Dark and bright-state polaritons in triple-Λ EIT system

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Abstract

Triple-Λ system is investigated using polariton theory. The role of dark and bright-state polaritons in the dynamics of the system is explained in detail. Time evolution of entanglement of single and three-photon EIT modes within the system is studied.

Introduction

Dark-state polariton (DSP), a collective excitation of atoms and light with vanishing population of excited states, is highly useful to trap light pulses in atomic coherence. Since the first report of Fleischhauer and Lukin [1, 2], the existance of DSP in single-Λ electromagnetically induced transparent (EIT) medium, there have been intense research on the application of EIT and other related nonlinear phenomena, in different configuration of atomic level structures, to quantum information processing[3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

Generation and transformation of multipartite entangled states are essential in quantum information processing. Different schemes were proposed to prepare multimode entangled states by four-wave mixing process [5, 13, 14, 15] and six-wave mixing process in Y system [16] and triple-Λ system [17]. Multi-Λ EIT system can be used to prepare and manipulate multimode entangled states [18, 19]. It was shown to be equivalent to single-Λ EIT system in [20], as only one specific linear combination of modes (called EIT mode) undergoes EIT process. Further a protocal was proposed to transfer quantum information between different optical modes and it was experimentally demonstrated [21].

Chong et al. used Sawada-Brout method [22] to study the properties of DSP in single and double-Λ atomic ensemble [23]. Their analysis was restricted to single-polariton sector. It was shown that DSP in double-Λ medium can be efficiently used for single-photon frequency conversion. Later this method was used to study the frequency conversion and entanglement in V configuration [24], double-Λ atomic ensemble trapped in a cavity [25] and in degenerate two level system [26].

In this paper, we have extended the polariton theory developed in [22] to triple-Λ EIT system. The linearized DSP solution of this system is obtained and the polariton dispersion curves within the system are plotted. The role of DSP and bright-state polaritons (BSPs) in the dynamics of the system is explained by including the decay of atomic levels. The time evolution of entanglement of single-photon and three-photon EIT modes within the system is studied. Lower order and higher order entanglement criteria are shown to be satisfied by single-photon and three-photon EIT modes respectively.

Description of triple-Λ system

We consider an ensemble of N atoms with triple-Λ level structure as shown in Figure 1. Two lower level states and the excited states are denoted by $|b\rangle$, $|c\rangle$ and $|a_j\rangle$ ($j = 1, 2, 3$) respectively. The transition $|b\rangle \leftrightarrow |a_j\rangle$ is coupled by quantized field $\hat{a}_{\mathbf{k}_j}$ with coupling strength g_j. The transition $|c\rangle \leftrightarrow |a_j\rangle$ is coupled by classical field with Rabi frequency Ω_j. This triple-Λ level structure can be realized in alkali atoms. For example, in ^{41}K, the two lower levels are the two hyperfine levels $|F = 1\rangle$ and $|F = 2\rangle$ of $4^2S_{1/2}$. The excited states are the hyperfine levels $|F = 1 \text{ or } 2\rangle$ of $n^2P_{1/2}$ ($n = 5, 6, 7$).

The operators are defined in momentum space. The atomic transition operator at position $\mathbf{r}$ is

$$\hat{\sigma}^{pq}_{\mathbf{r}} = |p\rangle_{\mathbf{r}}\langle q|_{\mathbf{r}}. \tag{1}$$

It is written in momentum space as

$$\hat{\sigma}^{pq}_{\mathbf{k}} = \frac{1}{\sqrt{N}} \sum_{\mathbf{r}} e^{i\mathbf{k}.\mathbf{r}} \hat{\sigma}^{pq}_{\mathbf{r}}. \tag{2}$$

The atomic Hamiltonian is ($\hbar = 1$)

$$\hat{H}_0 = \sqrt{N}\left[\omega_b \hat{I}_b + \omega_c \hat{I}_c + \sum_{j=1}^{3} (\omega_{a_j} \hat{I}_{a_j})\right], \tag{3}$$

where $\hat{I}_p = \hat{\sigma}^{pp}_{\mathbf{k}}$.

