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Universal Theory of General Invariance

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Abstract Through the introduction of a new principle of physics, we extend quantum mechanics by proposing three additional postulates. With them we construct a quantum theory where gravitation emerges from the thermodynamics of an entangled vacuum. This new quantum theory reproduces the observations of the ΛCDM model of cosmology, predicting the existence of massive vacua $M_{\text{on}} = \sqrt{\frac{\hbar c}{G}}$ and $M_{\text{off}} = \sqrt{\frac{\Lambda^2 \hbar^3 G}{c^5}}$. Finally, we propose an experiment for the formers direct detection.

Keywords emergent gravity · entanglement · holography · dark energy · dark matter · inflation · thermodynamic vacuum · emdrive

1 Motivation

At the largest length scales, the universe is described by a differentiable manifold whose dynamics are governed by Einstein's Field Equations. At the intermediate length scales, it's described by a smooth manifold, governed by Hamilton's equations of motion. At very small scales, the universe can no longer be described by a manifold. Rather at these scales, it's described by a Hilbert space whose dynamics are governed by the Schrodinger equation.

On the one hand, the universe is commutative at large scales while on the other hand, non-commutative at small scales. At the smallest length scale (the Planck scale) is spacetime commutative or non-commutative? If it's the former, there exists a deep asymmetry where there's a special scale in which the universe is describable only by a Hilbert space. If it's the latter, then how is spacetime smooth at large scales? This incongruity is very puzzling. How

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can the commutative structure of a smooth manifold be reconciled with the non-commutative structure of a Hilbert space?

During the early days of Quantum Field Theory (QFT), Heisenberg suggested the use of a non-commutative spacetime geometry at sufficiently small scales to introduce an effective UV cutoff [1]. In 1947, Synder formalized this idea by introducing a formulation of quantum mechanics on a non-commutative spacetime [2]. His reasoning was inspired by Heisenberg's commutation relations $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, where neither momentum nor position variables can be considered points belonging to a smooth manifold. By extending this concept to spacetime, Synder thought it possible to tame the divergences that plagued QFT.

Snyder's work was largely ignored because of the theoretical and experimental successes of renormalization techniques. It was, however, unknown at the time that these techniques would fail when taking gravitation into account. Von Neumann further formalized the idea of a non-commutative phase space by introducing topological spaces whose commutative C^*-algebras of functions are replaced by non-commutative algebras [3]. Rather recently, a low energy limit of string theory proposed that the commutation relations $[\hat{x}_i, \hat{x}_j] = 0$ be replaced by $[\hat{x}_i, \hat{x}_j] = i\theta_{ij}$, where θ_{ij} is a real-valued antisymmetric matrix with units of length squared [4]. Since then, non-commutative spacetime geometries have received significant attention.

The motivation for modifying the commutation relations was derived from the pathologies (UV and IR divergences) of QFT. The same reasoning can be applied to resolve the pathologies of General Relativity (GR) from within an extended quantum theory. Therefore, by modifying the commutation relations between spatial *positions* and *directions*, it's possible to introduce an effective UV/IR cutoff.

2 Fundamental Assumptions

Taking the above ideas a step further, we make the following fundamental assumptions:

1) Nature will always choose the structure of least description;
2) the Laws of Physics are scale-invariant and
3) do not allow infinite quantities.

Assumption 1) tells us if Nature must decide between a Hilbert space and a manifold, She will choose the simplest of the two. From the constructivist perspective, She selects the structure with the fewest relations required to construct Her physics. Assumption 2) tells us this structure is invariant with respect to scale transformations, implying one of these structures is an approximation of the other at some sufficient scale. Assumptions 1) and 2), together with 3), strongly suggests She will choose a locally finite-dimensional Hilbert space that's approximated by a smooth manifold at sufficiently large scales. With these assumptions in hand, we propose a new principle of physics.

