Article

Duality of Time and Perpetual Re-creation of Space

Mohamed Haj Yousef [1,†,‡]

[1] UAE University; mhajyousef@uaeu.ac.ae
† Physics Department, College of Science, United Arab Emirates University, Al-Ain, UAE

Abstract: Based on the Duality of Time hypothesis, a dynamic, granular and self-contained space-time is introduced and investigated. A new time-time or complex-time Euclidean space is defined and it is shown that the non-Euclidean Minkowski space is the first global approximation of this complex-time space in which the space-time interval becomes invariant between different inertial and non-inertial frames alike. Therefore, in addition to Lorentz factor, the equivalence principle is derived directly from the resulting discrete symmetry. To support this hypothesis, it will be applied to derive the mass-energy equivalence relation directly from fundamental classical principles. It will be also shown that the resulting dynamic quintessence could diminish the cosmological constant discrepancy by 117 orders of magnitude.

Keywords: speed of light invariance; arrow of time; complex time; quantum gravity; equivalence principle; cosmological constant problem

1. Introduction:

Relativity and its classical predecessor consider space and time to be continuous, whereas quantum mechanics is constructed on discrete quanta of energy and fields, albeit they still evolve in continuous background. Although both theories have passed many rigorous tests, they inevitably produce enormous contradictions when applied together in the same domain. This was manifested most clearly in the cosmological constant problem, which was described as "the worst theoretical prediction in the history of physics" [1]. Most scholars believe that this conflict may only be resolved with a successful theory of quantum gravity [2], which seems to be beyond the current scope of Relativity that is primarily based on dynamic space-time continuum.

Many innovative concepts have been investigated in this regard, but none has ever been able to settle the discrepancy [3]. The essence of the problem is that most of these theories are trying to implement some kind of quantization in continuous space-time background, which makes any prospective theory of gravity nonrenormalizable [4], thus impossible to test because it cannot make any meaningful predictions [5]. The two serious approaches to this critical problem are strings theory and loop quantum gravity. The first tries to develop an effective quantum field theory of gravity at low energies by postulating strings instead of point particles, while the latter uses spin networks to obtain granular space that evolves with time. This makes LQG closer to General Relativity that considers space-time to be dynamic, but here the smooth background is replaced by nodes [6], while strings theory still depends on the background continuum.

Nonetheless, the author believes that any successful Theory of Everything must not rely on either the continuum or discretuum structure of space-time. Rather, these two contrasting views must be the product of such theory, and they must become complementary on the microscopic and macroscopic scales. The only contestant that may fulfill this criterion is "Oneness", because on the multiplicity level things are either discrete or continuous, there is no other way. However, we need first to explain how the physical multiplicity can proceed from this metaphysical oneness, and then exhibit various discrete and continuous impressions. The key to resolve this dilemma is in understanding the "inner levels of time" in which "space" and "matter" are perpetually being generated and layered into the three spatial dimensions, which then evolve throughout the "outer level of time" that we encounter.

2. The Duality of Time postulate:

The Duality of Time, and the resulting dynamic creation of matter and space, is based on a previous research that presented an eccentric conception of time [7], [8], [9], [which include other references on the history and philosophical origins of this concept]. This hypothesis can be extracted here into the following postulate:

- At every instance of the outward normal level of time, space and matter are perpetually being re-created in one linear chronological sequence, which forms the inward level(s) of time that are also nested inside each lower dimension of space.

This means that at every instance of the **real flow of time** there is only **one metaphysical point**, that is the **unit of space-time geometry**, and the physical universe is a result of its perpetual recurrence in the inner level(s) of time, which is **perpetually re-creating** the three dimensions of space and what it may contain of matter particles, that then kinetically evolve throughout the outer normal level of time that we encounter. With regard to observers who are habitually living in this normal level of time, the process of re-creation is instantaneous, since it is performed by a massless metaphysical point, that is always moving at the speed of light, which is the only real speed in nature. Mass, energy and other physical properties, including velocity, acceleration and even the dimensions of space, are only observable on the outward level of time, as a result of the temporal coupling between at least two metaphysical points or instances.

It will be shown in section 4 how this single postulate leads at the same time to all the three principles of Special and General Relativity together, as well as Quantum Field Theory, including the first and second quantizations, of energy and fields, and hence this "third quantization" of space-time itself, which will eliminate all kinds of infinities that are normally encountered in the current background continuum models, due to singularities in the ill-defined Riemannian geometry [10].

Henceforth, the Duality of Time will be based on simple Euclidean geometry, but on complex-time space, because the perpetual re-creation in the inner level of time implies that normal time is actually imaginary or latent with relation to the real flow of time that is creating space and matter. Therefore, this **genuinely complex-time** will define a new discrete symmetry that allows expressing the (deceitfully continuous) non-Euclidean space-time in terms of its granular Euclidean complex-time space, whose granularity is expressed through the intrinsic properties of hyperbolic numbers ($\mathbb{H}$), i.e. without invoking Riemannian geometry. This will be discussed further in section 3.1, where it will be shown that Riemannian geometry on $\mathbb{R}^4$ is a special approximation of this complex-time Euclidean geometry on $\mathbb{H}^4$. This approximation, that lead to the non-Euclidean Minkowski space-time continuum of General Relativity, is necessary only when we consider space and matter to be coexisting together in (and with) time, i.e. all the dimensions are continuously existing together, thus causing the deceptive continuity of physical existence, while in fact they are being sequentially re-created in the inner levels of time before they evolve on the normal level. This means that each dimension of space is formed from the linear flow of time which has to be interrupted in order to make a new dimension, and this is achieved by multiplying with the imaginary unit which produces an abrupt rotation by $\pi/2$, creating a new dimension that is perpendicular on the previous level. It is this subtle property, that is a consequence of the duality nature of time, that makes space-time geometry genuinely discrete and complex, otherwise if we consider all the dimensions to be coexisting together it will be continuous and real. This will therefore have deep implications on the various fields of mathematics, such as geometry and number theory, because complex numbers are now genuinely natural while the reals are one of their special approximations.

Henceforth, we can clearly see how the perpetual re-creation can explain how the very dimensions of space are formed, as it will be discussed further in section 3 below. Furthermore, the linear progression of the real flow of time, that is sequentially creating the dimensions of space in

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the inner level, provides a straightforward explanation of the arrow of time, which will be explicitly expressed by equation 1 in section 3.1.

Additionally, the genuinely complex-time structure will be applied in section 4.2 to derive the mass-energy equivalence relation $E = mc^2$ directly from the principles of classical Newtonian Mechanics. This should provide a conclusive evidence to the hypothesis because it will be shown that an exact derivation of this experimentally verified relation is not possible without the inner levels of time, since it incorporates motion at the speed of light which leads to infinities on the physical level. All current derivations of this critical relation suffer from unjustified assumptions or approximations [11], [12], [13], [14] and [15], as was repeatedly acknowledged by Einstein himself [14], [16].

As an additional support to the Duality of Time postulate, we will also show in section 6 that the resulting dynamic quintessence will diminish the cosmological constant discrepancy by 117 orders of magnitude. This huge difference results from realizing that the modes of quantum oscillations occur in linear chronological sequence, and not all at the same time as it is currently treated in QFTs.

The perpetual re-creation is essentially equivalent to but more realistic than (eternal [17]) inflation [18], because it does not lead to any infinite hypothetical universes [19], [20], [21], yet it can easily solve many major problems in physics and cosmology. It would be too distracting to discuss all that in this introductory article, but the homogeneity problem, for example, will instantly cease under this postulate, since the universe, no matter how large it could be, is re-created sequentially in the inner level of time, so all the states are synchronized before they appear as one instance in the normal level. This would not have been conceivable without taking rigid space as flowing time in the inner level, and thus the outer time becomes latent to it. Without these two perpendicular levels of time, even motion at the speed of light would not be able to reach remote regions in the universe, which imposes the locality that prevents remote communication or synchronization. Therefore, without breaking the speed of light limit, this sequential re-creation can explain non-locality and various other critical quantum mechanical issues, some of which are outlined in other publications [9] [22].

Actually, the Duality of Time postulate is capable of taking the fundamental philosophical observations to a new profound level, beyond the physical boundaries, because the most basic principle of causality becomes a consequence of sequential metaphysical creation, and hence the conservation laws of momentum and energy. This will require a prolonged discussion in another article.

The Duality of Time does not replace or contradict the main exiting theories, but it exposes a deeper understanding of time that leads to metaphysical (non-background) Euclidean complex-time space with $(+,+,+,+)$ signature, which approximates to the non-Euclidean Minkowski space $(+,+,+,-)$ when we suppose continuous background space with physical dimensions. It is this approximation that lead to General Relativity, but Quantum Gravity can not be achieved without taking into account the new discrete symmetry of this granular complex-time geometry, which is also capable of explaining the other fundamental interactions in lower dimensions, as it will be outlined in section 4 below.

3. The dynamic, granular and self-contained space-time structure:

According to the above Duality of Time postulate, the dynamic universe is the succession of instantaneous discrete frames of space, that extend in the outward level of time that we normally encounter, but each frame is internally created in chronological sequence within the inward level that is the real flow of time. This is schematically demonstrated in Figure 1, where space is conventionally shown in two dimensions, as the (x,y) plane, and we will mostly consider the x axis only for simplicity.

In reality, however, we can conceive of at least **seven levels of time**, which *curl* into the **four dimensions of space-time**: $3D+1$, that are the three spatial and one temporal dimensions; since each spatial dimension is formed by two of the six inner levels, while the seventh is the outer normal time that we encounter (see also Figure 2 and section 4.1 below).