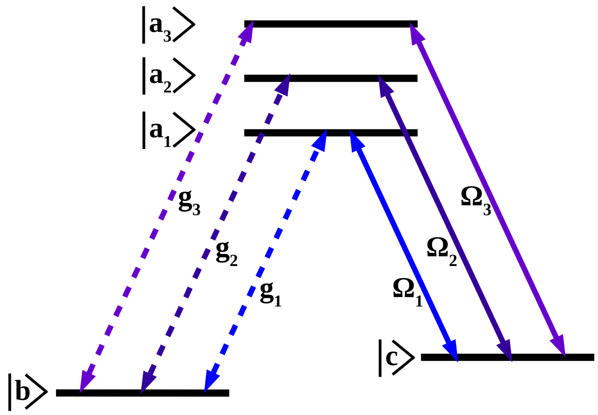


Figure 1: Triple-Λ level structure. The transition $|b\rangle \leftrightarrow |a_j\rangle$ is coupled by quantized field $\hat{a}_{\mathbf{k}_j}$ with coupling constant g_j and $|c\rangle \leftrightarrow |a_j\rangle$ is coupled by classical field with Rabi frequency Ω_j

The photon Hamiltonian is

$$\hat{H}_1 = \sum_{j=1}^{3} \sum_{\mathbf{k}_j} c|\mathbf{k}_j| \hat{a}^{\dagger}_{\mathbf{k}_j} \hat{a}_{\mathbf{k}_j}. \tag{4}$$

The interaction of atom with quantized fields is described by the minimal-coupling Hamiltonian

$$\hat{H}_2 = -\sqrt{N} \sum_{j=1}^{3} \sum_{\mathbf{k}_j} \Big[g_j \hat{a}_{\mathbf{k}_j} \hat{\sigma}^{a_j b}_{\mathbf{k}_j} + h.c \Big]. \tag{5}$$

The interaction of classical fields is described by the Hamiltonian

$$\hat{H}_3(t) = -\sqrt{N} \sum_{j=1}^{3} \Big[\Omega_j e^{i\omega_{L_j} t} \hat{\sigma}^{a_j c}_{\mathbf{k}_{L_j}} + h.c \Big]. \tag{6}$$

The total Hamiltonian of the system is

$$\hat{H}(t) = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3(t). \tag{7}$$

The time dependence of the Hamiltonian can be eliminated by unitary transformation with the following unitary operator.

$$\hat{U}_L(t) = exp\Big[i \sum_{j=1}^{3} \omega_{L_j} t \big(\sqrt{N} \hat{I}_{a_j} + \sum_{\mathbf{k}_j} \hat{a}^{\dagger}_{\mathbf{k}_j} \hat{a}_{\mathbf{k}_j} \big) \Big]. \tag{8}$$

The resultant time independent Hamiltonian is given by

$$\hat{H}_L = \sqrt{N} \Big\{ \omega_b \hat{I}_b + \omega_c \hat{I}_c + \sum_{j=1}^{3} \Big[(\omega_{a_j} - \omega_{L_j}) \hat{I}_{a_j} \Big] \Big\}$$

$$+ \sum_{j=1}^{3} \Big\{ \big(c|\mathbf{k} + \mathbf{k}_{L_j}| - \omega_{L_j} \big) \hat{a}^{\dagger}_{\mathbf{k}+\mathbf{k}_{L_j}} \hat{a}_{\mathbf{k}+\mathbf{k}_{L_j}} \Big\}$$

$$-\sqrt{N} \sum_{j=1}^{3} \Big\{ \Omega_{L_j} \hat{\sigma}^{a_j c}_{\mathbf{k}_{L_j}} + g_j \hat{a}_{\mathbf{k}+\mathbf{k}_{L_j}} \hat{\sigma}^{a_j b}_{\mathbf{k}+\mathbf{k}_{L_j}} + h.c \Big\}, \tag{9}$$

where $\mathbf{k} = \mathbf{k}_j - \mathbf{k}_{L_j}$.

The polariton excitation operator $\hat{A}^{\dagger}_{n\mathbf{k}}$ is defined such that

$$[\hat{H}_L, \hat{A}^{\dagger}_{n\mathbf{k}}] = \omega_n \hat{A}^{\dagger}_{n\mathbf{k}}, \tag{10}$$

where n enumerates different polaritons.

