A New Physical Model for Calculation of Atomic Mass

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According to the generally accepted physical model, the synthesis of the heavy elements may happen at a very high temperature in supernova explosions. In consequence of nuclear fusion, the supernova stars emit a very strong electromagnetic (EM) radiation, predominantly in the form of X-rays and gamma rays. The intensive EM radiation drastically decreases the masses of the exploding stars, directly causing mass defects of the nuclei. The general description of black body EM radiation is based on the famous Planck's radiation theory, which supposes the existence of independent quantum oscillators inside the black body. In this paper, it is supposed that in exploding supernova stars, the EM radiating oscillators can be identified with the nascent heavy elements losing their specific yields of their own rest masses in the radiation process. The final binding energy of the nuclei is additionally determined by strong neutrino radiation. Extending Planck's radiation law for discrete radiation energies, a very simple formula is obtained for the theoretical description of the measured neutral atomic masses.

Keywords: atomic mass calculation, Planck's radiation law, the origin of the elements, binding energy of the nuclei, new theoretical model of the nuclear synthesis.

1. Introduction

The theoretical determination of neutral atomic masses dates back to 1935, when C.F. von Weizsäcker [1] published his famous *liquid-drop model* for the calculation of nuclear masses. The model provides a general overview of masses and related stability of nuclei, and assumes the nucleus behaves in a gross collective manner, similar to a charged drop of liquid. The semi-empirical mass formula based on this phenomenological model was applied successfully mainly in the earlier period of nuclear physics. From the simple drop model one can easily calculate approximately the mass of a neutral atom by adding the number Z of electron masses to the mass of $X(Z,A)$ nuclei.

However, the liquid drop model does not give answers for many important questions related to the structure and forces inside the nuclei. The long-standing goal of nuclear physics has been to understand how the structure of nuclei arises from the interactions between the nucleons. It is known that nucleons are composed of quarks, but the important nuclear data have been derived yet only in rudimentary level relying to the quark interactions (by QCD).

The standard method for the modern calculation for light nuclei is based mainly on non-relativistic quantum mechanics. In the world, many realistic phenomenological models of two- and three-nucleon interactions have been developed by fits to nucleon-nucleon NN scattering data and the properties mainly of ^2H, ^3H, and ^4He. The non-relativistic Hamiltonian used typically contains two-body and three-body potentials. Different types of approximation method are now available in the literature to solve the few-body problems; nevertheless, parametric fitting of experimental data still remains necessary. The present paper does not investigate further the nuclear structure and forces, thus avoiding the difficult theoretical treatments and calculations. It focuses only on the birth of the nuclei in stars, and gives a very simple physical model for that.

It is widely known that the majority of the elements in the periodic table are synthesized in the stars. The synthesis of the heavy elements may happen only at very high temperature, for example, in supernova star explosions. In consequence of the nuclear fusion at high temperature, the supernova stars and of course, the ordinary stars, emit very strong electromagnetic (EM) radiation, predominantly in form of gamma and X-rays. In addition, the EM radiation is combined with strong neutrino radiation. The different intensive energy radiations continuously decreases the masses of the stars, directly causing mass defects of the nascent nuclei, and at least the strong binding of nuclei. The individual nuclei represent quantized black body oscillators; their frequencies are determined by their mass numbers. From this simple physical model, one can conclude that the binding energy curve of the nuclei is in immediate connection to the Planck's radiation law in a very high temperature region. In our paper, we have fitted the Planck's radiation law to the binding energy curve of the nuclei, supposing that the radiation frequency of an arbitrary nucleon is proportional to the root-square of its mass number.

2. Extension of Planck's Radiation Law

According to the Planck's radiation law the energy density of the EM radiation in function of the radiation frequency is

$$dE_f = \frac{2h}{c^2}\frac{f^3}{\exp(hf/kT)-1}df, \qquad (2.1)$$

where T is the absolute temperature, c is the speed of light, k is Boltzmann's constant, h is the Planck's constant and f is the radiation frequency. The new model for explaining the origin of the elements requires discrete radiation frequencies of the stars depending on the mass numbers of the nuclei. In classical electrodynamics, the radiation energy density of a simple dipole antenna is proportional to the frequency on the fourth power

$$E_f = cst. \times f^4. \qquad (2.2)$$

From the analogy, the discrete energy emitted by the individual nuclei at absolute temperature T must be

$$E_{rad}(Z,A) = cst. \times \frac{f^4(Z,A)}{\exp(hf/kT)-1}. \qquad (2.3)$$

One can suppose that this equation is a natural generalization of the Planck's radiation law for discrete radiation frequencies.