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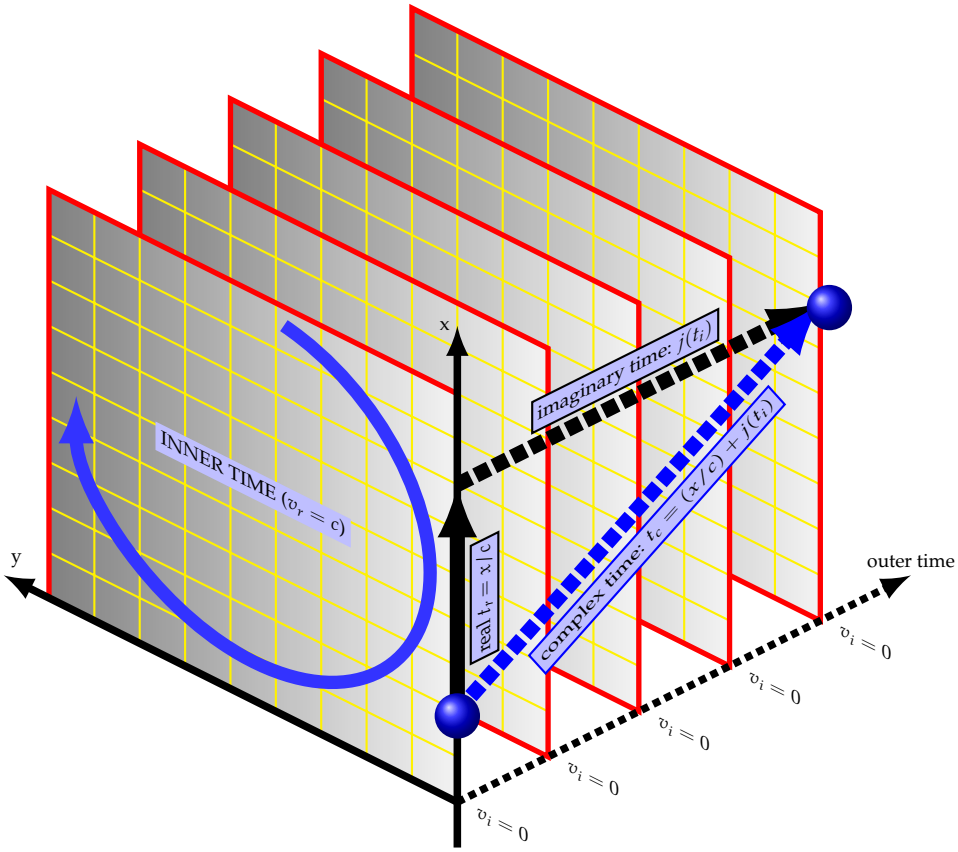

Figure 1. Representing the inner and outer levels of time together as genuinely complex time. In the real flow of time $t_r = \sqrt{(x/c)^2 + (y/c)^2 + (z/c)^2}$, or $t_r = x/c$ for motion on x-axis only, one frame of space is created in chronological sequence that then appears as one instance of the imaginary time t_i, so the total complex time is: $t_c = (x/c) + j(t_i)$, as split-complex or hyperbolic number.

Mathematically, this complex time can be represented by imaginary or complex numbers where space is treated as a plane or spherical wave, and time is the imaginary axis perpendicular to it.

The concept of imaginary time is already being used widely in various mathematical formulations in quantum physics and cosmology, without any actual justification apart from the fact that it is a quite convenient mathematical trick that is useful in solving many problems. As Hawking states: "It turns out that a mathematical model involving imaginary time predicts not only effects we have already observed but also effects we have not been able to measure yet nevertheless believe in for other reasons." [23].

Hawking, however, considers the imaginary time as something that is perpendicular to normal time that exists together with space, and that is how it is treated in physics and cosmology. According to the postulate of the Duality of Time, however, since space is now the real time, the normal time itself becomes genuinely imaginary.

Employing imaginary time is very useful because it provides a method for connecting quantum mechanics with statistical mechanics by using a Wick rotation, by $\pi/2$. In this manner we can find a solution to dynamics problems in n dimensions, by transposing their descriptions in $n+1$ dimensions, i.e. by trading one dimension of space for one dimension of time, which means substituting a mathematical problem in Minkowski space-time into a related problem in Euclidean space. Schroedinger equation and the heat equation are also related by Wick rotation. This method is also used in Feynman's path integral, which was extended in 1966 by DeWitt into gauge invariant functional-integral [24]. For this reason, there has been many attempts to describe quantum gravity in terms of Euclidean geometry [25], [26], because in this way it is possible to avoid singularities

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which are unavoidable in General Relativity, since it is primarily constructed on curved space-time continuum that uses Riemannian manifolds in which the geometry becomes ill-defined at the points of singularities.

3.1. The Euclidean complex-time space:

As we indicated above, the Duality of Time implies that normal time is actually imaginary or latent with relation to the real flow of time that is creating space and matter. Therefore, the non-Euclidean Minkowski space-time coordinates (x, y, z, t) will be a special approximation of the Euclidean space coordinates (x, y, z, jct_i), or time coordinates: $(x/c, y/c, z/c, jt_i)$ where $(x/c, y/c, z/c)$ or (t_x, t_y, t_z) represent the inner **real part of time**: $t_r = \sqrt{t_x^2 + t_y^2 + t_z^2} = \sqrt{(x/c)^2 + (y/c)^2 + (z/c)^2}$, and (jt_i) represents the outer **imaginary time**, so the total **complex time** is $t_c = t_r + jt_i$, where j is the split-imaginary unit which defines hyperbolic numbers, thus: $t_c \in \mathbb{H} \equiv \mathbb{R}(j)$, and its modulus is $|t_c| = \sqrt{t_r^2 - t_i^2}$. These hyperbolic numbers have been also called: tessarines, motors, bireal, perplex, semi-complex, or split-complex, but in this article, unless otherwise stated, *imaginary* and *complex* refer to these hyperbolic numbers $\in \mathbb{H}$.

Hyperbolic geometry was used in the early formulation of Special Relativity to represent Lorentz boost as a rotation in the imaginary plane, but it will be shown in section 4 below that under the new Euclidean complex-time frame, Lorentz transformations will be valid between inertial and non-inertial frames alike, because the dynamic relation between the real and imaginary parts of time implies that the instantaneous velocity in the imaginary time is always **zero** (see Figure 1), whether the object is accelerating or not. Therefore, in addition to Lorentz boost, this essential characteristic of the dynamic and granular complex-time structure, that defines a new discrete symmetry group, allows direct derivation of the equivalence principle that lead to General Relativity. This will be discussed further in sections 4, 4.1 and 4.4 below.

The representation of space-time, involving imaginary time, as Euclidean space having $(+,+,+,+)$ signature: (x, y, z, jct_i), was used in the early formulation of Special Relativity by Poincare [27] and even Minkowski [28], but because there were no substantial reasons to treat time as imaginary, Minkowski had to introduce the four-dimensional space-time: (x, y, z, t), with Lorentzian metric with signature $(+,+,+,-)$, in which time and space are treated equally. This four dimensional space later became necessary for General Relativity, due to the presence of gravity which required Riemannian geometry.

The non-Euclidean Minkowski space is therefore the first global approximation of the metaphysical reality, just as the Euclidean Minkowski space is a local approximation when the effect of gravity is neglected, while the Galilean space is the classical approximation for non-relativistic velocities. These three relative approximations are still serving very well in describing the respective physical phenomena, but they can not describe the actually metaphysical reality of the universe. As Hawking noticed: "In fact one could take the attitude that quantum theory and indeed the whole of physics is really defined in the Euclidean region and that it is simply a consequence of our perception that we interpret it in the Lorentzian regime." [25]. The Duality of Time explains exactly that the source of this deceptive perception is the fact that we do not witness the metaphysical perpetual re-creation process, but, being part of it, we always see the universe after it is created, so we imagine that our existence is continuous.

Without postulating the Duality of Time and the resulting perpetual re-creation of space, this concept of imaginary time does not have any genuine reality or justification outside the mathematical formulation, because both the Galilean space and Minkowski space-time consider space and time to be coexisting together, i.e. they both are real. The fact that each frame of the inner time (which constitutes space) appears as one instance on the outward time is what justifies treating time as imaginary with relation to space, thus perpendicular to it. In this dynamic creation of space in the complex time, the outward time is discrete and imaginary, while space becomes continuous with

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relation to the outer time, but this is only relative to the dimension in which the observer is situated, so for example: the $2D$ plane is itself continuous with relation to its inner dimensions but it forms one discrete instance with relation to the flow of time in the encompassing $3D$, which then appears internally continuous but discrete with regard to the encompassing outward time T. For this reason perhaps, although representing Minkowski space in terms of Clifford geometric algebra $G4 = G(M^4)$ employing bivectors [29], or even the spinors of complex vector space [30], allowed expressing the equations in simple forms, but it could not discover the intrinsic granularity of space-time without any background.

Discreteness implies interruption or discontinuity, and this is what the outer time is doing to the continuous flow of the inner time that is creating space and matter. This could not have been possible without introducing the inner levels of time in which the geometrical points of space and matter are being re-created perpetually in chronological sequence. Mathematically, this is achieved by multiplying the real flow of time with the imaginary unit, which produces an **abrupt rotation** by $\pi/2$, creating a new dimension that is perpendicular on the previous level. Multiplying with the imaginary unit again causes time to become real again, i.e. like space. Each space-time point, therefore, is the product of seven instances of time, the first six of which make the three spatial dimensions and the seventh is the outer time (see also Figure 2). It is not possible otherwise to imagine self-contained granular space-time without any background in which the topology could be defined, although LQG is one way to get around this obstacle [6].

3.2. The time-time frame:

This article will therefore adopt the complex Euclidean frame as the absolute reference: $(x/c, y/c, z/c, jt_i)$ or (t_x, t_y, t_z, jt_i), or simply: (t_r, jt_i). In this abstract complex frame, space and time are absolute, as they were originally treated in the classical Newtonian mechanics, but now empty space is **void**, because it does not have any physical reality, to differentiate it from vacuum that is the dynamic aether or quintessence. So, for void both the real and imaginary parts are null: $(0, 0)$, while for vacuum only the imaginary part is null: $(t_r, 0)$ which indicates infinite and inert space that is the ground state of matter particles that are then described by (t_r, jt_i), or: (c, jv), or also: (mc, jmv), which means that they are internally being re-created at the speed of light, that is the real part, and moving outwardly at the imaginary apparent velocity v that is given by equation 3. Vacuum is therefore when this apparent velocity, or momentum, becomes absolutely zero, both as the object's total velocity and any vector velocities of its constituents, and this corresponds to zero absolute temperature.

We can see from Figure 1 that space-time intervals can be obtained from: $s = \sqrt{x^2 + y^2 + z^2 - (ct_i)^2}$, or $s = \sqrt{r^2 - (ct)^2}$, because the modulus of hyperbolic or split-complex numbers $z = c + jy$ is calculated according to the formula: $|z| = \sqrt{x^2 - y^2}$.

Alternatively, instead of the classical space-time interval, we can use the new time-time interval which is the modulus of the complex time: $|t_c| = \sqrt{t_r^2 - t_i^2}$, where $t_r = \sqrt{(x/c)^2 + (y/c)^2 + (z/c)^2}$, or $t_r = x/c$ for motion on x-axis only.