If the system is initially in the ground state $|\mathbf{0}\rangle = |000\rangle|b\rangle$, the polariton excitation operator can be written as linear combination of $\hat{a}^{\dagger}_{\mathbf{k}+\mathbf{k}_{L_j}}$'s, $\hat{\sigma}^{a_j b}_{\mathbf{k}+\mathbf{k}_{L_j}}$'s and $\hat{\sigma}^{cb}_{\mathbf{k}}$ as follows.

$$\hat{A}^{\dagger}_{n\mathbf{k}} = \sum_{j=1}^{3} \Big[-\phi^{j-1}_{n\mathbf{k}} \sigma^{a_j b}_{\mathbf{k}+\mathbf{k}_{L_j}} \Big] + \phi^{3}_{n\mathbf{k}} \sigma^{cb}_{\mathbf{k}} + \sum_{j=1}^{3} \Big[\phi^{j+3}_{n\mathbf{k}} \hat{a}^{\dagger}_{\mathbf{k}+\mathbf{k}_{L_j}} \Big]. \tag{11}$$

From eq.(10) and (11), the effective Hamiltonian matrix of the system can be written as

$$\mathcal{H}_{\mathcal{L}} = \begin{bmatrix} (\omega_{a_1b}-\omega_{L_1}) & 0 & 0 & \Omega_1 & G_1 & 0 & 0 \\ 0 & (\omega_{a_2b}-\omega_{L_2}) & 0 & \Omega_2 & 0 & G_2 & 0 \\ 0 & 0 & (\omega_{a_3b}-\omega_{L_3}) & \Omega_3 & 0 & 0 & G_3 \\ \Omega_1^* & \Omega_2^* & \Omega_3^* & \omega_{cb} & 0 & 0 & 0 \\ G_1^* & 0 & 0 & 0 & (c|\mathbf{k}+\mathbf{k}_{L_1}|-\omega_{L_1}) & 0 & 0 \\ 0 & G_2^* & 0 & 0 & 0 & (c|\mathbf{k}+\mathbf{k}_{L_2}|-\omega_{L_2}) & 0 \\ 0 & 0 & G_3^* & 0 & 0 & 0 & (c|\mathbf{k}+\mathbf{k}_{L_3}|-\omega_{L_3}) \end{bmatrix} \tag{12}$$

where $G_j = \sqrt{N} g_j$.

The dispersion curves of polaritons are plotted in Figure 2 with the assumption that the classical fields are on resonance ($\omega_{L_j} = \omega_{a_jb}$, $\forall j$). There are seven dispersion curves corresponding to each polariton species. The closely spaced curves are shown clearly in the inset of Figure 2. The values taken for calculation are $\omega_{cb} = 100$, $\Omega_1 = 10$, $\Omega_2 = 15$, $\Omega_3 = 20$, $g_1 = 0.1$, $g_2 = 0.15$, $g_3 = 0.2$ and $N = 100$. The ratio (Ω_j/G_j) is taken to be same for all pairs of $|b\rangle \leftrightarrow |a_j\rangle$ and $|c\rangle \leftrightarrow |a_j\rangle$ transitions. All frequencies are taken in the unit of MHz. This same set of values is considered for all calculations in this paper.

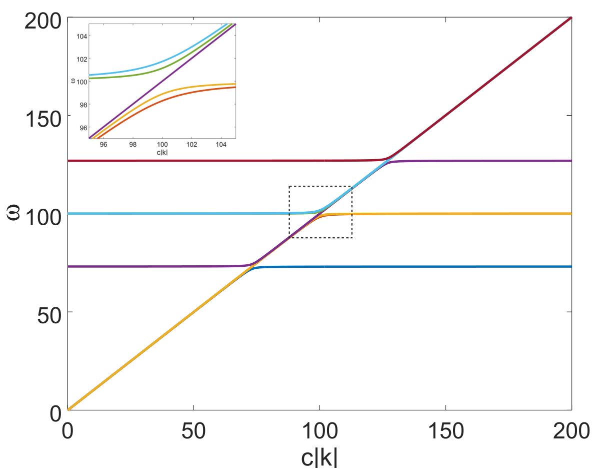


Figure 2: Dispersion curves of polaritons. The central portion covered by dotted lines is shown clearly in the inset. There are seven dispersion curves in total.