The Principle of General Invariance: *The laws of physics are scale-invariant within a fundamental UV/IR regime.*

Observational Motivation As our telescopes get more advanced and the surface of last scattering becomes more resolute, the primordial matter-spectrum appears to be converging to two numbers that follow a power-law of the form $\mathcal{P}(k) = Ak^n$ [5, 6]. Fluctuations of this surface appear to *only* contain scale-invariant Gaussian noise. This signal extends far beyond the surface of last scattering. Across all length and time scales, power-law phenomena are present. They appear in most (if not all) electronic devices [7–11], biological systems [12–15], sociological and psychological systems [16–20], and even the word frequency of this note [21]. Is the origin of power-law phenomena intimately related to structure formation in the early universe? Could the unreasonable ubiquity (or rather the inescapability) of the phenomena be a clue to the origins of spacetime?

3 Aim

Since Heisenberg's Matrix Mechanics formulation, there has been eight formulations of quantum mechanics (all of which produce the same predictions) with a dozen or so ways of interpreting them. This suggests there exists a more fundamental underlying theory; a quantum theory with additional postulates, containing entanglement degrees of freedom with a scale-invariant metric. The goal of this paper is to construct an extended quantum theory by 1) reformulating the $[\hat{x}_i, \hat{x}_j] = 0$ commutation relations such that they explicitly encode the entanglement degrees of freedom, 2) replace the commutative relations *between* the directions of 3-space with the noncommutative Clifford algebra, and 3) construct a locally finite-dimensional Hilbert space with a scale-invariant metric on the entanglement structure. In doing so, we aim to demonstrate emergent spacetime and thus gravitation from quantum entanglement [22, 23].

From a quantum information perspective, our toy model is a quantum data structure. Our approach, therefore, carries a familial resemblance to the thought that our universe is a quantum computer. Rather than proposing the universe is a quantum computer [24–26], we propose the universe is a quantum computer program. Spacetime from this perspective is then equivalent to the way in which all degrees of freedom are configured in a computable background independent structure.

The ultimate goal of this work is to *explicitly* derive this structure from three additional postulates, such that it is not only entirely consistent with the ΛCDM model of cosmology but also produces new predictions beyond the Standard Model.

4 Postulates

I. Two points in spacetime are entangled if they don't commute and are not entangled if they do commute.

$$[\hat{x}_i, \hat{x}_j] = il_p^2 \hat{E}_{ij} \tag{1}$$

where

$$\hat{E} = \begin{cases} 1, \ \hat{x}_i \text{ is adjacent to } \hat{x}_j \\ 0, \qquad \text{else.} \end{cases} \tag{2}$$

Postulate I. proposes the entanglement degrees of freedom are indistinguishable from the commutation relations of spacetime. When two points are entangled, they no longer commute and therefore no longer belong to a smooth manifold. Since at large scales spacetime is commutative, we immediately deduce that as the universe expands, $\hat{E}$ becomes sparse and sufficiently approximated by a smooth manifold. Extrapolating from this implies the universe began in a state of maximal entanglement, where all points are entangled with one another. The arrow of time is then identified with $\hat{E}$ flowing from a dense to a sparse state.

If the indices of $[\hat{x}_i, \hat{x}_j]$ are continuous, then spacetime requires an *uncountably* infinite number of degrees of freedom to specify. Therefore, diagonalizing $\hat{E}$ would require an infinitely powerful computer. We regard such an apparatus as unphysical, implying our 'spacetime' Hilbert space is locally finite-dimensional.

II. For every quantum mechanical operator $\hat{\mathbf{A}}$ there exists an underlying graph $\mathcal{G}$. When spacetime is in superposition, the vertices $\mathcal{V} = \{e_1, e_2\}$ of $\mathcal{G}$ are elements of the Clifford algebra $Cl(2)$. The commutation relations are given as $e_{ii}^2 = 1$ and $e_i e_j = -e_j e_i$ with imaginary constant $i_2 = e_1 e_2$. When spacetime is not in superposition, the vertices $\mathcal{V} = \{e_1, e_2, e_3\}$ of $\mathcal{G}$ are elements of the Clifford algebra $Cl(3)$, with imaginary constant $i_3 = e_1 e_2 e_3$.

