New idea of the Goldbach conjecture *

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Abstract

A new idea of the Goldbach conjecture has been studied, it is that the even number is more bigger, the average form of the sum of two primes are more larger too. And then, we prove that every sufficiently large even number is the sum of two primes.

key words: The Goldbach conjecture; prime number; prime number theorem; the average form of the sum of two primes

∗ Supported by the National Natural Science Foundation of China.

I. INTRODUCTION

It is well known that Christian Goldbach had first conjectured the famous Goldbach conjecture, it is that every even number greater than 2 can be the sum of two primes. In 1966, Chen Jingrun had proved that every sufficiently large even integer is a sum of a prime and a product of at most two primes [1]. Why is this problem hard to prove? Because the prime number theorem [2, 3, 4, 5, 6] is

$$\Pi(x) \approx x/logx \ , \tag{1}$$

the distance of two adjoining prime numbers is about $logx$, it is infinite if the number x is infinite. And we have not the analytic form of the prime theorem.

Why can they verified the Goldbach conjecture in very large number by computer [7, 8]? This is the reason that we put forward the next new idea.

II. THE NEW IDEA OF THE GOLDBACH CONJECTURE

Because $\Pi(x) \approx x/logx$, there are about $\frac{x}{2logx}(\frac{x}{logx}+1)$ even numbers which are one odd prime number plus one odd prime number in the region $[6, 2x]$, there are only x even numbers in the region $[2, 2x]$, so the average forms of the even number which is in the region $[2, 2x]$ are $\frac{1}{2logx}(\frac{x}{logx}+1) \approx \frac{x}{2(logx)^2}$. That is to say that there is infinite form of the sum of two prime numbers if the even number x is infinite.

We list the partial even number x and the form $n_{form}(x)$ of the sum of two primes in Tab. (I).

So we conjecture that the even number greater than 2 can be the sum of two primes, the form $(n_{form}(x))$ of the sum of two primes about has $n_{form}(6x) \approx 2n_{form}(6x+2) \approx 2n_{form}(6x+4) \approx \frac{9x}{(log(6x))^2}, (x \in N \ \text{ and } \ x \geq 16)$. And the even number is ten times of the original even number x, $n_{form}(10x)$ must be more larger too, about has $\frac{10log(x)log(x)}{(log10+logx)^2}n_{form}(x)$, that is to say that $n_{form}(10x) \approx \frac{10(logx)^2}{(log10+logx)^2}n_{form}(x)$.

III. THE POSSIBLE PROOF OF THE GOLDBACH CONJECTURE

While $n \in N$ and $n \geq 5000$, we assume that every even number in the region $[4, 2n]$ is the sum of two prime numbers. If the even number $2n+2$ is the sum of two prime numbers too, the Goldbach conjecture is proved.

TABLE I: The even number x and the forms $n_{form}(x)$ of the sum of two primes.

$x(n_{form}(x))$	$x(n_{form}(x))$	$x(n_{form}(x))$	$x(n_{form}(x))$	$x(n_{form}(x))$
96(7)	98(3)	100(6)	102(8)	104(5)
106(6)	108(8)	110(6)	112(7)	114(10)
996(37)	998(16)	1000(28)	1002(36)	1004(18)
1006(17)	1008(42)	1010(25)	1012(23)	1014(39)
9996(255)	9998(99)	10000(127)	10002(197)	10004(99)
10006(92)	10008(192)	10010(191)	10012(99)	10014(209)
99996(1303)	99998(605)	100000(810)	100002(1423)	100004(627)
100006(630)	100008(1209)	100010(831)	100012(681)	100014(1235)

Considering the even number $2n$ has the form $n_{form}(2n)$ of the sum of two primes, one can get $n_{form}(2n) \gg 1$. Use $x_{2n,i1}$ and $x_{2n,i2}$ ($x_{2n,i2} \geq x_{2n,i1}$) to represent the ith pair of primes which are $x_{2n,i1} + x_{2n,i2} = 2n$, one can have $n - x_{2n,i1} = x_{2n,i2} - n = d_i \geq 0$ and $x_{2n,i2} - x_{2n,i1} = 2d_i$ ($n - 3 \geq d_i \geq 0$). Choosing the non-zero minimum $d_{min}(n - 3 \geq d_{min} \geq 1)$, (i) because the even number $2n - 2d_{min}$ can be the sum of two prime numbers ($x_{2n,min1} + x_{2n,min1} = 2n - 2d_{min}$), so the even number $2n - 2d_{min} + 2d_{min} = 2n$ can be the sum of two prime numbers too ($x_{2n,min1} + x_{2n,min2} = 2n$); (ii) because the even number $2n$ can be the sum of two prime numbers ($x_{2n,min1} + x_{2n,min2}$), so the even number $2n + 2d_{min}$ can be the sum of two prime numbers too ($x_{2n,min2} + x_{2n,min2} = 2n + 2d_{min}$). Because we do not set fixed value about n and the even number $2n - 2d_{min} + 2j$ ($d_{min} \geq j \geq 1$) is the sum of two primes, so the even number $2n + 2j$ ($d_{min} \geq j \geq 1$) is the sum of two primes too.

That is to say that if every even number in the region $[4, 2n]$ is the sum of two prime numbers, then the even number $2n+2j$ ($d_{min} \geq j \geq 1$) is the sum of two primes too. So all the even number greater than 2 can be the sum of two primes.

IV. THE FORM $n_{form}(x)$ OF THE SUM OF TWO PRIMES

The prime number p_i ($p_i \neq 2$ and $p_i \neq 3$) has $\frac{p_i}{3}$ with a remainder of 1 or 2, $p_{1,i}$ denotes $\frac{p_{1,j}}{3}$ with a remainder of 1 and $p_{2,j}$ denotes $\frac{p_{2,j}}{3}$ with a remainder of 2. The amount of primes in $[0, x]$

is about $\frac{x}{logx}$, the amount of $p_{1,i}$ or $p_{2,j}$ in $[0, x]$ is about $\frac{x}{2logx}$, so the average form $n_{form}(x)$ of the sum of two primes has

$$n_{form}(6x) \approx 2n_{form}(6x+2) \approx 2n_{form}(6x+4) \approx \frac{9x}{(log(6x))^2}, (x \in N \ \ and \ \ x \geq 16) \ . \quad (2)$$

And one can obtain

$$\frac{n_{form}(10x)}{n_{form}(x)} \approx \frac{10(logx)^2}{(log10 + logx)^2} \ . \quad (3)$$

V. CONCLUSION

The even number greater than 2 can be the sum of two primes, the form ($n_{form}(x)$) of the sum of two primes about has $n_{form}(6x) \approx 2n_{form}(6x+2) \approx 2n_{form}(6x+4) \approx \frac{9x}{(log(6x))^2}, (x \in N$ and $x \geq 16)$. And $n_{form}(10x) \approx \frac{10log(x)log(x)}{(log10+logx)^2} n_{form}(x)$. Similarly, every odd number greater than 5 can be the sum of three prime numbers, and the odd number is more larger, the average value of the forms of the sum is more still.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants No. 11465011, the Foundation of Department of science and technology of Guizhou Province of China under Grants No. J[2014]2150, the Foundation of the Guizhou Provincial Education Department of China under Grants No. KY[2016]104, the Visiting Scholars Program of Western China. Thanks are given for Xiaokai He, Xiongjun Fang, Yanjun Liu, Hongwei Yu, Jin Wang and Cong Zhu’s help.

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