For $\omega_{L_j} = \omega_{a_jb}$ $(\forall j)$ and for small detuning of quantized fields, the linearized DSP solution is given by

$$\omega_{\mathbf{k}} = \omega_{cb} + \frac{\sum_{j=1}^{3} \left|\frac{\Omega_j}{G_j}\right|^2 c\delta\mathbf{k}_j}{1+\sum_{j=1}^{3}\left|\frac{\Omega_j}{G_j}\right|^2}, \tag{13}$$

$$\phi_{n\mathbf{k}}^{j-1} = \frac{\Omega_j}{|G_j|^2} \frac{c\delta\mathbf{k}_j + \sum_{i=1, i\neq j}^{3} \left|\frac{\Omega_j}{G_j}\right|^2 c(\delta\mathbf{k}_j - \delta\mathbf{k}_i)}{1+\sum_{j=1}^{3}\left|\frac{\Omega_j}{G_j}\right|^2} \phi_{n\mathbf{k}}^3, \tag{14}$$

$$\phi_{n\mathbf{k}}^{j+3} = -\frac{\Omega_j}{G_j}\phi_{n\mathbf{k}}^3, \quad j = 1,2,3 \tag{15}$$

where $\delta\mathbf{k}_j = \left(|\mathbf{k}_j| - \frac{\omega_{a_jb}}{c}\right)$.

When all the quantized fields are on resonance there exists an eigenstate proportional to $\left(0,\quad 0,\quad 0,\quad 1,\quad -\dfrac{\Omega_1}{G_1},\quad -\dfrac{\Omega_2}{G_2},\quad -\dfrac{\Omega_3}{G_3}\right)^T$. This state corresponds to resonant DSP. The resonant dark-state polariton excitation operator is given by

$$\hat{A}^{\dagger}_{0\mathbf{k}} = \sin\theta\hat{\sigma}^{cb}_{\mathbf{k}} - \frac{\cos\theta}{R}\left[\sum_{j=1}^{3}\frac{\Omega_j}{G_j}\hat{a}^{\dagger}_{\mathbf{k}+\mathbf{k}_{L_j}}\right], \qquad (16)$$

where $\sin\theta = \dfrac{1}{\sqrt{1+R^2}}$ and $\cos\theta = \dfrac{R}{\sqrt{1+R^2}}$ with $R = \sqrt{\sum_{j=1}^{3}\left|\dfrac{\Omega_j}{G_j}\right|^2}$.

The field term in eq.(16) is the linear combination of quantized modes that exhibits EIT in triple-Λ system as shown in [20]. Upon acting on the ground state, it results in dark states

$$|D^m_{\mathbf{k}}\rangle = \left[\hat{A}^{\dagger}_{0\mathbf{k}}\right]^m|\mathbf{0}\rangle \;\; ; \;\; (m = 1, 2, 3, ...). \qquad (17)$$

Role of DSP and BSPs

Decay of atomic levels is not considered so far. The two lower levels of triple-Λ system could be chosen as the two hyperfine ground states of alkali atoms. Hence we ignore the decay of lower levels. When the decay of excited states are included, the angular frequencies $\omega_{a_j b}$'s in the matrix $\mathcal{H_L}$ have to replaced by $(\omega_{a_j b} - i\Gamma_j)$'s, where Γ_j is the decay constant of the level $|a_j\rangle$. Finding analytic expression for the eigenvalues of this matrix is difficult. However, in the case where all the quantized and classical fields are on resonant, the eigenvalues can be written in the following form

$$\omega_0 = \omega_{cb} \qquad (18)$$

$$\omega_l = \omega_{cb} + \alpha_l f_l(\Omega_u, G_u, \Gamma_u) - i f'_l(\Gamma_v),$$
$$(l = 1, 2, \ldots, 6) \;\; \& \;\; (u, v = 1, 2, 3), \qquad (19)$$

where α_l is a real coefficient, $f_l(\Omega_u, G_u, \Gamma_u)$ is a function of Ω_u's, G_u's and Γ_u's and $f'_l(\Gamma_v)$ is a function of Γ_v's alone.

The eigenvalues ω_0 and ω_l's correspond to DSP and BSPs respectively. The time evolution of polariton operators can be written as

$$\hat{A}^{\dagger}_0(t) = e^{-i\omega_{cb}t}\hat{A}^{\dagger}_0(0)$$
$$\hat{A}^{\dagger}_l(t) = e^{-i\omega_{cb}t}e^{-i\alpha_l f_l(\Omega_u, G_u, \Gamma_u)t}e^{-f'_l(\Gamma_v)t}\hat{A}^{\dagger}_l(0) \qquad (20)$$

Henceforth we ignore the label of wave vectors. The j^{th} quantized mode is denoted by $\hat{a}_j$.

