavior. Moreover, the separation of movements on interaction scales in the dynamics equation implies a non-differentiable hydrodynamical model. Finally, such an approach was applied both to one and two body problems and, via numerical simulation, to the morphogenesis of gravitational structures. Consequently, quantization at intragalactic (Solar System) and extragalactic scales (Tifft's effect) as well as certain modifications of Newton's force occur. At the same time there is a tendency to form gravitational structures at any epoch, without inflationary phase.

Introduction

One of the main open problems of gravitation is that of the formation and evolution of structures from the action of gravitation at different scales (intragalactic scale, extragalactic scale etc). However, up to this time the galaxy formation theory is not a robust theory able to account the detailed properties of galaxies, for understanding stars formation etc [1]. "The same can be said of the formation of planetary systems, as it is now demonstrated by the discovery of extra-solar planetary systems with properties that were unexpected from the previously accepted models of formation" [2] (see e.g. Jupiter – size planets very close to the central star and highly elliptic orbits [3]). Moreover, at the intragalactic and at extra galactic scales the formation and evolution of gravitational structures is strongly interconnected with that of dark matter.

"The existence of large quantities of unseen matter is a necessary ingredient in the standard approach, since, in its absence, the self-gravitational attraction of visible matter is insufficient for galaxies to form within the known age of the Universe. However, while the anomalous dynamical effects (flat rotation curves of spiral galaxies, velocity dispersion of clusters of galaxies, etc) and gravitational lensing effects that the dark matter hypothesis attempts to explain are established, dark matters itself escapes any detection" [2]. We note that the General Relativity Theory is not giving a complete answer to all the problems previously mentioned.

On the other hand the gravitational systems which display chaotic behaviour are recognized to acquire self-similarity (spatial and temporal structures seem also to appear). Moreover, the gravitational systems manifest strong fluctuations at all possible scales [2, 4-11]. One of the most impressive examples is the fruitful suggestion made by Laskar that the inner planetary system (telluric planets) is chaotic [4-8]. It was also experimentally observed the regularity of the distribution of planets, satellites and asteroids in the Solar System [4-9]. Since the non-differentiability appears as a universal property of these systems, it is necessary to construct a non-differentiable gravitation, [2, 10, 11]. In such a conjecture, by considering that the complexity of the gravitational interactions is replaced by non-differentiability, it is no longer necessary to use the whole classical "arsenal" of quantities of standard gravitation (differentiable gravitation through General Relativity). One model which treats the gravitational interactions in the previously mentioned manner is the Scale Relativity Theory (SRT) [2, 10, 11].

In the present paper, using the non-differentiable hydrodynamics approach of the SRT, a method different from the one of Nottale [2, 10], some applications in the study of gravitational interactions are given.

Mathematical Model

Taking into account the complexity of the phenomena implied in the morphogenesis of the gravitational systems, we assume that the dynamics of these systems imply the fractal structure of space [2, 10, 11]. If such an assumption works, then the dynamics of the gravitational systems in a fractal space are described by the covariant derivative [18-20]:

$$\frac{\hat{d}}{dt}=\frac{\partial}{\partial t}+\left(\hat{\mathbf{V}}\cdot\nabla\right)-i\mathrm{D}\Delta,\mathrm{D}=D\left(dt\right)^{\left(\frac{2}{D_F}\right)-1} \tag{1}$$

where $\hat{\mathbf{V}}$ is the complex speed field

$$\hat{\mathbf{V}}=\mathbf{V}_D-i\mathbf{V}_F \tag{2}$$

Here $\mathbf{V}_D$ is the standard classical speed (differentiable speed), which is independent of scale resolution (*dt*), while the imaginary part, $\mathbf{V}_F$, is a new quantity arising from non-differentiability (the fractal speed), which is resolution-dependent; D is a structure coefficient, characteristic to the fractal-non-fractal transition, scale resolution and fractal dimension D_F dependent, and Δ is the Laplace operator.

We note that the use of any Kolmogorov or Haussdorff definitions [24 – 26] can be accepted for fractal dimension, but once a certain definition is admitted, it should be used until the end of analyzed dynamics. Moreover, our operator given by Eq. (1) is more general that the one of Nottale from SR [2, 10, 11]. Indeed, for movements on fractal curves with $D_F=2$ (compatible with Brownian type movements) the operator given by Eq. (1) takes the form from SRT [2, 10, 11]

$$\frac{\hat{d}}{dt}=\frac{\partial}{\partial t}+\left(\hat{\mathbf{V}}\cdot\nabla\right)-iD\Delta$$

Applying the operator given in eq. (1) to the complex speed field given by eq. (2) and accepting the Newton's second generalized principle [2, 10, 11], in the form $\frac{\hat{d}\hat{\mathbf{V}}}{dt}=-\nabla U$, we obtain a Navier-Stokes-type equation:

$$\frac{\hat{d}\hat{\mathbf{V}}}{dt}=\frac{\partial\hat{\mathbf{V}}}{\partial t}+(\hat{\mathbf{V}}\cdot\nabla)\hat{\mathbf{V}}-i\mathrm{D}\Delta\hat{\mathbf{V}}+\nabla U=0 \tag{3}$$

where $U=GM_0m_0/r$ is the gravitational scalar potential with *G* the Newton's constant, M_0 the rest mass of the gravitational source, m_0 the rest mass of the test particle and *r* the source - test particle distance.