The most important task was to determine the mathematical relation between the radiation frequency and the arbitrary nucleon. Our original goal was to calculate the neutral atomic mass values; therefore, it was supposed that the radiation frequency mainly depends on its mass; *i.e.* on the mass number A. The connection between the mass and its frequency is very simple

$$E = mc^2 = ma^2\omega^2 = 4\pi^2 ma^2 f^2, \qquad (2.4)$$

according to the Special Relativity Theory (SRT). From this relation, it is obvious that the square frequency that can be associated with the atom is very nearly proportional to the mass number of the atom

$$f^2(A) = cst. \times AM_0. \qquad (2.5)$$

3. New Mass-Formula for the Neutral Atoms

The formula finally found for the calculation of atomic masses is the next

$$\begin{aligned} M(Z,A) &= AM_0 + M_{rad}(A) + \\ &+ M_{as}(Z,A) + M_p(Z,A), \end{aligned} \qquad (3.1)$$

where $M(Z,A)$ is the calculated atomic mass, AM_0 is the atomic mass before the nuclear synthesis, $M_{rad}(A)$ is the mass defect caused by the EM and gamma-radiation. The last two terms depend on the atomic number Z (*number of protons*): $M_{as}(Z,A)$ is the "asymmetry" mass (energy) and $M_p(Z,A)$ is the "pairing" mass (energy).

Now we give the value of the M_0 mass, let it be the experimental mass of the neutron. The mass defect of the atoms caused by the EM radiation can be written into the next form

$$\begin{aligned} M_{rad}(A) &= -C_{rad}\frac{f^4(A)}{B^f-1} = \\ &= -C_{rad}\frac{A^2M_0^2}{B^{\sqrt{AM_0}}-1}, \end{aligned} \qquad (3.2)$$

where the Eq. (2.5) is used for the radiation frequency of the atom having mass number A. In this equation the constants C_{rad} and B will be determined by a fitting procedure using experimental mass data of the neutral atoms. During the fitting procedure we have found a better expression for the radiation term (formally similar to gravity)

$$\begin{aligned} M_{rad}(A) &= -C_{rad}(A-1.5)^2 M_0^2 / R(A); \\ R(A) &= B^{\sqrt{(A-1.5)M_0}} - 1; \ (A \geq 2), \end{aligned} \qquad (3.3)$$

where $R(A)$ is proportional to the atomic radius having mass number A.

The asymmetry mass (energy) is related to the *Pauli extension principle* what is given by

$$M_{as}(Z,A) = C_{as}M_0^2\left(\frac{A-2Z}{A+3}\right)^2, \qquad (3.4)$$

where C_{as} is fitting parameter. The last term in the new atomic mass formula is the pairing mass (energy) what is the consequence of the spin-coupling of the nuclei:

$$M_p(Z,A) = -\frac{1}{2}C_pM_0^2\frac{(-1)^Z+(-1)^{A-Z}}{R(A)}. \qquad (3.5)$$

This term connects to observation that the nuclei having even number of protons and even number of neutrons (even-Z, even-N), or, in short even-even nuclei, are most abundant and more stable. The odd-odd nuclei are the least stable, while even-odd and odd-even nuclei are intermediate in stability. Due to the Pauli exclusion principle the nucleus would have a lower energy if the number of protons with spin up were equal to the number of protons with spin down. This is also true for neutrons.