Thus it can be also readily seen from Figure 1 that the modulus of this complex time equals negative the proper time as defined in Special Relativity:

$$\begin{aligned} |t_c| &= |(x/c) + j(t_i)| \\ &= \sqrt{(x/c)^2 - t_i^2} \\ &= t_i\sqrt{(x^2/t_i^2)/c^2 - 1} \\ &= t_i\sqrt{v^2/c^2 - 1} \\ &= -t_i/\gamma \end{aligned} \tag{1}$$

Hence:

$$\tau = -|t_c| = -t_i/\gamma \tag{2}$$

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The reason why the modulus of complex time: $|t_c| = -\tau$, or its imaginary part: $t_i = -\gamma\tau$, is negative with relation to proper time: $\tau = t/\gamma$, where t is the normal measured time, is because the (imaginary) flow of time in the outward level t_i is interrupting and delaying the real flow of time that is creating space and matter: t_r, so the actual (net value of) time is always smaller than the real time: $|t_c| = \sqrt{t_r^2 - t_i^2} \leqq t_r$.

This essential characteristic is what makes time restricted into one direction, because it means that t_i can never exceed t_r: $0 \leq t_i \leq t_r$, because they both belong to one single existence that is flowing either in the real inward dimension which is creating the continuous space or in the outward direction which forms the imaginary discrete time, not the two together, otherwise they both would be real as we are deceived.

This is also equivalent to: $0 \leq |v| \leq c$, thus when: $t_i \longrightarrow t_r = x/c$, we get: $v = x/t_i \longrightarrow c$. If $t_i = 0$ we have flat infinite space without time, which is the state of vacuum: $(t_r, 0)$, so this imaginary time acts like a resistance against the perpetual creation of space, and its interruption, i.e. going in the outward dimension of time, is what causes the inertial mass m_0, which then effectively increases with speed according to: $m = \gamma m_0$ (as it will be also proved in sections 4.2, 4.3 and 5 below), and when the outward imaginary time approaches the inner time, the apparent velocity v approaches the speed of light c, and $m \longrightarrow \infty$. On the other hand, the apparent velocity v can not exceed c because it is the average of all instantaneous velocities of all individual geometrical points N that constitute the particle, which are always fluctuating between rest and the speed of light, as we have seen in section 4. So by definition v is capped by c, as can be also seen from equation 3.

Furthermore, this important property, that the outward time is effectively negative with respect to the real flow of time, will be inherited by the velocity, momentum and even energy; all of which will be similarly negative in relation to their real counter part. It is this fundamental property that will enable the derivation of the relativistic momentum-energy relation, the equivalence principle between inertial and gravitational masses, in addition to allowing energy and mass to be imaginary, negative and even multidimensional. This will be discussed further in sections 4.1, 4.3 and 5, respectively.

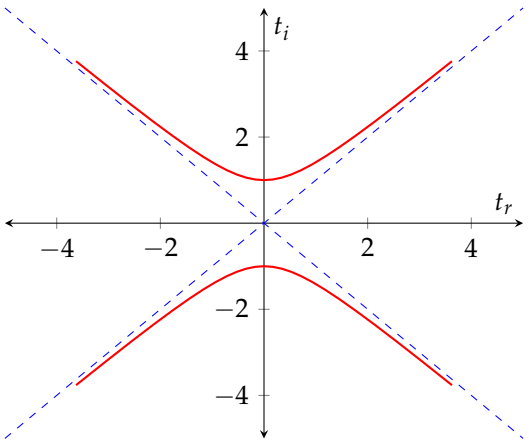

Figure 2. The arrow of time is represented by the upper branch of the hyperbola $a(t_r^2 - t_i^2) - b = 0$, while the lower branch is the reverse arrow of time, and when these two branches are connected to form a degenerate conic, the two opposing directions of time form one dimension of space. This condition occurs when $a = 1$ and $b = 0$, thus $t_r^2 - t_i^2 = 0$ or $t_i = t_r$, which corresponds to $v_r = c$.

This essential observation also provides the fundamental reason of the arrow of time, although the **reverse arrow of time** can still be conceived as the other disconnected branch of the hyperbola. When these two opposite arrows of time meet, the hyperbola (that can be described by the general equation: $a(t_r^2 - t_i^2) - b = 0$) turns into degenerate conic and these two opposite time directions become one dimension of space. This condition occurs when $a = 1$ and $b = 0$, hence the above

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equation becomes: $t_r^2 - t_i^2 = 0$, or: $t_i = t_r$, which corresponds to $v_r = c$ which is the state of vacuum or the ground state of matter as we mentioned in this section above.

4. Deriving the three principles of relativity:

The famous Michelson-Morley experiment in 1887 proved that light travels with the same speed regardless whether it was moving in the direction of the movement of the Earth or perpendicular to it [31]. This unexpected null result initiated active research that eventually led to Special Relativity in 1905 [32]. The speed of light in vacuum is then considered the maximum speed which anything in the universe can attain. Photons and other massless elementary particles propagate in vacuum at this terminal speed, regardless of the motion of the source or reference frame of the observer. However, even though this was confirmed by many experiments, there is yet no theoretical account that could explain the reasons behind this constancy and invariance of the speed of light.

As demonstrated in Figure 1 and explained further in section 4.2, one of the striking conclusions of the sequential re-creation in the inner level of time is the fact that it conceives of only two primordial states: **vacuum** and **void**, whose spatial and temporal superposition is producing the multiplicity of intermediary states that appear in the cosmos as matter particles. Vacuum, or empty space, is the continuous existence in the inner level of time, and void is the discrete existence in the outer level. These two extreme states can be conceived as abstract forms but only a relative superposition of them can ever be observed or measured as relative events in space-time, or inner-outer time that is the complex-time that will be discussed further in the following section 3.

For normal observers who are habitually living (as discrete instances) in the outer time, the continuous existence of vacuum appears moving at the terminal speed (of light) since it does not suffer the inertia that is experienced by massive particles and objects as they advance in the outer time. This inertia is the resistance that results from interrupting the perpetual creation in the inner time in order to appear as discrete instances in the outer time. That is why the outer time is imaginary with relation to the inner time that is the real flow of creation. Therefore, this dynamic and complex nature of time is the fundamental fact that underlines the emergence of dimensions. In other words: *vacuum* is the continuous and infinite existence which forms the inner real part of time, and whenever this existence is interrupted it makes a new dimension that appears as a discrete instance on the outer imaginary level which is then described as *void* since it does not last for more than one instance, before it is re-created again in a new state that may resemble the previous perished states.

This means that in the real flow of time in which space and matter are always being re-created, the individual geometrical points can either be at rest (in the outer/imaginary time) or moving at the speed of light (in the inner/real time), while the apparent limited velocities of compound particles and objects (in the complex time, which forms the physical dimensions of our space-time) are the temporal average of this spatial combination that may also dynamically change as they are progressing over the outward ordinary time direction.

This model, therefore, brings back the same classical concept of aether but in a novel manner that does not require it to affect the speed of light, because it is now the background space itself, being granular and created dynamically in time, and not something in a fixed background continuum that used to be called vacuum. On the contrary, this dynamical creation in the inner time provides a simple ontological reason for the constancy and invariance of the speed of light, which is so far considered as an axiom, that has been experimentally verified but not yet proven in any theoretical sense.

Consequently, there is no gradual motion in the common sense that the object leaves its place to occupy new adjacent places, but it is successively re-created in those new places, i.e. motion occurs as a result of discrete change and not infinitesimal transmutation, so the observed objects are always at rest in the different positions that they appear in (see also Figure 1). This is the same conclusion of the Moving Arrow argument in Zeno's paradox, which Bertrand Russell described as: "It is never moving, but in some miraculous way the change of position has to occur between the instants, that is to say, not at any time whatever." [33] [34].

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Therefore, the universe is always coming to be in "zero time" (on the outward level), and its geometrical points are sequentially fluctuating between existence and nonexistence (or vacuum and void), which means that the actual instantaneous speed of each point in space can only change from $v_i maginary = 0$ to $v_r eal = c$, and vice versa. This instantaneous abrupt change of speed does not contradict the laws of physics, because it is occuring sequentially in the inner levels of time before the physical objects are formed. This fluctuation is the usual process that is encountered by the photons of light and other massless particles on the normal outward level of time. Hence, this model of perpetual creation is extending this process onto all other massive particles and objects, but on the inner level of time where each point is still massless because it is metaphysical, while "space" and "mass" and other physical properties are actually generated from the temporal coupling, or entanglement, of these geometrical points, which is exhibited only on the outward level of time.

Accordingly, normal limited velocities of massive physical particles and objects are a result of the spatial and temporal superposition of the various dual-states velocities of their individual points N:

$$|v| = \frac{\sum_N < 0|c >}{N} \Rightarrow 0 \leqq |v| \leqq c \tag{3}$$

Individually, each point is massless and it is either at rest or moving at the speed of light, but collectively they have some non-zero inertial mass m, energy E, and limited total apparent velocity v given by equation 3. This means that all frames are effectively at rest in the normal (imaginary) level of time, and there is no difference between inertial and non-inertial frames, thus there is even no need to introduce the second principle of Special Relativity (which says that the laws of physics are invariant between inertial frames) neither the equivalence principle that lead to General Relativity. These two principles, which are necessary to derive Lorentz boost and Einstein's field equations, will follow directly from the complex-time geometry as it will be shown in sections 4.1 and 4.4 below. Furthermore, it will be shown in section 4.2 that this essential fact that results from the genuinely complex nature of time is the only way that allows exact mathematical derivation of the mass-energy equivalence relation ($E = mc^2$) directly from the fundamental classical principles.

In this way, the Duality of Time postulate and the resulting perpetual re-creation in the inner level of time can explain at once all the three principles of Special and General Relativity and transform it into full quantum field theory because it is now based on discrete instances of dynamic space, without any background. Additionally, the other fundamental forces could now be interpreted in terms of this new complex-time geometry, but in lower dimensions: $2D + 1$, $1D + 1$, and $0D + 1$, while gravity is in $3D + 1$.

4.1. Lorentz boost:

It was originally shown by Poincare [27] that by using the mathematical trick of imaginary time, Lorentz transformation becomes a rotation between inertial frames. For example, if the space coordinates of an event in space-time relative to one frame are $X = ct + jx$, then its (primed) coordinates $X' = ct' + jx'$ with respect to another frame, that is moving with uniform velocity $v < c$ with respect to the first frame, are: $X' = Xe^{-j\phi} = [ct + jx][\cosh(\phi) + j\sinh(\phi)]$, where $\tanh(\phi) = \sinh(\phi)/\cosh(\phi) = v/c = \beta$, and since: $\cosh^2(\phi) - \sinh^2(\phi) = j^2 = 1$, then: $\gamma = \cosh(\phi) = 1/\sqrt{1 - v^2/c^2} = 1/\sqrt{1 - \beta^2}$.