Postulate II. proposes Holography [27–38] is indistinguishable from a lossless compression of the degrees of freedom in 3-space to 2-space. In our construction, the vertices of $\hat{E}$ are sent to the graphs $\mathcal{G}$ such that they contain the position operators $(\hat{x}, \hat{y}, \hat{z})$. The vertices of this graph are the algebraic constants of $Cl(3)$, with commutation relations *between* the $\hat{x}$, $\hat{y}$, and $\hat{z}$ directions. Since this relation doesn't commute, it's impossible to measure the precise position of a particle with respect to more than one axis.

III. The operator-valued power-spectrum $\mathcal{P}[\hat{\mathbf{A}}(\mathcal{G})] \equiv |\mathcal{G}\rangle$, generates a Hilbert space $\mathcal{H}$ of dimension $|\mathcal{V}|$ with metric eigenfunction $|g\rangle = |g_+\rangle + |g_-\rangle$.

Postulate III. proposes a way to take $\mathcal{G}$ and construct a locally finite-dimensional Hilbert space with a scale-invariant metric. Presumably, this would be sufficient to explain structure formation in the early universe.

In GR, the field equation $G^a_b = \kappa T^a_b$ is solved for the metric tensor g_{ab} for a given source T^a_b. Similarly, the field equation proposed below is solved for the metric eigenfunction $|g\rangle = |g_+\rangle + |g_-\rangle$ given the source $|\mathcal{G}\rangle$.

5 The Entanglement Field Equation

To construct a field equation for the entanglement degrees of freedom we look to Postulate III. As a small example, consider a traceless 2×2 adjacency matrix with eigenvalues $\lambda_1 = -\lambda_2$. Disregarding the coefficient matrix gives

$$\mathcal{P}(\lambda) = \lambda^{\theta/\pi}(1 + e^{i\theta}). \tag{3}$$

When $n(= \theta/\pi)$ is integral the above is a real number. When n is non-integral, the above is a complex number. When $\mathcal{P}(\lambda)$ is multiplied by its coefficient matrix, the entries become the number of walks of length n from vertices i to j. Therefore, the continuation of n assigns the complex phase $e^{i\theta}$ to a continuum of walks specified by the adjacency matrix. This analytic continuation allows the construction a Hilbert space that's finite in its spatial degrees of freedom $\mathcal{V}$ and infinite in its rotational degrees of freedom θ. In general, the entanglement field equation is defined as

$$|\mathcal{G}\rangle = \mathbf{L}^+ |g_+\rangle + \mathbf{L}^- |g_-\rangle \tag{4}$$

where the plus and minus super and subscripts denote a summation over the positive and negative eigenvalues of $\mathcal{G}$. The left-hand side of this equation describes the topology of a patch of spacetime, where the right-hand side describes all possible ways of moving through this spacetime. We derive the above and demonstrate with proof $\langle g|g\rangle < \infty$ [1].

6 Information-theoretic Operators

To solve the entanglement field equation, we look to Postulates I & II. In the $Cl(2)$ basis there is only one non-empty graph to choose from. Likewise, in the $Cl(3)$ basis there are only five non-empty graphs to choose from. We demonstrate with proof [2] that N independent quantum harmonic oscillators in the $Cl(3)$ basis takes the vertices of $\hat{E}$ and sends them to

$$\begin{aligned} \hat{G} &= \frac{1}{2m_{\text{vacc}}}\hat{\mathbf{P}} \circ \hat{\mathbf{P}} + \frac{1}{2}m_{\text{vacc}}\omega^2\hat{\mathbf{Q}} \circ \hat{\mathbf{Q}} \\ &= \frac{1}{2m_{\text{vacc}}}\hat{\mathbf{p}} \otimes \hat{\mathbf{p}} + \frac{1}{2}m_{\text{vacc}}\omega^2\hat{\mathbf{q}} \otimes \hat{\mathbf{q}} \end{aligned} \tag{5}$$

where

$$\hat{\mathbf{p}} = -e_1 e_2 e_3 \hbar \nabla, \quad \hat{\mathbf{q}} = \hat{x} e_1 + \hat{y} e_2 + \hat{z} e_3 \tag{6}$$

with the Hamiltonian

$$\hat{H}_{\text{vacc}} = \operatorname{tr} \hat{G} \tag{7}$$

and with m_{vacc} unknown. The 'atoms' of the decompressed spacetime have the graph-theoretic bit representations