In order to understand the role of DSP and BSPs in the dynamics of the system, we consider a simple case in which there is a single photon in the mode $\hat{a}_1$. The initial state of the system is given by

$$|\psi(0)\rangle = \hat{a}^{\dagger}_1|\mathbf{0}\rangle = \sum_{n=0}^{6}\phi^{4*}_n A^{\dagger}_n|\mathbf{0}\rangle. \qquad (21)$$

The state of this system at any time t can be written as

$$|\psi(t)\rangle = |BS\rangle + |DS\rangle, \qquad (22)$$

where $|DS\rangle$ is dark state spanned by the basis vectors $\{|\mathbf{0}\rangle = |000\rangle|b\rangle,\ |\mathbf{4}\rangle = |000\rangle|c\rangle,\ |\mathbf{6}\rangle = |010\rangle|b\rangle,\ |\mathbf{7}\rangle = |001\rangle|b\rangle\ \&\ |\mathbf{5}\rangle = |100\rangle|b\rangle\}$, $|BS\rangle$ is bright state and it is spanned by $\{|\mathbf{1}\rangle = |000\rangle|a_1\rangle,\ |\mathbf{2}\rangle = |000\rangle|a_2\rangle\ \&\ |\mathbf{3}\rangle = |000\rangle|a_3\rangle\}$.

From the time evolution of polariton operators

$$|\psi(t)\rangle = e^{-i\omega_{cb}t}\left[\phi^{4*}_0\hat{A}^{\dagger}_0(0) + \sum_{l=1}^{6}e^{-i\alpha_l f_l(\Omega_u, G_u, \Gamma_u)t}e^{-f'_l(\Gamma_v)t}\phi^{4*}_l\hat{A}^{\dagger}_l(0)\right]|\mathbf{0}\rangle$$
$$= C_1|\mathbf{1}\rangle + C_2|\mathbf{2}\rangle + C_3|\mathbf{3}\rangle + C_4|\mathbf{4}\rangle + C_5|\mathbf{5}\rangle + C_6|\mathbf{6}\rangle + C_7|\mathbf{7}\rangle \qquad (23)$$

where,

$$C_r = \sum_{l=1}^{6}e^{-i\alpha_l f_l(\Omega_u, G_u, \Gamma_u)t}e^{-f'_l(\Gamma_v)t}\phi^{4*}_l\phi^{r-1}_l \quad ; \quad (r = 1, 2, 3) \qquad (24)$$

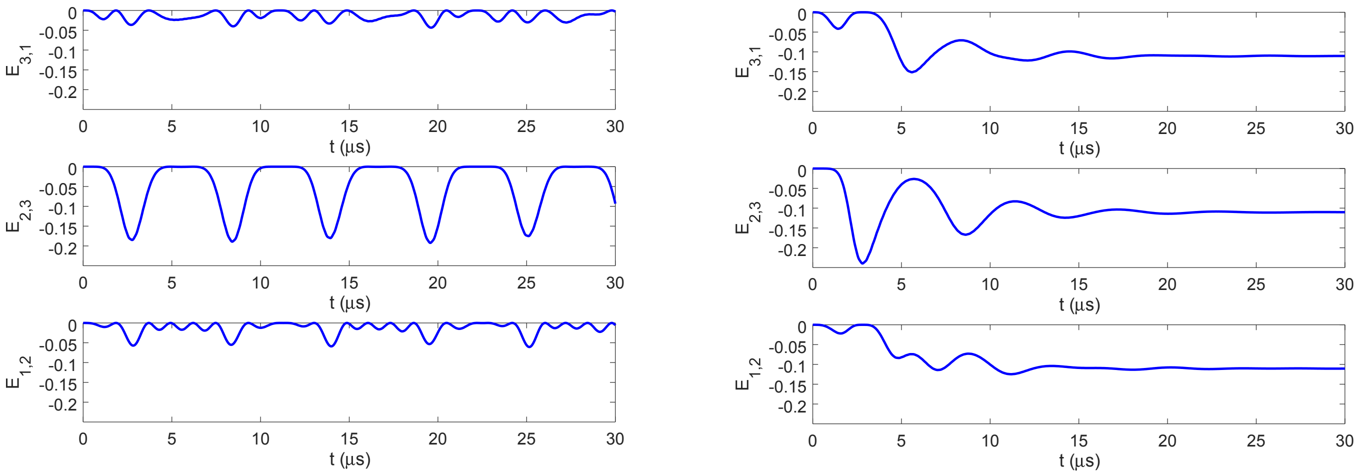
$$C_s = \phi^{4*}_0\phi^{s-1}_0 + \sum_{l=1}^{6}e^{-i\alpha_l f_l(\Omega_u, G_u, \Gamma_u)t}e^{-f'_l(\Gamma_v)t}\phi^{4*}_l\phi^{s-1}_l \quad ; \quad (s = 4, 5, 6, 7) \qquad (25)$$