In the complex-time frame of the Duality of Time postulate, however, the outer time is the (genuinely) imaginary part, while the real part is the inner time that constitutes space, thus the time coordinates: $T = t_r + jt_i = (x/c) + jt_i$ is used instead of space coordinates: $X = ct + jx$. Therefore, the above rotation equations will still be valid but with time, rather than space, coordinates: $T' = Te^{-j\phi} = [t_r + jct_i][\cosh(\phi) + j\sinh(\phi)]$, and then the speed of light will be the ground state, or the *rest speed*, so when the apparent imaginary velocity v on the outer time is zero there is still the

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real speed that is the constant and invariant speed of light, or in general: $V = c + jv \Longrightarrow \beta = v/c = \tanh(\phi), \gamma = \cosh(\phi) = c/\sqrt{c^2 - v^2}$. See also Figure 1, and also Figure 3.

Using the concept of split-complex time, we can easily derive Lorentz boost $\gamma = 1/\sqrt{1 - v^2/c^2}$ for example by calculating the proper time τ as it can be readily seen from Figure 1, which is replicated in Figure 3 that represents complex velocity, for better clarity, and also because we want to stress the fact that the apparent (imaginary) motion in any direction is in fact interrupting the real motion in the inner time that is creating space at the speed of light, so that in the end the *actual velocity* is: $v_a = \sqrt{c^2 - v^2}$, which means that everything in space is intrinsically moving at the speed of light in the real flow of time, and its apparent motion in the outward time is causing an apparent (or imaginary) slow down from this real speed, so that when the object is not moving at all it is actually still moving at the speed of light, which becomes zero in relation to other stationary objects because they are all moving at this same real speed.

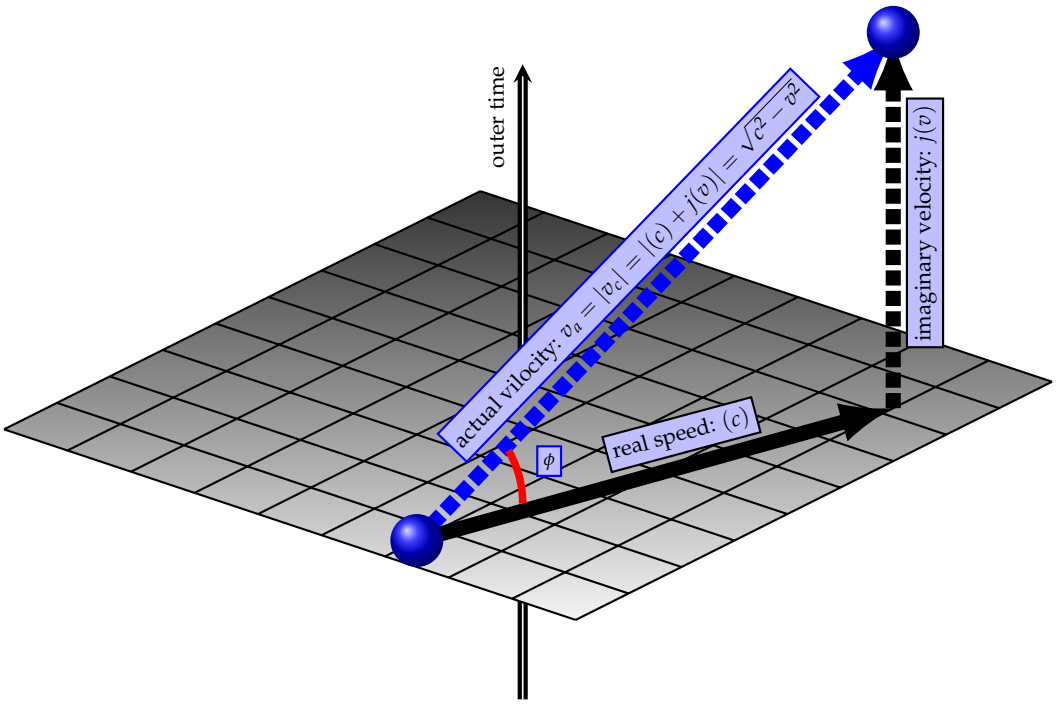

Figure 3. The actual velocity v_a is the modulus of split-complex velocity that combines the real speed $v_r = c$ with the imaginary velocity $v_i = jv$, calculated according to hyperbolic numbers, thus: $v_a = |v_c| = |v_r + jv_i| = |c + jv| = \sqrt{c^2 - v^2}$, from which we can easily calculate Lorentz boost as $\gamma = c/v_a = \cosh\phi$.

Lorentz boost is therefore the ratio of the real velocity c over the actual velocity v_a, which is equal $\cosh\phi$, as demonstrated in Figure 3:

$$\begin{aligned} \gamma &= c/v_a \\ &= \frac{c}{\sqrt{c^2 - v^2}} \\ &= 1/\sqrt{1 - v^2/c^2} \\ &= \cosh(\phi) \end{aligned} \tag{4}$$

4.2. The mass-energy equivalence relation:

In addition to explaining the constancy and invariance of the speed of light and merging it with the second and third principles of Relativity, the most significant implication of the Duality of Time

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postulate is that it appears to be the only way to explain the equivalence and transmutability between mass and energy ($E = mc^2$). Einstein gave various heuristic arguments for this relation without ever being able to prove it in any theoretical way [35], neither did anyone else. Based on Doppler effect and Maxwell's theory of radiation, the reasoning that he gave in his 1905 original derivation [11] was questioned by Planck [12] and shown to be faulty [13]. In 1907, Einstein acknowledged the controversy over his derivation [14], and in the following forty years, he produced more than half dozen proofs that all suffer from unjustified assumptions or approximations. He never succeeded in producing a valid general proof [15]. In 1955 he wrote in a letter to Carl Seelig: "I had already previously found that Maxwell's theory did not account for the micro-structure of radiation and could therefore have no general validity." [16]. In 1990, Rohrlich tried to apply the relativistic Doppler shift, but he also had to introduce various approximations in order to reach the final equation [36]. Various other attempts have been made in this regard but until now there is no exact derivation of this famous formula.

It can be readily seen from Figure 4 that the transmutability between mass and energy can only occur in the inner levels of time, because it must involve motion at the speed of light that appears on the normal level of time as instantaneous, hence the same Relativity laws become inapplicable since they prohibit massive particles from moving at the speed of light, in which case the mass $m = \gamma m_0$ would be infinite. In the inner levels of time, however, this would be the normal behavior because the geometrical points are still massless, and their continuous coupling and decoupling is what generates mass and energy on the inner and outer levels of time, respectively.

As we introduced in section 3.1 above, the normal limited velocities of massive physical particles and objects are a result of the spatial and temporal superposition of the various dual-state velocities of their individual points. This superposition of instantaneous velocities occurs in the inner levels of time where individually each point is massless and it is either at rest or moving at the speed of light, but collectively they have some non-zero inertial mass m, energy E, and limited total apparent velocity v, which can be calculated from equation 3.

Based on this metaphysical behavior in the inner levels of time, we will provide in the following three simple methods to derive the mass-energy equivalence relation directly from the classical equation of mechanical work $E = \int_0^x F.dx$. The first two methods (sections: 4.2.2, 4.2.3) that lead to equations 9 and 10 are equivalent, and they respectively involve integration (or rather: summation) in the inner time when velocity changes abruptly from zero to c, or when the mass changes also abruptly from zero to m (or vice versa). This is obviously not allowed on the normal level of time when dealing with physical objects that have mass. The third method in section 4.2.7 is obtained from the geometry of Figure 5 which gives the relativistic (effective) mass equation $m = \gamma m_0$ and the relativistic energy-momentum equation 15, as will be explained further in section 4.1 and Appendix A.

In normal classical mechanics, the kinetic energy is the work done in accelerating a particle during the infinitesimal time interval dt, and it is given by the dot product of force F and displacement dx:

$$\begin{aligned} E &= \int_0^x F.dx \\ &= \int_0^t F.vdt \\ &= \int_0^t \frac{d(mv)}{dt}.vdt \end{aligned} \tag{5}$$

Thus:

$$E = \int (v^2 dm + mvdv) \tag{6}$$

Now if we assume mass to be constant, so that: $dm = 0$ (we will discuss relativistic mass further in section 4.2.4 below), we will get:

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$$E_k = \int (v^2 dm + mvdv) = 0 + m \int_0^v vdv \tag{7}$$

4.2.1. The classical kinetic energy (in the normal time):

So in the classical view of apparently continuous existence, when we consider both space and time to be real, i.e. when we consider an infinitesimally continuous and smooth change in speed from zero to v, the result of this integration will give the standard equation that describes the kinetic energy of massive particles or objects moving in the normal level of time:

$$E_k = \frac{1}{2} mv^2 \tag{8}$$

The reason why we are getting the factor of "half" in this equation is because the velocity increases gradually with time, which makes the integration equals to the area of the triangle as demonstrated by the first arrow in Figure 4.

4.2.2. Method I (abrupt change of speed in the inner time):

The mass-energy equivalence relation: $E = mc^2$ (without the "half") can now be easily obtained from the same integration in equation 7 if, and only if, we suppose that the object, whose mass is m, moves from rest to c, or vice versa, in "zero time", which of course will contradict the laws of physical motion because the acceleration would be infinite, and hence the force and the energy. Light does in fact behave in this manner, for example: in pair production, or when emitted or absorbed; but the photon is massless, unlike other particles and objects which have mass and suffer inertia and acceleration.

By introducing the duality of time and the resulting perpetual re-creation, this problem is solved because the conversion between mass and energy takes place sequentially, in the inward level, over all the massless geometrical points that constitute the particle, and this whole process appears as once instance in the outer level, as demonstrated in Figure 1.

So by integrating equation 7 directly from zero to c, which then becomes summation because it is an abrupt change, since the change in the outward time is zero, and here we consider $dm = 0$, since the apparent velocity does not change in this case, but we will also discuss relativistic mass in section 4.2.4 below; thus we get:

$$\begin{aligned} E &= \int (v^2 dm + mvdv) \\ &= 0 + m \int_0^c v.dv \\ &= m \sum_0^c v.dv = mc^2 \end{aligned} \tag{9}$$

The difference between the above two cases that result in equations 8 and 9 is demonstrated in Figure 4, where in the first case the integration that gives the kinetic energy E_k is the area of the triangle below the gradual arrow (1), while in the second case it is the area of the rectangle below the right angle arrow (2).