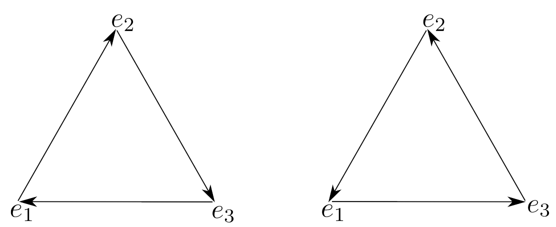

Fig. 1 The vertices of the graphs are elements of the Clifford algebra $Cl(3)$. The orientations of the edges are representative of a measured spin value of +1/2 and -1/2.

with the background independent operator-valued adjacency matrix

$$\hat{\mathbf{Q}} = \begin{bmatrix} \hat{x}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{y}e_2}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{z}e_3}e_1 \\ \sqrt{\hat{y}e_2}\sqrt{\hat{x}e_1}e_2 & \hat{y}e_2 & \sqrt{\hat{y}e_2}\sqrt{\hat{z}e_3}e_2 \\ \sqrt{\hat{z}e_3}\sqrt{\hat{x}e_1}e_3 & \sqrt{\hat{z}e_3}\sqrt{\hat{y}e_2}e_3 & \hat{z}e_3 \end{bmatrix}. \tag{8}$$

In the $Cl(2)$ basis,

$$\hat{G} = \frac{1}{2m_{\text{vacc}}} \hat{\mathbf{p}} \cdot \boldsymbol{\sigma} + \frac{1}{2} m_{\text{vacc}} \omega^2 \hat{\mathbf{q}} \cdot \boldsymbol{\sigma} \tag{9}$$

where

$$\hat{\mathbf{p}} = -e_1 e_2 \hbar \nabla, \quad \hat{\mathbf{q}} = \hat{x} e_1 + \hat{y} e_2 + \hat{z} e_3, \quad \boldsymbol{\sigma} = \sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3 \tag{10}$$

with a minimized Hamiltonian

$$\hat{H}_{\text{vacc}} = \operatorname{tr} \hat{G} = 0. \tag{11}$$

The 'atoms' of this compressed spacetime have the qubit representation

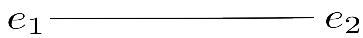

Fig. 2 The vertices of this graph are elements of the Clifford algebra $Cl(2)$. The edge of this graph has no orientation, representing a superposition of the spin +1/2 and -1/2 values.

with the operator-valued adjacency matrix

$$\hat{\mathbf{q}} \cdot \boldsymbol{\sigma} = \begin{bmatrix} -\hat{z} & \hat{x} + e_2 e_1 \hat{y} \\ \hat{x} + e_1 e_2 \hat{y} & \hat{z} \end{bmatrix}. \tag{12}$$

7 Emergence of Time

Since in the $Cl(3)$ basis spin-1/2 is stored by the orientations of the edges of the graph, $(\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}, \hat{\mathbf{q}} \otimes \hat{\mathbf{q}})$ is regarded as *bitspace*. Since in the $Cl(2)$ basis spin-1/2 is stored by both the vertices and edges, $(\hat{\mathbf{p}} \cdot \boldsymbol{\sigma}, \hat{\mathbf{q}} \cdot \boldsymbol{\sigma})$ is regarded as *qubitspace*. Bitspace carries two copies of $SO(3)$ and qubitspace carries two copies $SU(2)$. The *qubitmorphisms* $(\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}, \hat{\mathbf{q}} \otimes \hat{\mathbf{q}}) \longrightarrow (\hat{\mathbf{p}} \cdot \boldsymbol{\sigma}, \hat{\mathbf{q}} \cdot \boldsymbol{\sigma})$, together with the *bitmorphisms* $(\hat{\mathbf{p}} \cdot \boldsymbol{\sigma}, \hat{\mathbf{q}} \cdot \boldsymbol{\sigma}) \longrightarrow (\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}, \hat{\mathbf{q}} \otimes \hat{\mathbf{q}})$, generates the