In eq.(23), the sum of first three terms constitute the bright state and the sum of remaining terms is the dark state. BSPs govern the time evolution of both the bright and dark states whereas the contribution of DSP is only to dark state and it is stationary. If we consider the expectation value of a dynamical variable in this state, the temporal oscillation of its value, in the transient regime, is governed by BSPs. However this oscillation is damped out by the incoherent decay of excited states and the system is driven to steady state. The steady state value is completely determined by DSP.

To show this we investigate the evolution of entanglement of the modes in the state $|\psi(t)\rangle$. In order to detect the entanglement we use the following inequality [27]

$$E_{i,j} = \langle N_{a_i} N_{a_j} \rangle - |\langle \hat{a}_i \hat{a}_j^\dagger \rangle|^2 < 0 \tag{26}$$

for each pair of modes. If this inequality is satisfied then the modes are necessarily entangled. This is lower order criteria, as each term is only up to the fourth order in the creation and annihilation operators [28]. Numerical evaluation of this inequality for each pair of modes without and with decay of excited states are plotted in Figure 3(a) and (b) resectively. Without decay $E_{i,j}$ takes negative values periodically for all three pairs. When decay of excited states are considered, the periodic oscillation is damped. The single photon EIT mode in triple-Λ system is W state. Hence $E_{i,j}$ takes a constant value of $-(1/9)$ for all pairs of modes in the steady state.



(a)

(b)

Figure 3: Lower order entanglement criteria for each pair of modes (a) without decay and (b) with decay

Three-Photon EIT modes

Now we consider three photons, one photon in each mode of the triple-Λ system, in the presence of three classical fields. The state of the system at time t is written as

$$|\psi'(t)\rangle = \hat{a}_1^\dagger(-t)\hat{a}_2^\dagger(-t)\hat{a}_3^\dagger(-t)|\mathbf{0}\rangle \tag{27}$$

The state of the three-photon EIT mode can be obtained from eq.(27) by projecting the atomic subspace to the ground state. By using time evolution of polaritons, it can be written as

$$\begin{aligned} |EIT\rangle = {} & C_{300}|300\rangle + C_{210}|210\rangle + C_{201}|201\rangle + C_{120}|120\rangle \\ & + C_{102}|102\rangle + C_{111}|111\rangle + C_{021}|021\rangle + C_{012}|012\rangle \\ & + C_{030}|030\rangle + C_{003}|003\rangle \end{aligned} \tag{28}$$

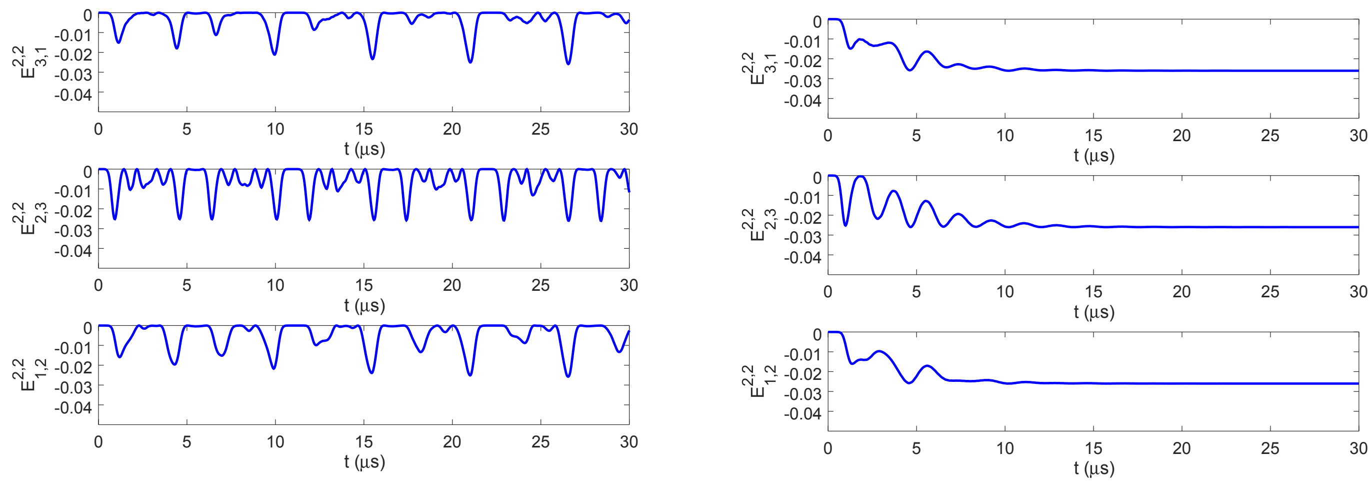
The expressions of C_{ijk} are given in Appendix. The following higher order entanglement criteria is used to detect the entanglement of three-photon EIT mode.