4. Numerical Results

The supposed new atomic mass formula contains only five fitting parameters, which are the next

$$M_0,\ C_{rad},\ C_{as},\ C_p,\ B. \qquad (4.1)$$

The extensive theoretical studies have shown that these parameters are not independent of each other. Surprisingly, the last four parameters depend only on a single parameter Q

$$\begin{aligned} C_{rad} &= Q^5/2,\ C_{as} = Q, \\ C_p &= Q^4/2,\ B = 1+Q. \end{aligned} \qquad (4.2)$$

The new mass formula (3.1) was fitted to nearly 2000 measured neutral atomic masses obtained from the publication of G. Audi and A.H. Wapstra [2]. The results of the fitting procedure are

$$M_0 = 934.470122...\ MeV \qquad (4.3)$$

$$Q = 0.220530193...\text{dimensionless}. \qquad (4.4)$$

The accuracy of the new atomic mass formula was determined by the relative standard deviation (n = number of the involved isotopes)

$$\sigma = \sqrt{\frac{1}{n-1}\sum_i \left(\frac{M_{i,calc} - M_{i,\exp}}{M_{i,\exp}}\right)^2} = \tag{4.5}$$

$$= 1.55\ldots \times 10^{-4}.$$

The obtained radiating nucleon mass M_0 is about *5 MeV* less than the known rest mass of the neutron. Physical explanation of this fact that at a very high fusion temperature, the masses of neutrons decreases by this average value. The missing part of the neutron mass appears in the energy of the thermal radiation field. Taking this into account, we introduce the concept of *total binding energy* of the nuclei

$$E_B(Z,A) = A\left(M_0 - M_N\right) + M_{rad}(A) + \\ + M_{as}(Z,A) + M_p(Z,A) < 0, \tag{4.6}$$

where

$$M_N = 939.565413\ldots\ MeV \tag{4.7}$$

Figure 1. shows the energy components per nucleon calculated by our nuclear radiation model.

Figure 2. shows the calculated rays of nuclei (in proportional sense) by our nuclear radiation model based on Eq.(3.3).

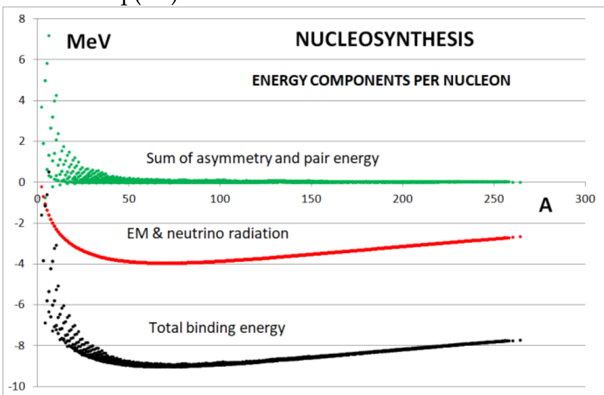

Figure 1.: The energy components per nucleon of the new nuclear radiation model vs. atomic mass

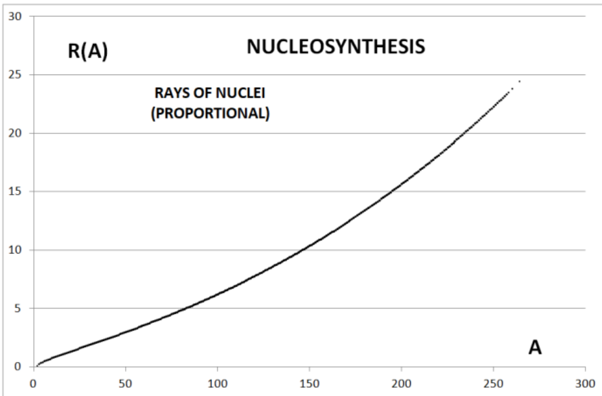

Figure 2.: The calculated nuclei rays (proportional) by the new nuclear model vs. atomic mass

5. Conclusions

Based on our new successful atomic mass formula, we have concluded that at extreme high temperature nuclear synthesis can be physically described exclusively following the generalized Planck's radiation law for discrete frequencies. In Planck's model it is supposed that the *black body oscillators* are independent of each other, and have a Maxwell-Boltzmann energy distribution. Already in earlier nuclear physics, there was some experience showing that all the nuclei inside the atom are weakly bound. This experimental fact was also proved theoretically in the present work.

The accuracy of the here-introduced atomic mass formula is comparable with the accuracy of the nuclear liquid drop model by von Weizsäcker. As is known, the nuclear drop model has five independent fit parameters, in contrast of only two independent fit parameters of the present nuclear model.

References

[1] C.F. von Weizsäcker **Z. Phys.** 96, p. 431, (1935).

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