$$\begin{aligned} &SU(2) \times SU(2) \longrightarrow SO(4) \\ &SO(4) \longrightarrow SO(3) \times SO(3) \end{aligned} \tag{13}$$

representation of 4-dimensional Euclidean space $\mathbb{R}^4$, thereby rendering this extended quantum theory Lorentz invariant and therefore relativistic. This extenstion can therefore be considered a *finite* QFT. From Postulate I., spacetime is constructed by entangling N independent quantum harmonic oscillators (gluing the algebraic constants of respective graphs) in the $Cl(3)$ and $Cl(2)$ bases. Therefore, for arbitrary N our spacetime is written down in $Cl(3)$ as a $3N \times 3N$ block matrix and in $Cl(2)$ as a $2N \times 2N$ block matrix.

Since the off-diagonal elements of $\hat{G}$ encode all possible configurations of the entanglement structure $\hat{E}$, and since the diagonal elements store the total energy of the system, for all configurations of the entanglement structure, the total energy of the system remains constant

$$\forall \delta \hat{E}, \quad \delta \operatorname{tr} \hat{G} = 0 \tag{14}$$

and therefore

$$\delta \hat{E} \cong t. \tag{15}$$

This can be considered a *strict* emergence of time (an isomorphism between changes in $[\hat{x}_i, \hat{x}_j]$ and t). This, however, only demonstrates a local conservation of energy in the $Cl(3)$ basis (energy is automatically conserved in the $Cl(2)$ basis). Transitions from $Cl(2)$ to $Cl(3)$ have the Hamiltonian representation $\hat{H}_{\text{vacc}} |g\rangle \longrightarrow \hat{H} |g\rangle$, yielding $0 \longrightarrow E |g\rangle$. This is a gross violation of a global conservation of energy. We deem this unphysical and demand energy to be conserved globally.

Endowing the $Cl(3)$ graph with a hyper-edge $\mathcal{E}$, allows a one-to-one correspondence between the fundamental charges

$$+1 \longleftrightarrow \{e_1, e_2, e_3\},\ +1/3 \longleftrightarrow \{e_1, e_3, e_2\},\ +2/3 \longleftrightarrow \{e_2, e_1, e_3\}$$
$$-1/3 \longleftrightarrow \{e_2, e_3, e_1\},\ -2/3 \longleftrightarrow \{e_3, e_1, e_2\},\ -1 \longleftrightarrow \{e_3, e_2, e_1\},\ 0 \longleftrightarrow \{\emptyset\}.$$

$\hat{G}$ and therefore $\hat{H}_{\text{vacc}}$ are invariant under spin, charge, and parity conjugation (SCP symmetry)

$$\begin{aligned} \hat{\mathbf{Q}} &\longrightarrow \hat{\mathbf{Q}}^\top \\ \text{off-diag } \hat{\mathbf{Q}} &\longrightarrow -\text{off-diag } \hat{\mathbf{Q}} \\ \text{diag } \hat{\mathbf{Q}} &\longrightarrow -\text{diag } \hat{\mathbf{Q}}, \end{aligned} \tag{16}$$

which can all be conveniently expressed as

$$\hat{\mathbf{Q}} \longrightarrow -\hat{\mathbf{Q}}^\top. \tag{17}$$

Since the actions on $\hat{E}$ are the turning on and off of entanglement between the qubits and bits that hold spacetime together, and since these actions are indistinguishable from time, reversing these actions is equivalent to reversing time

$$\delta\hat{E} \longrightarrow \delta^{-1}\hat{E}. \tag{18}$$

The *bitmorphism* takes every point on a symplectic manifold and sends it to a graph in $\mathcal{G}Cl(3)$. Thus, $\mathcal{G}Cl(3)$ is an infinite-dimensional Hilbert space invariant under the actions of $SO(3)$. The *qubitmorphism* takes every graph in $\mathcal{G}Cl(3)$ and sends it to a graph in $\mathcal{G}Cl(2)$; a 2-dimensional Hilbert space invariant under the actions of $SU(2)$. For energy to be conserved globally, transitions between $\mathcal{G}Cl(3)$ and $\mathcal{G}Cl(2)$ must be one-to-one and onto. Thus, $\mathcal{G}Cl(3)$ must be 3-dimensional. This implies $\mathcal{G}Cl(2)$ is the bounded region of $\mathcal{G}Cl(3)$.