4.2.3. Method II (generating mass in the inner time):

We can also arrive to the mass-energy equivalence relation directly from the starting equation 7 in an alternative manner if we consider a sudden decoupling, or disentanglement, of the geometrical points that couple together in order to constitute the physical particle that appears in the outer level of time with inertial mass m moving at an apparent (imaginary) velocity v, so when these geometrical points are disentangled to remain at their real speed (of light), the mass m converts back into energy E while the apparent velocity does not change, because this process is happening in the inward level of

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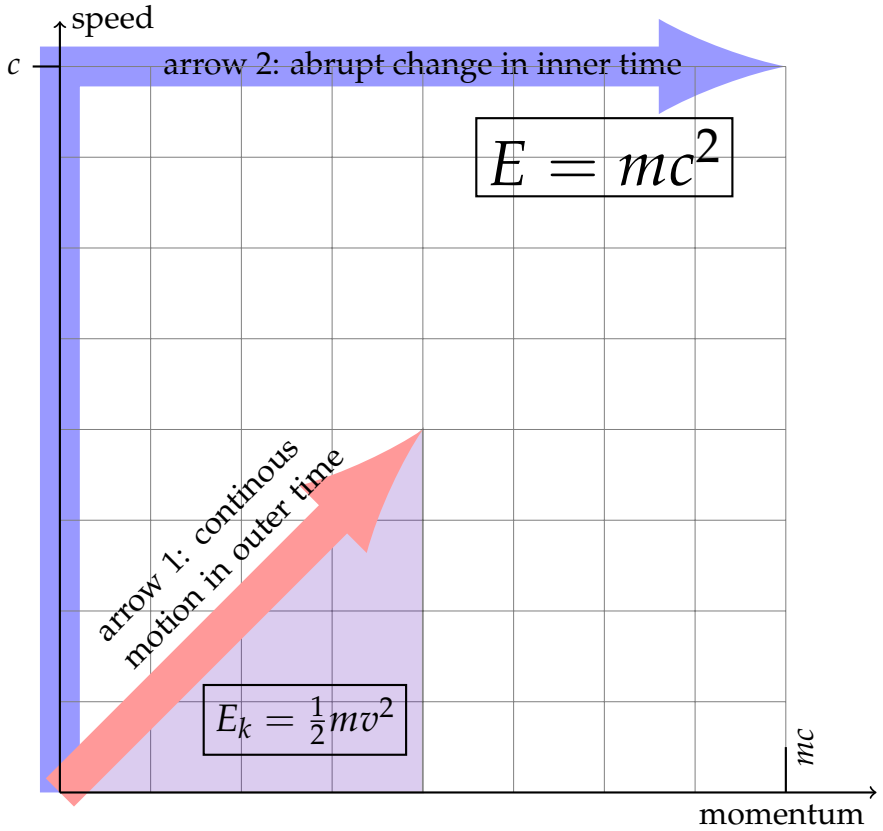

Figure 4. Arrow 1: gradual change of speed in the outer level of time, leading to the classical equation of kinetic energy $E_k = \frac{1}{2}mv^2$. Arrow 2: abrupt change from zero to c in the inward level of time, leading to the mass-energy equivalence relation $E = mc^2$, which can not happen on the normal level of time because physical objects may not move at the speed of light.

time which appears outwardly as instantaneous. Thus, if we put $dv = 0$ in equation 7 and integrate over mass from m to zero (where $v = c$), or vice versa, we get:

$$E = \int (v^2 dm + mvdv) = c^2 \int_0^m dm + mv(0) = mc^2 \tag{10}$$

Unlike the classical case in equation 8 where the change in speed occurs in the normal outward level of time, these simple derivations (in equations 9 and 10) would not have been possible without considering the inner levels of time, which appears outwardly as instantaneous.

4.2.4. Note (relativistic mass):

If we want to consider mass to be variable with speed as in early Special Relativity, and distinguish between rest mass: m_0 and relativistic mass: m, according to the standard equation that uses Lorenz factor: $m = \gamma m_0 = m_0/\sqrt{1 - v^2/c^2}$, then we can arrive to the equation $E = m_0c^2$ by calculating the derivative dm/dv which in this case will not be equal to zero as we required in equation 7 above. However, the above relativistic equation of mass ($m = \gamma m_0$) is only obtained based on the same mass-energy equivalence relation that we are trying to prove here, so in this case it will be a circular argument. Therefore, the two equations: $E = m_0c^2$ and $m = \gamma m_0$ are equivalent, and deriving one of them will lead to the other. See also Appendix A for how to derive the $E = m_0c^2$ from $m = \gamma m_0$.

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4.2.5. The wave-particle duality:

We can conclude, therefore, that on the highest existential level, there is either energy in the form of massless active waves moving at the speed of light in the inner level of time, or passive mass in the form of matter particles that are always instantaneously at rest in the outer level of time, not the two together; that is what happens in the real flow of time. The various states of objects and particles are some spatial and temporal superposition of these two primordial states of their metaphysical constituents, so some particles will be heavier than others, and some will have more kinetic energy. In any closed system, such as an isolated particle, atom, or even objects, the contributions to this superposition state come from all the states in the system that are always fluctuating between mass and energy, corresponding to $v_i = 0$ and $v_r = c$, respectively, so on average the total state is indeterminate, or determined only as a probability distribution, as far as it is not detected or measured.

Consequently, everything in the universe is always fluctuating between particle state and wave state, or mass and energy, which can be appropriately written as: $m_0c^2 \rightleftharpoons hf$. This means that a particle at rest with mass m_0 can be excited into a wave with frequency f, and the opposite is correct when the wave collapses into particle. Either zero mass at the speed of light, or zero energy at rest, or: either energy in the active existence state or mass in the passive nonexistence state. The two cannot happen together on the primary level of time, but a mixture or superposition of various points is what causes all the phenomena of motion and interaction between particles with limited velocities and energy on the outward level of time.

4.2.6. The effective mass:

Therefore, even when the object is moving at any speed that could be very close to c, at the time of measurement its instantaneous velocity is always zero: $v_i = 0$, and its mass will still be the same rest mass: m_0, because it is only detected as a particle, while its kinetic energy will be given by its relativistic mass: $E_k = \frac{1}{2}mv^2$, and then its total energy equivalence, with relation to an observer moving at a constant (apparent) velocity v, will be given by:

$$E = E_0 + E_k = m_0c^2 + \frac{1}{2}mv^2 \tag{11}$$

Thus, with the help of Lorentz factor $\gamma = \sqrt{1 - v^2/c^2}$, we could get rid of the confusion between "rest mass" and "relativistic mass" and just call it mass m since the above equation 11 describes energy and not mass:

$$E = E_0 + E_k = mc^2 + \frac{1}{2}\gamma mv^2 \tag{12}$$

Which means that the mass of any particle is always the rest mass, it is not relativistic, but its energy is relativistic, primarily because energy is related to time and motion or velocity. However, since we have been using m_0 all over this article, we will keep using it as the rest mass, and $m = \gamma m_0$ as the effective mass, unless stated otherwise.

4.2.7. Method III (from the definition of momentum):

The total relativistic energy in equation 11 can also be obtained by integrating the starting equation 6 over the inner and outer time together, since in the inner time the rest energy $E_0 = m_0c^2$, or mass m_0, is generated at the speed of light c as a result of the instantaneous coupling between the geometrical points which constitutes the particle of mass m_0 (thus $dv = c$ and $dm = m_0$ in the inner time), and in the outer time the kinetic energy $E_k = \frac{1}{2}mv^2$ is generated as this mass m_0 moves gradually so its apparent velocity changes by dv, which corresponds to increasing the effective mass from m_0 to m, thus we can integrate:

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$$\begin{aligned} E &= \int (v^2 dm + mvdv) \\ &= \int_{0,0}^{c,m_0} v^2 dm + \int_{0,m_0}^{v,m} mvdv \end{aligned} \tag{13}$$

Thus we get the same equation 11 or 12:

$$\begin{aligned} E =& m_0c^2 + \frac{1}{2}(m)(v^2) - \frac{1}{2}(m_0)(0) \\ =& m_0c^2 + \frac{1}{2}mv^2 \end{aligned} \tag{14}$$

This equation can also be given in the general form that relates the relativistic energy and momentum (see Appendix A for how to convert between these two equations):

$$\begin{aligned} E =& \sqrt{(m_0c^2)^2 + (\gamma m_0 cv)^2} \\ =& \sqrt{(m_0c^2)^2 + (pc)^2} \end{aligned} \tag{15}$$

This last equation, that is equivalent to equation 14, will be also derived directly from the Duality of Time postulate in section 4.3, while in Appendix A we show how to derive it classically from equation 12 starting from the effective mass relation: $m = \gamma m_0$.

Again, however, a fundamental derivation of this relativistic energy-momentum equation 15 is also not possible without the Duality of Time postulate. All the current derivations in the literature rely on the effective mass relation: $m = \gamma m_0$, which is equivalent to the same relation we are trying to derive (see above and also Appendix A), while finding this equation from the four-momentum expression, or space-time symmetry, relies on induction rather than rigorous mathematical formulation.

4.3. Deriving the relativistic energy-momentum relation

In addition to the previous methods that derived the simple form of the mass-energy equivalence: $E = m_0c^2$ in equations 9 and 10 above, the relativistic energy-momentum equation: $|\vec{P}| = E/c = \sqrt{(m_0c)^2 + (\gamma m_0 v)^2}$ can be derived according to the Duality of Time postulate directly from the fundamental definition of momentum, as mass times velocity: $p = mv$.

According to the Duality of Time postulate, an object with mass m, moving at a constant velocity v, is continuously being re-created at the speed of light $v_r = c$ in the inner time t_r, before it appears as an instance in the imaginary outer time t_i where its instantaneous velocity is $v_i = 0$ but the re-creation makes it appear to be moving at an average velocity v. So in the inner time it behaves as a wave with instantaneous velocity c, and in the outer time it behaves as a particle with effective mass $m = \gamma m_0$ and instantaneous velocity $v_i = 0$ while its apparent velocity is obtained by averaging these two instantaneous velocities over all its metaphysical points N, given by equation 3 above. Therefore, the real momentum in the inner time is $p_r = h/\lambda = m_0c$, while the normal kinetic momentum, which is the imaginary part, is $p_i = \gamma m_0 v$. Thus the total complex momentum is: $p_c = p_r + jp_i$, whose modulus is: $|p_c| = \sqrt{p_r^2 - p_i^2}$.