$$E_{i,j}^{2,2} = \langle \hat{a}_i^{\dagger 2} \hat{a}_i^2 \hat{a}_j^{\dagger 2} \hat{a}_j^2 \rangle - |\langle \hat{a}_i^2 \hat{a}_j^{\dagger 2} \rangle|^2 \ < \ 0 \tag{29}$$

This expression is plotted for all three pairs of modes without and with decay in Figure 4(a) and (b) respectively. $E_{i,j}^{2,2}$ takes negative values for each pairs of modes. It oscillates temporally when decay is not included. With decay it is driven to take a constant negative value. Hence this is genuinely entangled state. However, if the lower levels were not chosen to be hyperfine ground states of alkali atoms, then the decay of lower levels would affect the entanglement of EIT mode.

Conclusion

In summary, we have extended the Sawada-Brout-Chong method to triple-Λ system. The role of dark and bright state polaritons in the dynamics of the system is studied in detail by including the decay of excited states. Single-photon and three-photon EIT modes are shown to be genuinely entangled. The influence of decay of excited states is only to drive the system towards the steady state. It does not affect the entanglement of EIT modes. It is easy to show that multi-Λ system can be used to transfer entangled modes between different frequency channels without any loss of entanglement. Hence this system could be employed in processing quantum information protocals.



(a) (b)

Figure 4: Higher order entanglement criteria for each pair of modes (a) without decay and (b) with decay

Appendix

The expression of coefficients C_{ijk} are

$$
\begin{aligned}
C_{300} &= [M]_{55}[M]_{65}[M]_{75},\\
C_{210} &= [M]_{56}[M]_{65}[M]_{75} + [M]_{55}[M]_{66}[M]_{75}\\
&\quad +[M]_{55}[M]_{65}[M]_{76},\\
C_{201} &= [M]_{57}[M]_{65}[M]_{75} + [M]_{55}[M]_{65}[M]_{77}\\
&\quad +[M]_{55}[M]_{67}[M]_{75},\\
C_{120} &= [M]_{55}[M]_{66}[M]_{76} + [M]_{56}[M]_{66}[M]_{75}\\
&\quad +[M]_{56}[M]_{65}[M]_{76},\\
C_{102} &= [M]_{55}[M]_{67}[M]_{77} + [M]_{57}[M]_{65}[M]_{77}\\
&\quad +[M]_{57}[M]_{67}[M]_{75},\\
C_{111} &= [M]_{57}[M]_{66}[M]_{75} + [M]_{57}[M]_{65}[M]_{76}\\
&\quad +[M]_{56}[M]_{65}[M]_{77} + [M]_{56}[M]_{67}[M]_{75}\\
&\quad +[M]_{55}[M]_{66}[M]_{77} + [M]_{55}[M]_{67}[M]_{76},\\
C_{021} &= [M]_{57}[M]_{66}[M]_{76} + [M]_{56}[M]_{67}[M]_{76}\\
&\quad +[M]_{56}[M]_{66}[M]_{77},\\
C_{012} &= [M]_{56}[M]_{67}[M]_{77} + [M]_{57}[M]_{67}[M]_{76}\\
&\quad +[M]_{57}[M]_{66}[M]_{77},\\
C_{030} &= [M]_{56}[M]_{66}[M]_{76}, C_{003} = [M]_{57}[M]_{67}[M]_{77}.
\end{aligned}
$$

with $M = e^{-i\mathcal{H}_{\mathcal{L}}t}$. When the decay of excited states are considered ω_{a_jb} are replaced by $\omega_{a_jb} - \Gamma_j$ in $\mathcal{H}_{\mathcal{L}}$.

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