8 Emergence of Measurement

.Postulate III. allows the construction of a locally finite-dimensional Hilbert space from the entanglement structure, if and only if the information-theoretic operators are traceless. Since we demand degrees of freedom can neither be created nor destroyed, the diagonal elements of $\hat{\mathbf{Q}}$ and $\hat{\mathbf{P}}$ must be traced out and stored; not as operator-valued numbers but as real-valued numbers. This, then, implies transitions from $\mathcal{G}Cl(2)$ to $\mathcal{G}Cl(3)$ are quantum-to-classical, suggesting $\mathbf{p}$ and $\mathbf{q}$ are stored in the observer's memory. From this perspective, the collapse of the wavefunction can be considered a *flow* of qubits in $\mathcal{G}Cl(2)$

to bits in $\mathcal{G}Cl(3)$. This quantum-classical phase transition is therefore indistinguishable from the act of observation, suggesting the observer is an arbitrary information processing and storage device.

In 1936, two formal systems of computation were developed independently of one another, Church's λ-calculus [39–41] and the Turing machine [42–44]. Shortly after their development, Church and Turing proved both of these systems formally equivalent, in what is now known as the Church-Turing thesis [39–44]. In 1958, Curry found a remarkable connection between the typed combinators of the λ-calculus and representations of proofs in first-order logic [45–47]. In 1969, Howard discovered a correspondence between deductive proofs and certain typed λ-terms [47]. The Curry-Howard correspondence draws a deep connection between mathematical proofs and computer programs. Inspired by the λ-calculus, we propose the following *measurement calculus* as a bookkeeping device for the observer's memory. The flow from $\mathcal{G}Cl(2)$ to $\mathcal{G}Cl(3)$ is abstracted as

$$\mathcal{O}.(\hat{\mathbf{q}} \cdot \boldsymbol{\sigma})(\hat{\mathbf{Q}}) = \mathcal{O}.\mathbf{q} \tag{19}$$

which yields the mapping [1]

$$\mathcal{O} : \hat{\mathbf{q}} \cdot \boldsymbol{\sigma} \longrightarrow \left\{ \hat{\mathbf{Q}} \;\middle|\; \mathcal{O} \supset \mathbf{q} \right\}. \tag{20}$$

Here, an observer $\mathcal{O}$ takes the position-qubit $\hat{\mathbf{q}} \cdot \boldsymbol{\sigma}$ and transforms it to the position-bit $\hat{\mathbf{Q}}$ by taking its position data $\mathbf{q}$ to memory. Stated another way, qubits stored in $\mathcal{G}Cl(2)$ are decompressed to bits stored in $\mathcal{G}Cl(3)$. This 3-dimensional Hilbert space is where the observer's memory resides, suggesting the only physically meaningful degrees of freedom are precisely the degrees of freedom that the observer stores. Effectively, the role of the observer is to move the system from its zero-point energy state in $\mathcal{G}Cl(2)$ to an excited state in $\mathcal{G}Cl(3)$.

Our construction doesn't require the existence of an observer. Devoid of any observers, the universe remains frozen in its zero-point energy state, retaining dynamics through changes in its entanglement structure. Time, however, only becomes meaningful in $\mathcal{G}Cl(3)$, explaining why the collapse of the wavefunction appears instantaneous.