This total momentum is obtained by integrating $dp = d(mv)$ between $(0,0)$ and (c, m_0) on the inner level of time where the particle is created, or actually being perpetually re-created, at the speed of light, and this term makes the real part of the complex momentum. Then we integrate between $(0, m_0)$ and (v, m) on the outer level of time where the particle whose mass is m_0 gains an apparent velocity v and thus its effective mass increases from m_0 to m, and this term makes the imaginary part of the complex momentum $p_c = p_r + jp_i$:

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$$\begin{aligned} p_c &= \int_{0,0}^{m,v} d(mv) \\ &= \int_{0,0}^{m,v} (vdm + mdv) \\ &= \int_{0,0}^{c,m_0} vdm + j\int_{0,m_0}^{v,m} mdv \end{aligned} \tag{16}$$

The first term gives us the real momentum $p_c = \int_{0,0}^{c,m_0} vdm = m_0 c$, while the second term gives the imaginary momentum $p_i = \int_{0,m_0}^{v,m} mdv = 0(m - m_0) + m(v - 0) = mv$. Thus:

$$p_c = p_r + jp_i = m_0 c + jmv = m_0 c + j\gamma m_0 v \tag{17}$$

Hence the modulus of this total complex momentum is:

$$|p_c| = \sqrt{(m_0 c)^2 - (\gamma m_0 v)^2} \tag{18}$$

Again we notice here that the contribution of the kinetic momentum $p_i = mv = \gamma m_0 v$ is negative with relation to the real momentum $p_r = m_0 c$, just like the case of t_i and v_i as we have seen in sections 3.1 and 4.1, and as it will be also the case for energy as we shall see in section 5 further below.

These equations above, with the negative sign, do not contradict the current equations, such as: $E = \sqrt{(mvc)^2 + (m_0 c^2)^2}$ which treat energy as scalar and do not realize its complex dimensions (see also section 5 below). Practically, in any mass-energy interaction or conversion, the negative term will be converted back to positive because when the potential energy $m_0 c^2$ is released, in nuclear interactions for example, this means that it has been converted from the inner level of time where it behaves like mass into the outer level to become kinetic energy. In other words: the absorption and emission of energy or radiation, nuclear interactions, or even the acceleration and deceleration of mass, are simply conversions between the inner and outer levels of time.

Therefore, to obtain the relativistic energy (equations 12 or 15), we can work with equation 18, that gives the magnitude of the four-momentum, as it is usually treated in Special Relativity (see also Appendix A), i.e.:

$$\begin{aligned} E &= c|\vec{P}| = c|p_c| \\ &= \sqrt{(m_0 c^2)^2 - (\gamma m_0 vc)^2} \\ &= \sqrt{(m_0 c^2)^2 - (pc)^2} \end{aligned} \tag{19}$$

Alternatively, we can simply add the inner rest energy $m_0 c^2$ to the outer (kinetic) relativistic energy $\frac{1}{2}mv^2 = \frac{1}{2}\gamma m_0 v^2$, since they are treated as scalars, thus we get the same equation 12 which also leads to equation 15, as it is derived in Appendix A.

This derivation of the relativistic energy-momentum relation from the fundamental definition of momentum $p = mv$ is based on the Duality of Time concept, by taking into account the complex nature of time, as hyperbolic numbers, which is why the contribution of the imaginary term appears here as negative in equation 19.

Therefore, when we take into account the complex nature of time as we described in sections 3.1 and 4.1 above (Figures: 1 and 3), energy and momentum will be also complex. This will introduce an essential modification on the relativistic energy-momentum equation which will lead to the derivation of the equivalence principle and allow energy to be imaginary, negative and even multidimensional. This will be discussed further in the following sections 4.4 and 5.

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4.4. The equivalence principle

In moving from Special to General Relativity, Einstein observed the equivalence between the gravitational force and the inertial force experienced by an observer in a non-inertial frame of reference. This is the same as the equivalence between active gravitational and passive inertial masses, which has been later accurately tested in many experiments [37], [38], but there is no direct mathematical derivation to this principle apart from the famous spacecraft accelerator thought experiment which relies on induction.

When Einstein combined this equivalence principle with the two principles of Special Relativity, he was able to predict the curved geometry of space-time, which is directly related to its contents of energy and momentum of matter and radiation, through a system of partial differential equations known as Einstein field equations.

We explained in section 4.2 above that an exact derivation of the mass-energy equivalence relation is not possible without postulating the Duality of Time, because it requires motion at the speed of light which is not possible on the physical level. It turned out that this complex character of time is also required to obtain a direct derivation of the equivalence principle, which is in fact equivalent to $E = mc^2$ or $m = \gamma m_0$, because they are all a result of the fact that space and matter are being perpetually re-created in the inner time, i.e. fluctuating between the particle state and wave state, and this is the only way that lead to a self-contained granular and dynamic space-time without any background, as we explained in sections 3 and 3.1 above.

The representation of the *complex momentum* in Figure 5 that is based on the complex time frame as illustrated in Figure 1 indicates that the modulus of the complex momentum $|p_c| = |p_r + jp_i| = |mc + jmv| = \sqrt{(mc)^2 - (mv)^2}$ should be invariant between inertial and non-inertial frames alike, because effectively all objects are always at rest in the outer level of time as we showed in section 3 above. This means that momentum is always conserved **even when the velocity changes**, i.e. as the object accelerates between non-inertial frames!

Actually, without this exotic property of momentum it is not possible at all to obtain an exact derivation of $m = \gamma m_0$ which is equivalent to $E = m_0c^2$ as we mentioned in section 4.2 above and in Appendix A below. These experimentally verified equations are correct if, and only if, the total momentum $|p_c|$ is always conserved. For example when the object accelerates from zero to v, we get:

$$\begin{aligned} &\sqrt{(mc)^2 - (mv)^2} = m_0c \Rightarrow \\ &m = m_0c/\sqrt{c^2 - v^2} = \gamma m_0 \end{aligned} \tag{20}$$

Furthermore, it is this exotic property that makes space-time itself dynamic and quantized without any background, because this requires that particles and objects are always at rest in their instantaneous locations, as Zeno indicated about two millenniums ago [34], since we explained in section 4 above that there is no real gradual motion but a discrete change without transmutation [see also: [9]: ch. vii].

The invariance of momentum between non-inertial frames is conceivable despite the change in velocity, because it means that as the velocity increases, the gain in kinetic momentum $p_i = mv$ (that is the imaginary part) is compensated by the increase in the effective mass: $m = \gamma m_0$ due to motion, which causes the real part $p_r = mc$ also to increase, but since $p_c = p_r + jp_i$ is hyperbolic, thus its modulus $\sqrt{(mc)^2 - (mv)^2}$ remains invariant. That is why it is necessary to use split-complex numbers here, because of their unique way of calculating the modulus, that is equivalent to and much easier than using Riemannian manifolds.

Therefore, in addition to the previous methods in equations 9 and 10, and the relativistic energy-momentum relation in equation 19, the mass-energy equivalence relation: $E = m_0c^2$ can now be deduced from equation 20 as it is shown in Appendix A below, because, as we mentioned in section 4.2.4 above, the equations: $E = m_0c^2$ and $m = \gamma m_0$ are equivalent, and the derivation of one of them

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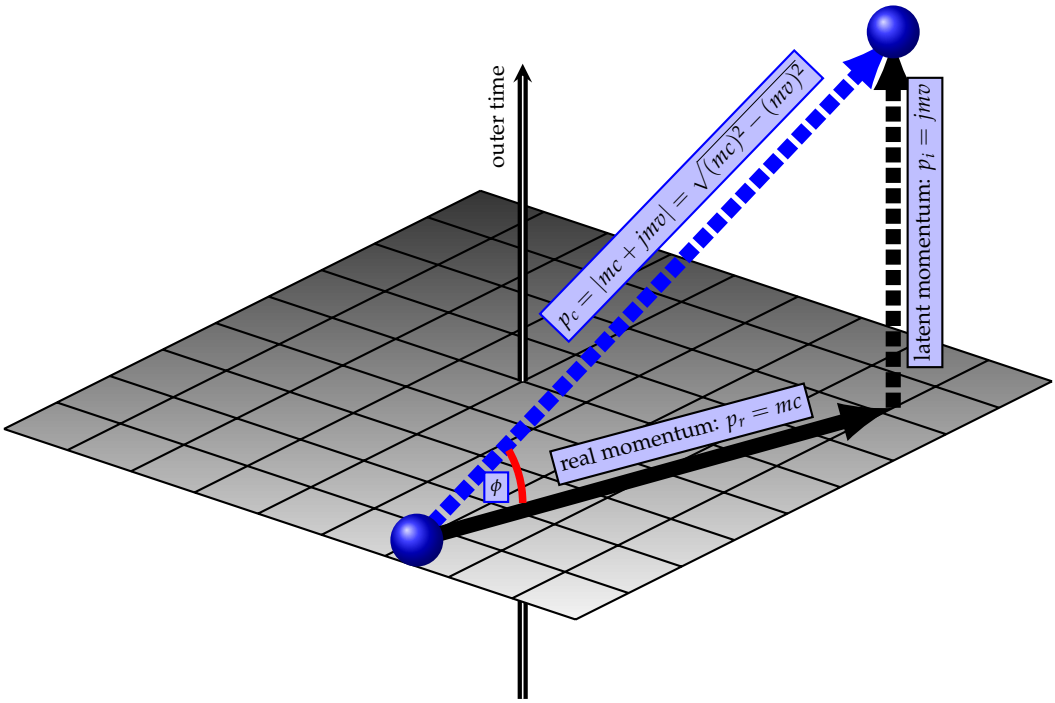

Figure 5. The complex-momentum p_c is combined from the real part that is the *rest momentum* in the inner time $p_r = mc$, with the imaginary part that is the normal kinetic momentum $p_i = mv$ on the outer level of time, thus: $|p_c| = |mc + jmv| = \sqrt{(mc)^2 - (mv)^2}$.

leads to the other. Without this concept of genuinely imaginary time, this essential equation can not be derived.

The absolute conservation of momentum can be easily concluded from Figure 3 simply by multiplying the velocity with mass on the inner and outer levels, but because of its importance we modified it here into Figure 5 and since this essential modification leads also to the mathematical derivation of the equivalence principle of General Relativity, which is currently only obtained by induction, from the famous spacecraft accelerator thought experiment.

This conservation of momentum under acceleration leads directly to the equivalence principle of General Relativity, because it means that the total (complex) force: $F_c = \frac{dp}{dt}$ must have two components; one that is related to acceleration as v changes in the outer time t_i, which is the imaginary part, and this is the accelerating force $F_i = ma = mdv/dt$, while the other is related to the change in effective inertial mass $m = \gamma m_0$, or its equivalent energy $E = mc^2$, which is manifested as the curvature of the geometry of space which is being re-created in the inner time t_r, and this is the gravitational force F_r; and these two components must be equivalent so that the total resulting complex momentum remains conserved. This also means that the inertial mass is itself the gravitational mass.

We can then write:

$$\begin{aligned} F_c &= F_r + jF_i \Rightarrow \\ |F_c| &= \frac{d(\sqrt{m^2c^2 - m^2v^2})}{dt} = 0 \end{aligned} \tag{21}$$

This equation gives the equivalence between the gravitational force and the pseudo-force experienced by an observer in a non-inertial frame. From this conservation of complex momentum we should be able to find the law of gravitation and the stress-energy-momentum tensor which leads to Einstein's field equations of General Relativity. This important conclusion requires further

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investigation, but the most important issue here is that this equivalence principle should apply to all fundamental forces and not only to gravity, as it will be investigated in another article [39].