Equation (3) means that at any point of any non-differentiable path, the local acceleration term, $\partial_t\hat{\mathbf{V}}$, the non-linear (convective) term, $(\hat{\mathbf{V}}\cdot\nabla)\hat{\mathbf{V}}$, the dissipative term, $\Delta\hat{\mathbf{V}}$, and the force term, ∇U, make their balance. Therefore, in a fractal space the gravitational system can be assimilated with a "rheological" fluid with imaginary viscosity, $iD\left(dt\right)^{(2/D_F)-1}$, whose dynamics is described by the complex speed field $\hat{\mathbf{V}}$.

If the motions of the gravitational system are irrotational:

$$\nabla\times\hat{\mathbf{V}}=0,\nabla\times\mathbf{V}_D=0,\nabla\times\mathbf{V}_F=0 \tag{4}$$

we can choose $\hat{\mathbf{V}}$ of the form:

$$\hat{\mathbf{V}} = -2i\mathrm{D}\nabla \ln \psi$$

For $\psi = \sqrt{\rho}e^{iS}$, with $\sqrt{\rho}$ the amplitude and S the phase of ψ, the complex speed field given by eq. (2) takes the explicit form:

$$\begin{aligned} \hat{\mathbf{V}} &= 2\mathrm{D}\nabla S - i\mathrm{D}\nabla \ln \rho \\ \mathbf{V}_D &= 2\mathrm{D}\nabla S \\ \mathbf{V}_F &= \mathrm{D}\nabla \ln \rho \end{aligned} \tag{5}$$

By substituting eq. (5) in eq. (3) and separating the real and the imaginary parts, up to an arbitrary phase factor which may be set to zero by a suitable choice of the phase of ψ, we obtain:

$$m_0[\frac{\partial \mathbf{V}_D}{\partial t} + (\mathbf{V}_D \cdot \nabla)\mathbf{V}_D] = -\nabla(Q+U) \tag{6}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}_D) = 0 \tag{7}$$

with Q the non-differentiable potential:

$$Q = -2m_0\mathrm{D}^2 \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = -\frac{m_0\mathbf{V}_F^2}{2} - m_0\mathrm{D} \cdot \nabla \cdot \mathbf{V}_F \tag{8}$$

and m_0 the rest mass of the fluid ”entity”. Equation (6) is the law of momentum conservation and equation (7) is the law of probability density conservation. These equations define the non-differentiable hydrodynamics (NH). We note that the Nottale hydrodynamics model can be obtained from NH model for movements of fluid entities on the fractal curves with $D_F = 2$ compatible with Brownian type motions.

One Body Problem through the Non-differentiable Hydrodynamics

Let us apply the previous formalism to study the dynamics in the Solar System. We consider a planet (or generally, a test particle) in the gravitational field of the Sun, U, and in the collective field of an ensemble of planets (more generally of particles) and assume that this system is chaotic. Our conjecture is that the effect of chaos on large time scales can be summarized by a Brownian motion type process, with a diffusion coefficient D (chaotic effect on the considered test-particle of all the other celestial bodies has as result motions on non-differentiable curves of fractal dimension $D_F = 2$). A particularization of equations (6), (7) and (8) in the case of stationary motion in the gravitational field U gives:

$$\begin{aligned} &\nabla^2\sqrt{\rho} = -\frac{1}{2m_0D^2}\left(E - \frac{1}{2}m_0\mathbf{V}_D^2 + \frac{GM_0m_0}{r}\right)\sqrt{\rho}, \nabla \cdot (\rho\mathbf{V}_D) = 0 \\ &0 < r < \infty, 0 \le \delta \le \pi, 0 \le \varphi \le 2\pi, \\ &\rho(r=\infty,\delta,\varphi) = 0, \rho(r,\delta,\varphi)_{\delta \le \pi/2} = \rho(r,\delta+\pi/2,\varphi) \end{aligned} \tag{9}$$

where E is an energy constant. Using the method from [14], the eigenvalues of the energy

$$E_n = -2m_0D^2\frac{1}{a^2n^2}, \; a = \frac{4D^2}{GM_0}, n = 1,\ 2,\ 3 \tag{10}$$