Since for every position-qubit, there are two copies of the position-bit, and since they both go to memory, implies there exists a conjugate observer in a mirror universe. In this conjugate universe all spin up measurements become spin down measurements, all matter is replaced with antimatter, the positions of all particles are reflected, and time is reversed. Therefore, the total energy

[1] *notation* The left-hand side of the measurement calculus $\mathcal{O}.(a)(b)$, denotes a transformation of a into b (in a black box). The right-hand side $\mathcal{O}.(c)$, denotes the information traced out from b ($c = \operatorname{tr} b$) and sent to the observer's memory. The internal configuration of the observer is a sub-configuration of $\hat{E}$ in $\mathcal{G}Cl(2)$.

of the universe and its conjugate is exactly zero. From the perspective of the observer and their conjugate, both universes evolve forward in time, forever remaining causally disconnected from one another.

The measurement process is then characterized by $\mathcal{O}$ measuring along the e_i direction of $\mathcal{G}Cl(2)$ and $\mathcal{O}^\dagger$ measuring along the e_j direction of $\mathcal{G}Cl(2)$. Upon measurement, $\mathcal{O}$ observes $\hat{\mathbf{Q}}$ and $\mathcal{O}^\dagger$ observes $-\hat{\mathbf{Q}}^\top$. The collapse of ψ then specifies the state of $\mathcal{O}$'s memory and the collapse of $\psi^\dagger$ specifies the state of $\mathcal{O}^\dagger$'s memory. The $\mathcal{O}$-calculus for this "coupleverse"

$$\mathcal{O}.\mathbf{q} = \mathcal{O}^\dagger.\mathbf{q} \tag{21}$$

yields the one-to-one mapping

$$\mathcal{O}, \mathcal{O}^\dagger : \hat{\mathbf{q}} \cdot \boldsymbol{\sigma} \longrightarrow \left\{ \hat{\mathbf{Q}} \circ \hat{\mathbf{Q}} \;\middle|\; \mathcal{O}, \mathcal{O}^\dagger \supset \mathbf{q}, \mathbf{q} \right\}. \tag{22}$$

Here, an observer $\mathcal{O}$ and $\mathcal{O}^\dagger$ with memory specified by the collapse of ψ and $\psi^\dagger$, measure along opposite directions in $\mathcal{G}Cl(2)$, decompressing the position-qubit $\hat{\mathbf{q}} \cdot \boldsymbol{\sigma}$ to the position-bits $\hat{\mathbf{Q}} \circ \hat{\mathbf{Q}}$, taking the diagonal of each position-bit to $\mathcal{O}$ and $\mathcal{O}^\dagger$'s memory. Prior to measurement, the contents of the observers memory is specified by the expectation value

$$\langle \hat{q} \rangle = \langle \psi | \, \hat{q} \, | \psi \rangle \tag{23}$$

with the probability of $\mathcal{O}$ and $\mathcal{O}^\dagger$ having a non-empty memory of

$$\langle \psi | \psi \rangle = 1. \tag{24}$$

A transition from $\mathcal{G}Cl(2)$ to $\mathcal{G}Cl(3)$ is a lossless decompression of the spatial degrees of freedom stored by $\mathcal{G}Cl(2)$. Since this transition is one-to-one it has an inverse. Therefore, a transition from $\mathcal{G}Cl(3)$ to $\mathcal{G}Cl(2)$ is a lossless compression of spatial degrees of freedom. This, in turn, implies the collapse of the wavefunction is reversible. The inverse transition takes the measured quantities $\mathbf{q}$ and restores the initial state $\hat{\mathbf{q}} \cdot \boldsymbol{\sigma}$. The inverse $\mathcal{O}$-calculus

$$\mathcal{O}^{-1}.(\mathbf{q})(\hat{\mathbf{Q}}) = \mathcal{O}^{-1}.(\hat{\mathbf{q}} \cdot \boldsymbol{\sigma}) \tag{25}$$

yields the one-to-one mapping

$$\mathcal{O}^{-1} : \mathbf{q}, \mathbf{q} \longrightarrow \left\{ \hat{\mathbf{Q}} \circ \hat{\mathbf{Q}} \;\middle|\; \mathcal{O}^{-1} \supset \hat{\mathbf{q}} \cdot \boldsymbol{\sigma} \right\}. \tag{26}$$

Here the position data is taken from $\mathcal{O}$ and $\mathcal{O}^\dagger$'s memory, transformed back to the position-bits, compressed to the position-qubits, and sent to $\mathcal{O}^{-1}$'s memory, uncollapsing the wavefunction. Since $\mathcal{G}Cl(2)$ is the bounded region of $\mathcal{G}Cl(3)$, $\mathcal{O}$'s memory is completely encoded in the bulk and $\mathcal{O}^{-1}$'s memory is completely encoded on the boundary.