5. Multidimensional energy

In the same manner as in the above geometrical representation of time, velocity and momentum in Figures 1, 3 and 5, that are based on the complex-time frame, we can now give more dimensionality to energy, that is already hinted in the current formulation of Special and General Relativity, for example through the relativistic energy-momentum equation 15 and the experimental fact that the cosmological constant is positive. Since it is intimately related to time, energy has to have complex, and even multiple intersecting dimensions in accordance with the dimensions of space and matter which are generated in the inner levels of time before they evolve throughout the outer level. We must notice straightforward, however, that not all these levels of energy are associated with mass which is only a property of $3D$ space. In lower dimensions, energy should rather be associated with the corresponding coupling property, such as the electric and color charges. Therefore, it is expected that negative mass is only possible in $4D$ spatial dimensions, as it has been already anticipated before [40], [41].

It is clear initially that, just like time, velocity and momentum that were discussed above, when we take the complex nature of time into account, the kinetic energy $\frac{1}{2}\gamma m_0 v^2$ in equation 11, or pc in the relativistic energy-momentum equation 15, becomes negative with relation to the potential energy mc^2 stored in mass m. Therefore the energy E in equation 13 becomes complex E_c with real E_r and imaginary E_i parts. The real part E_r represents re-creation through the change in mass dm, and the imaginary part E_i represents the kinetic evolution of this mass in the outer time through the change in the apparent velocity dv:

$$\begin{aligned} E_c &= \int (v^2 dm + jmvdv) \\ &= \int_{0,0}^{c,m_0} v^2 dm + j \int_{m_0,0}^{m,v} mvdv \end{aligned} \tag{22}$$

The real part is $E_r = m_0 c^2$ and the imaginary part is $E_i = \gamma m_0 vc = pc$, thus we get:

$$\begin{aligned} |E_c| &= \sqrt{(m_0 c^2)^2 - (\gamma m_0 vc)^2} \\ &= \sqrt{(m_0 c^2)^2 - (pc)^2}; p = \gamma m_0 v \end{aligned} \tag{23}$$

This negative contribution of the kinetic energy, however, does not falsify the current equations 11 and 15, but it means that the potential energy and the kinetic energy are in different levels of time and the conversion of potential energy into kinetic energy is like the conversion from the inner time into the outer time, so when they are in the outer time they are added together as in the previous equations because they become both in the outer level of time.

Again, just as it is the case with the absolute conservation of momentum that we have seen in section 4.4 above, energy is also always conserved, even when the apparent velocity v changes, since the instantaneous velocity v_i in the outer level of time is always zero as we have seen in section 3 and Figure 1 above. As it is the case for momentum, this absolute conservation of energy is conceivable because it means that as the velocity changes, the change in kinetic energy $E_i = mvc$ (that is the imaginary part) is compensated by the change in the effective mass: $m = \gamma m_0$ due to motion, which causes the real part of energy $E_r = mc^2$ also to change accordingly, but since $E_c = E_r + jE_i$ is hyperbolic, thus its modulus $\sqrt{(mc^2)^2 - (mvc)^2}$ remains invariant between all frames. This means that:

$$|E_c| = \sqrt{(mc^2)^2 - (mvc)^2} = m_0 c^2 \tag{24}$$

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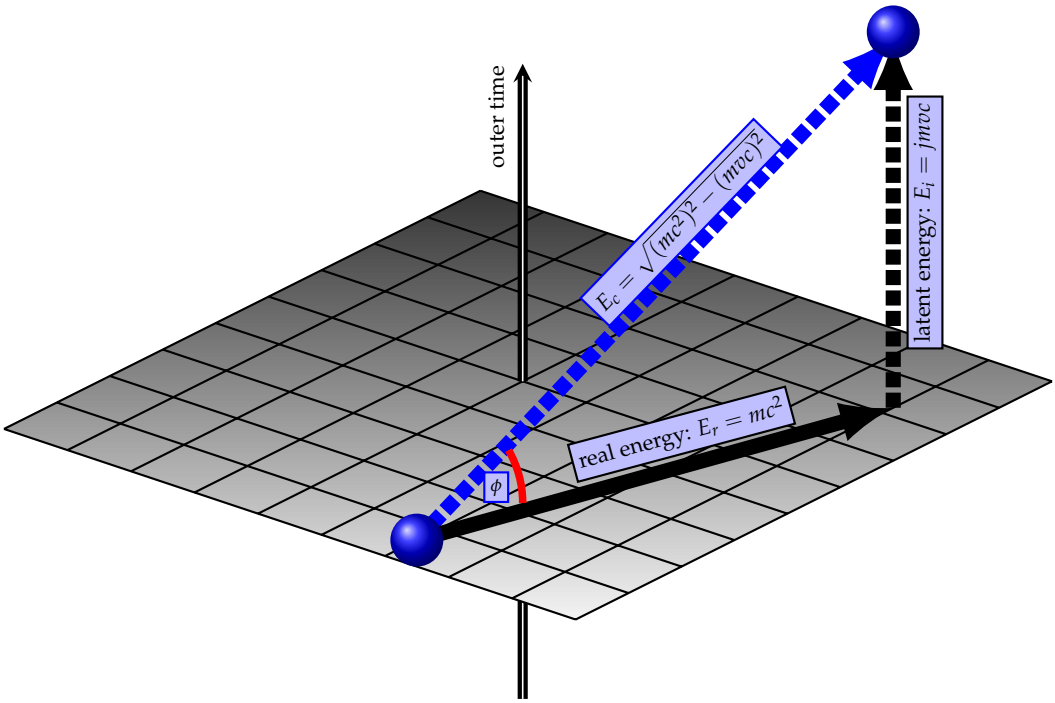

Figure 6. The complex energy is obtained by the combination of the real energy E_r, that corresponds to the re-creation of space and matter in the inner level of time, which then evolve in the outer time, thus the normal kinetic energy becomes imaginary E_i or latent with relation to E_r.

This equation provides an additional method to derive the mass-energy equivalence, because the left side in this equation can be reduced to mc^2/γ:

$$\begin{aligned} \sqrt{(mc^2)^2 - (mvc)^2} &= mc\sqrt{c^2 - v^2} \\ &= mc^2\frac{\sqrt{c^2 - v^2}}{c} \\ &= mc^2/\gamma \Rightarrow \\ m &= \gamma m_0 \end{aligned} \tag{25}$$

So when we combine the two equations: 24 and 25, we get the effective mass relation: $m = \gamma m_0$ that is equivalent to $E = m_0c^2$ as we have seen above and in Appendix A.

6. Aether and the cosmological constant problem:

Aether was used in ancient and medieval science as a thin transparent material that fills the upper spheres where planets swim. The concept was used again in the 18th century to explain the propagation of light and gravitation. This continued until the late 19th century in what is called: luminiferous aether, or light-bearing aether, which was needed to allow the apparently wave-based light to propagate through empty space.

The concept of aether was contradictory because this medium must be invisible, infinite and without any interaction with physical objects. Therefore, it was completely discarded after the null results of Michelson–Morley experiments in 1887, which clearly showed that light travels with the same speed regardless whether it was moving in the direction of the movement of the Earth or perpendicular to it [31].

After the development of Special Relativity, aether theories became scientifically obsolete, although Einstein himself said that his model could itself be thought of as an aether, since empty space now has its own physical properties [42]. In 1951, Dirac reintroduced the concept of aether in

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an attempt to address the perceived deficiencies in current models [43], thus in 1999 one proposed model of dark energy has been named: quintessence [44], or the fifth fundamental force [45].

As a scalar field, the quintessence is considered as some form of dark energy which could provide an alternative postulate to explain the observed accelerating rate of the expansion of the universe, rather than Einstein's original postulate of cosmological constant [46], [47].

However, we have seen in sections 3, 3.2 and 4 above that the perpetual re-creation in the inner level of time brings back the same classical concept of aether, while at the same time it explains the constancy and invariance of the speed of light because aether is now the background space itself, that is created dynamically in time. This maximum cosmological speed limit is basically the speed of creation which can never be exceeded, although nonlocal interactions, such as quantum tunneling [48] and the EPR [49], are still possible because there is no motion in the common sense that the object gradually leaves its place to occupy new adjacent places, but it is successively re-created in those new places, which could be at the other side of the universe [22], [34].

The classical concept of aether was rejected because it required ideal properties that could not be attributed to any hypothetical medium that was thought to be filling the otherwise empty space background which was called vacuum. With the new dynamic creation, however, those ideal properties of aether can be explained, because it is no longer something filling the vacuum, but it is vacuum itself, that is perpetually being created at the speed of light. Thus its state is described by: $(c, 0)$ as we explained in section 3.2 above, which indicates infinite and inert space that is the ground state of matter particles that are then described by (c, jv), whereas the absolutely empty space is now called void and its state is $(0, 0)$.

Therefore, in contrast to the classical view, aether is now the ground state of the geometrical points of space, and particles are its excited states, which is the same view of Quantum Field Theory. Yet, as it in General Relativity, the new concept of space-time, that results from this complex-time geometry, is dynamic and affected by the presence of matter, but it is now intrinsically granular and self-contained without any background.

6.1. The cosmological constant problem(s)

In order to make the universe static, Einstein first added the cosmological constant to his field equations so that the attraction of matter $\frac{1}{2}Rg_{\mu\nu}$ may balance the cosmological constant repulsion $\blacksquare g_{\mu\nu}$:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \blacksquare g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{26}$$

This postulate was later discarded after Hubble's discovery of the expansion of the universe in 1929. Nevertheless, in 1998, after the discovery of the accelerating universe from distant Type Ia supernovae [50], [51], in addition to the data from the cosmic microwave background and large galaxy redshift surveys [52], astronomers were able to measure the evolution of the expansion rate of the universe which lead to the current standard model of cosmology, the Lambda-Cold-Dark-Matter model $(\Lambda - CDM)$.

Galactic observations and large scale cosmology [53] put an upper bound on the value of the cosmological constant in the range $|\Lambda| < 10^{-56}m^{-2}$, which means that the vacuum energy density: $|\rho_{vac}| < 10^{-47}GeV^4$, according to cosmological parameters from SDSS and WMAP [54], and the ratio between the energy density due to the cosmological constant and the critical density of the universe is currently estimated at $\Omega_\Lambda = 0.6911 \pm 0.0062$ [55].

However, as early as 1960s, Quantum Field Theory predicted that quantum fluctuations would make huge contributions to the vacuum energy, much larger than the energy due to the cosmological constant which was known to be either zero or very small [56]. The problem became critical after the development of inflationary models in the 1980s, since the cosmic inflation is driven by vacuum energy [10].