and the eigenfunctions

$$\rho_{nlm} = C_{nlm}\left(\frac{2r}{na}\right)^{2l} e^{-\frac{2r}{na}}\left[L_{n+l}^{2l+1}\left(\frac{2r}{na}\right)P_l^m(\cos\delta)\right]^2$$
$$C_{nlm} = \left(\frac{2}{na}\right)^3 \frac{(n-l-1)!}{2n[(n+l)!]^3}\frac{2l+1}{4\pi}\frac{(l-|m|)!}{(l+|m|)!} \tag{11}$$

result. In the previous relations L_{n+l}^{2l+1} are the associated Laguerre polynomials, P_l^m are the associated Legendre polynomials, n is the gravitational equivalent of the principal quantum number, l is the gravitational equivalent of the orbital quantum number and m is the gravitational equivalent of the magnetic quantum number. For physical reasons, only the following combination of gravitational quantum numbers, are acceptable $0 \le l \le n-1, -l \le m \le +l$. Choosing the parameter D of the form $D = GM_0 / 2w$, where w is a fundamental constant that has the dimension of a speed, the macroscopic gravitational coupling constant $\alpha_g = w/c$, with *c* the speed of light into the vacuum, is introduced [2, 15]. Then, for $w \approx 144 km/s$ and $\alpha_g \approx 2072$ [15] quantization in Solar System is obtained: specific energies $\varepsilon_n = E_n / m_0 = -G^2M_0^2 / 8D^2n^2$, orbital speeds $w_n = w/n$, semi-major axes $a_n = an^2$ etc.

Through an extension of the previous method in the two body problem with comparable rest mass, the quantization in galaxy pairs can be obtained (Tifft's effect [16]).The inertial forces $m_0\mathbf{V}_D \cdot \nabla\mathbf{V}_D$ appear for *m*=0 states (static states), in which the Newton and non-differentiable forces balance each other:

$$-\nabla U - \nabla Q_{nm} = 0, -\nabla Q_{nm} = \frac{m_0 D^2}{ar^2} = \frac{GM_0 m_0}{r^2}$$

For $m \neq 0$ states (dynamical states) the inertial, Newton and non-differentiable forces are in balance in every field points:

$$-m_0\mathbf{V}_D \cdot \nabla\mathbf{V}_D - \nabla U - \nabla Q_{nm} = 0$$
$$-\nabla Q_{nm} = 4m_0 D^2\left(\frac{1}{ar^2} - \frac{m^2}{r^3\sin^2\delta}\right) = \frac{GM_0 m_0}{r^2} - m_0\left(\frac{GM_0}{w}\right)^2 \frac{m^2}{r^3\sin^2\delta}$$

One can notice that non-differentiability of the movement curves introduces, through the non-differentiable force, extra-terms at Newton's force. In figure 1a, b we show the behaviour of the gravitational system both in the Newton's case (m=0, see fig. 1a) and in the non-differentiable one (m=1, see fig. 1b), corresponding to the non-differentiable force. Therefore, the universal quantization in various gravitational systems, the rotation curves of galaxies, the speed dispersion in clusters of galaxies etc. [2, 10, 17] are determined, not only through Newton's potential, but also through the non-differentiable potential Q_{nm}.

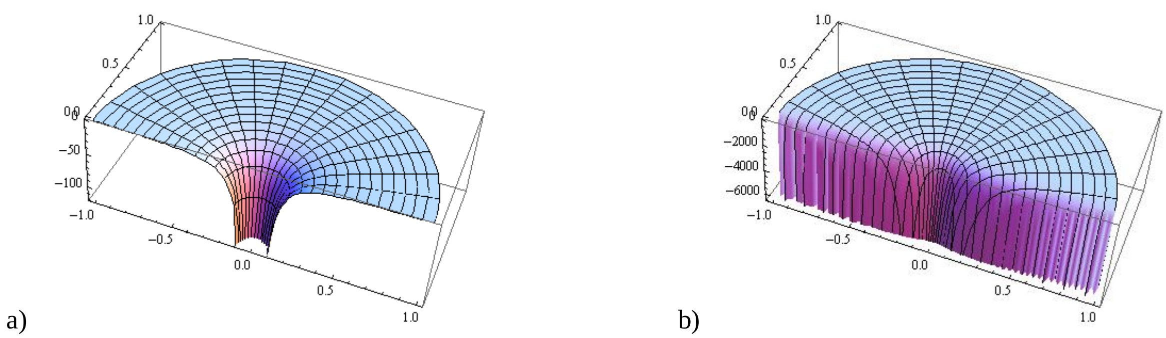

Figure 1: Dependence of the normalized gravitational force F_m on the normalized coordinates $(r/a, \delta)$ (a) for m=0, and (b) m=1.

The non-differentiable potential (8) comes from the non-differentiability of the movement curves and has to be treated as a kinetic term, not as a potential term.