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As Weinberg showed [10], there are many problems associated with the cosmological constant. The observed acceleration of the expansion simply means that the cosmological constant is about 120 orders of magnitude smaller than the density of vacuum energy, the Planck density, that can be calculated from QFTs. This is known as the problem of smallness. Moreover, being so small, why it just happens to have exactly the value that makes its density similar to the average matter density, which is called the coincidence problem.

In QFTs, the vacuum energy density is due to the zero-point energy of quantized fields, which originates from the quantization of the simple harmonic oscillations of a particle with mass. The zero-point energy of the Klein-Gordon field is infinite, but a cutoff at Planck length is enforced, because it is generally believed that General Relativity does not hold for distances smaller than this length: $\ell_P = \sqrt{\hbar G/c^3} = 1.616229(38) \times 10^{-35} m$, which corresponds to Planck energy: $E_P = \sqrt{\frac{\hbar v^5}{G}} \approx 1.2209 \times 10^{19} GeV$.

The origin of vacuum energy density in QFT according to the Standard Model of particles is the electromagnetic, the weak and the strong interactions. Here we shall consider only the quantum theory of electromagnetic interactions, since it is the most successful example of a quantum field theory. The quantum harmonic oscillator has a non-vanishing zero-point energy $E_0 = \frac{1}{2}\hbar\omega_k$. Therefore, the total zero-point energy can be expressed by [57]:

$$E = \delta^3(0) \int_\omega d^3k \frac{1}{2}\hbar\omega_k, \tag{27}$$

where ω_k is the frequency and k is the wave-number of a continuum of plane-wave modes. The infinite delta-function $\delta^3(0)$ can be regularized by introducing a box of volume V, thus: $\delta^3(k) = \frac{1}{2\pi^3}\int d^3x e^{ikx}$, which tends to $\frac{V}{8\pi^3}$ when k tends to 0 [57]. The vacuum energy density can now be obtained by:

$$\begin{aligned} \rho_{vac} &= \frac{E}{V} \\ &= \frac{1}{V}\sum_k \frac{1}{2}\hbar\omega_k \\ &\approx \frac{\hbar}{2\pi^2 c^3}\int_0^{\omega_{max}} \omega^3 d\omega \\ &= \frac{\hbar}{8\pi^2 c^3}\omega_{max}^4 \end{aligned} \tag{28}$$

This follows when the $V \longrightarrow \infty$ so that the energy density does not depend on the box quantization [58], and by applying the cutoff at Planck length/energy, since $E_P = \hbar\omega_{max}$, so we get: $\rho_{vac} = \frac{E_P^4}{8\pi^2\hbar^3 c^3}$, which gives us: $\rho_{vac} \sim 10^{76} GeV^4$. Comparing this theoretical value with the observed as we stated above $\rho_{vac} \sim 10^{-29} GeV^4$, we find a 120 orders of magnitude discrepancy.

6.2. Quick overview of the suggested solutions

The smallness of the cosmological constant became a critical issue after the development of cosmic inflation in the 1980s, because the different inflationary scenarios are very sensitive to the actual value of ρ_{vac}. Many solutions have been suggested in this regard, as it was reviewed by Weinberg [10] and Sanhi [59]. These suggestions include various modifications on either GR or QFT or the way they are linked together.

The quintessence models suggest that the vacuum density may in fact be time-dependent, while other models involve attempts to quantize gravity, which Weinberg believe to be 'the most promising' although they usually involve mathematically ill-defined procedures. In the supersymmetric theory, because each particle has a superpartner, the fermion and boson contributions to the vacuum energy would cancel to zero. However, since we do not observe such superpartners in nature, so the supersymmetry must be broken [56]. String theory has also failed to give a plausible solution.

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According to Witten [60], since the problem really involves quantum gravity, string theory is the only framework for addressing it. However, the cosmological constant problem has historically been a main obstacle of making string theory more realistic [56]. Finally, the proposal of an anthropic solution is based on the idea that our universe is embedded in a larger multi-verse, so we happen to live in the right universe whose cosmological constant is compatible with conditions for life forms to evolve. If the cosmological constant is positive and too large, the universe will enter an expanding phase too early without the formation of galaxies. If it is negative and too large the universe will collapse too fast [61].

6.3. The Duality of Time solution

Because the difference is so huge, none of the above speculations came ever close to solving the puzzle. The 10^{120} discrepancy is many orders of magnitudes larger than the number of all atoms in the universe, which is called Eddington number $N_{edd}10^{80}$ [62]. This indicates that there is something substantially wrong in our understanding of the quantum processes at the atomic level. It would not be strange, therefore, if we postulate that this huge number of atoms is not real! Yet since we clearly observe multiplicity in our normal level of time, it remains that they are multiplied in the inner levels of the real flow of time.

According to the Duality of Time postulate, the huge discrepancy in the cosmological constant is diminished and even eliminated because the vacuum energy should be calculated from the average of all states, and not their collective summation as it is currently treated in QFTs. This means that we should divide the vacuum energy density in equation 28 by the number of modes included in the unit volume. Since we took Planck length as the cutoff, this number is:

$$\begin{aligned} N &= (\frac{2\pi}{\ell_P})^3 \\ &= 8\pi^3/(1.616229 \times 10^{-35})^3 \\ &\approx 10^{117}. \end{aligned} \tag{29}$$

This will reduce the discrepancy between the observed and predicted values of ρ_{vac} from 10^120 into $\approx 10^3$ only. The remaining small discrepancy could now be explained according to quintessence models as summarized in section 6.2 above, but a more accurate calculation is needed first because all the current methods to calculate ρ_{vac} are approximate. This will also depend on the actual cutoff value, which resembles the quanta of space that could be different from Planck length.

Actually, according to the initial predictions, the quanta of space could potentially take any arbitrarily small value, depending on the degrees of freedom or number of dimensions as we discussed in section 3.1 above. This value would be larger in empty space and smaller in dense materials, and thus it would very small in the early history of the universe. That is why quintessence models are promising in this regard, since they are formulated on scalar fields with time-dependent equation of state, while Einstein's cosmological constant postulate is characterized with fixed energy density, which means that the ratio of energy pressure p_q to mass density ρ_q is: $w_q = p_q/\rho_q = -1$, while this equation of state for quintessence varies with time. It was found in 2004 that the equation of state had possibly crossed the cosmological constant boundary from above to below. This no-go theorem is called the Quintom scenario and it requires at least two degrees of freedom for dark energy models [63].

7. Conclusion

The postulate of the Duality of Time, and the resulting perpetual creation of space and matter in the inner levels of time, provide simple solutions to various major problems in physics and cosmology.

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The problem in most current models is that they suppose the continuous existence of matter in background space, which makes it impossible to think of fully quantized space and time. By introducing the inner levels of time, we obtained a self-contained, granular and dynamic space-time without any background.

We showed how this concept of the inner levels of time lead to the same principles of Special and General Relativity together, but with a granular space-time structure that could explain Quantum Gravity as well as the other fundamental interactions in terms of its complex geometry.

Therefore, the Duality of Time does not replace or contradict any exiting theory, but it exposes a deeper understanding of the duality nature of time, which lead to a genuinely complex (hyperbolic) time that can be expressed as a metaphysical (non-background) Euclidean space with (+, +, +, +) signature, which approximates to the non-Euclidean Minkowski space (+, +, +, -) on the physical level (when we suppose a continuous space background).

In addition to its inherent arrow of time, the postulate of the Duality of Time also provides realistic explanation of various quantum mechanical phenomena, but in this article we could not discuss this topic at length. We showed, however, that this model is more realistic than eternal inflation, because it leads to homogeneous and finite universe, and the resulting dynamic quintessence reduces the cosmological constant discrepancy by at least 117 orders of magnitude.

In order to develop this model further we need to examine more closely the flow of inner time and how it dynamically creates the three spatial dimensions. There are strong indications that this will explain the hierarchy problem since gravity is exhausted in the volume whereas other forces are exhibited in lower dimensions. These and other issues are investigated in another forthcoming article [39].

Appendix Appendix A:

Starting from equation 6 above:

$$E = \int (v^2 dm + mvdv), \tag{30}$$

Now we can find dm/dv by differentiating $m = \gamma m_0 = m_0/\sqrt{1 - v^2/c^2}$:

$$\begin{aligned} \frac{dm}{dv} &= \frac{m_0 v/c^2}{\sqrt[3/2]{1 - v^2/c^2}} \\ &= \frac{mv/c^2}{1 - v^2/c^2} \\ &= \frac{mv}{c^2 - v^2} \end{aligned} \tag{31}$$

From this equation we find:

$$mvdv = c^2 dm - v^2 dm \tag{32}$$

Replacing in equation 30 we get:

$$\begin{aligned} E &= \int (v^2 dm + c^2 dm - v^2 dm) \\ &= \int c^2 dm \\ &= mc^2 \end{aligned} \tag{33}$$

This method, however, can not be considered a mathematical validation of the mass-energy equivalence relation $E = mc^2$, because the starting equation $m = \gamma m_0$ is not derived by any other fundamental method from Relativity, other than being analogous to the equations of time dilation and length contraction: $t = \gamma t_0$, $L = L_0/\gamma$.

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Using the same equation $m = \gamma m_0$ with $E = mc^2$; thus: $E = m_0c^2/\sqrt{1 - v^2/c^2}$, we can also derive the relativistic energy-momentum relation, by squaring and doing some modifications:

$$\begin{aligned} E^2 &= \frac{m_0^2c^4}{1 - v^2/c^2} \\ &= \frac{m_0^2c^4}{1 - v^2/c^2} + \frac{m_0^2c^2v^2}{1 - v^2/c^2} - \frac{m_0^2c^2v^2}{1 - v^2/c^2} \end{aligned} \tag{34}$$

From this equation we get:

$$E^2 = c^2\frac{m_0^2v^2}{1 - v^2/c^2} + \frac{m_0^2c^4 - m_0^2c^2v^2}{1 - v^2/c^2} \tag{35}$$

Thus:

$$\begin{aligned} E^2 &= p^2c^2 + m_0^2c^2\frac{c^2 - v^2}{1 - v^2/c^2} \\ &= (pc)^2 + (m_0c^2)^2 \end{aligned} \tag{36}$$

Or:

$$\begin{aligned} E &= \sqrt{(pc)^2 + (m_0c^2)^2} \\ &= c\sqrt{(mv)^2 + (m_0c)^2} \end{aligned} \tag{37}$$

Again, since this derivation relies originally on the equation: $m = \gamma m_0$, it can not be considered a mathematical validation of the mass-energy equivalence relation.

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