For a special form of the non-differentiable potential [18-20], using the equations (6)-(8) for plane symmetry, we can suggest another solution for the morphogenesis of the gravitational structures, without any need for an inflationary phase. Thus, introducing the normalized coordinates and imposing adequate initial boundary conditions [18-20], in results the numerical solutions from Fig. 2a-c.

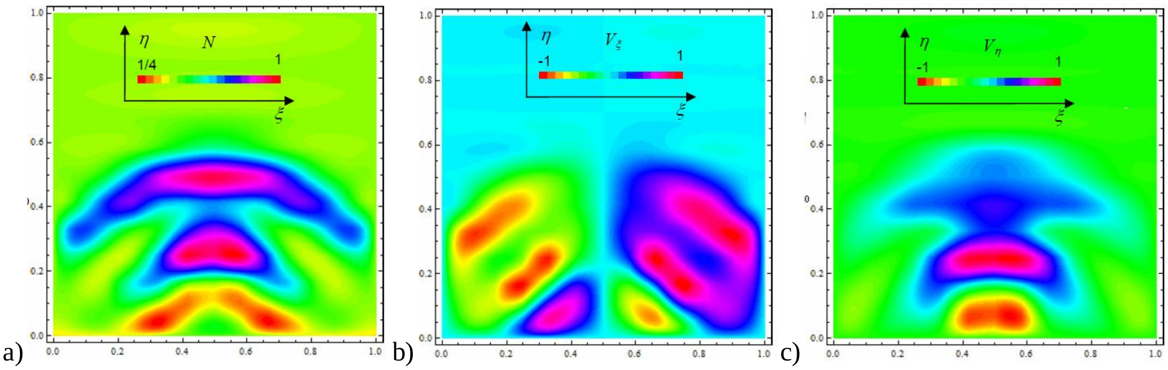

Figure 2: Dependence of the normalized density field N (a), normalized speed fields V_ξ (b), V_η (c), at normalized time $\tau = 1/2$ on the normalized spatial coordinates (ξ, η) .

Inspection of these numerical solutions shows the following: i) the normalized density field is of soliton-package-type [22]. Such numerical solution can explain for example the mass distribution of planets in the inner and outer of our solar system; ii) the normalized speed field V_ξ is symmetric with respect to the symmetry axis of the spatio-temporal Gaussian; iii) vortices and shock waves type are induced at the periphery of structure for the normalized speed field V_η . Therefore, the non-differentiability of the space at large scales involves a transformation of the equations of motion into those of a macroscopic non-differentiable hydrodynamic system. As a consequence, there is a tendency to form gravitational structures at any epoch: these gravitational structures are described by the probability density distributions given by the square of the modulus of the probability amplitudes, which are solutions of this non-differentiable hydrodynamic system. The non-differentiable approach is fundamentally different from the classical one. The loss of determinism of individual trajectories is compensated by determinism of gravitational structures. At each epoch, stationary solutions may correspond to the shape of the non-differentiable potential and the limiting and matching conditions. These gravitational structures also evolve (as given by the time-dependent non-differentiable hydrodynamic system) in correspondence with the evolution of the environment [2].

Conclusions

Assuming that the motions of a test particle in a gravitational field take place along continuous but non-differentiable curves the following conclusions result: i) A gravitational structure implies not only celestial bodies (test particles, gravitational field sources etc.) correlated by Newtonian forces, but also the environment in which they evaluate, assimilated as a complex fluid whose particles are moving on continuous but non-differentiable curves. This environment, that can’t exist as an entity separated from celestial bodies, is described by a special hydrodynamic type non-differentiable potential which works simultaneously with the standard gravitational potential (Newtonian potential). The non-differentiable potential is induced by the non-differentiability of the movement curves of the fluid particles. ii) The dynamics of a gravitational structure are described by an equation for a complex speed field and exhibit rheological properties (memory). iii) The separation movements on the interaction scales implies a non-differentiable hydrodynamics which contains, at differentiable scale, the law of momentum conservation and, at non-differentiable scale, the law of probability density conservation. iv) Applying the non-differentiable hydrodynamics approach to the study of a test particle dynamics in a gravitational field, quantization in Solar System, i. e. at intragalactic scale, result. Moreover, applying the same methodology in the two body problem with comparable rest mass, quantization in galaxy pairs for example, i. e. at the extragalactic scale, are obtained (Tifft's effect). v) The non-differentiable potential induces a Newtonian type force for the static gravitational states or a modified Newtonian type force (extra-terms at standard gravitational force) for the dynamical gravitational states. vi) There is a tendency to form gravitational structures at any epoch without any need for an inflationary phase given the fractal structure of space. These gravitational structures are described by the probability density distributions given by the square of the modulus of the probability amplitudes, which are numerical solutions of the non-differentiable hydrodynamic system.

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