9 Emergence of Spacetime

Suppose we prepare $\hat{E}$ as a false singularity. This is done by entangling all points of spacetime with all other points, e.g. $\hat{E}$ is a complete graph. Now suppose we cut this graph into two pieces, labeling them G and H. The length of $|g\rangle$ is proportional to the number of eigenvalues and therefore the number of vertices of G and H. If the graph is cut into a small and large piece, they will be far away from each other. If the graph is cut into two pieces of similar size, they will be close together. Therefore, dilations are generated by cutting the graph such that $|V(G)|$ is strictly decreasing with respect to $|V(H)|$ and contractions are generated if $|V(G)|$ is approaching $|V(H)|$.

Suppose we now treat $\hat{E}$ as a finite-state machine. If at every iteration 2^N entanglement degrees of freedom are turned off, such that $|V(G)|$ is strictly decreasing with respect to $|V(G)|$, then observers in this patch of spacetime will experience an acceleration away from each other. If at every iteration 2^N entanglement degrees of freedom are turned on, such that $|V(G)|$ is approaching $|V(H)|$, then observers in this patch of spacetime will experience an acceleration towards each other. Therefore, from the equivalence principle, the rate of entanglement sources gravitation and the rate of disentanglement sources anti-gravitation.

9.1 Solution to the Entanglement Field Equation

We demonstrate with proof [3] that for the position-qubit in $\mathcal{G}Cl(2)$, the metric eigenfunction at the Planck scale is

$$g(\delta) = l_p^{\delta/2\pi}(1 + e^{i_2\delta/2}) \tag{27}$$

where δ is a free parameter, l_p is the Planck length, and i_2 is the imaginary constant in $Cl(2)$. Choosing

$$\delta_{\text{off}} = -\frac{\pi k_B c^3}{G\hbar} \quad \text{and} \quad \delta_{\text{on}} = -\pi k_B \Lambda \tag{28}$$

gives the holographic entropies

$$S_{\text{off}} = -\frac{k_B c^3}{4G\hbar} \ln\left(\frac{G\hbar}{c^3}\right) \quad \text{and} \quad S_{\text{on}} = -\frac{k_B \Lambda}{4} \ln\left(\frac{G\hbar}{c^3}\right) \tag{29}$$

which are interpreted as the entropy of disentanglement and entanglement, respectively. The Bekenstien-Hawking entropy [27–38, 48–58] is then derived as the total entropy of the entanglement structure

$$S_{BH} = \sum_{(i,j)\in\hat{E}} S_{\text{on}} + S_{\text{off}}. \tag{30}$$

$g(\delta_{\text{on}})$ and $g(\delta_{\text{off}})$ become a coupled system of equations of state for $\hat{E}$, depending on pressure, area, and temperature

$$g_{21} = g(P_{21}, A_p, T) = A_p^{-k_B \Lambda/4}\left(1 + \exp\left\{\frac{-l_p e_2 e_1 P_{21}}{3\pi^2 T}\right\}\right) \tag{31}$$

$$g_{12} = g(P_{12}, A_p, T) = A_p^{-k_B c^3/4G\hbar}\left(1 + \exp\left\{\frac{-l_p e_1 e_2 P_{12}}{3\pi^2 T}\right\}\right). \tag{32}$$

Here, T is the Unruh temperature [59–61], A_p is the Planck area, e_1 and e_2 are the spatial directions of $\mathcal{G}Cl(2)$, and P_{12} and P_{21} are the pressures

$$P_{21} = \frac{\Lambda\hbar a}{l_p c} \quad \text{and} \quad P_{12} = -\frac{c^2 a}{l_p G}. \tag{33}$$

With the pressure-force relation and Newton's 2nd Law, the vacuum masses

$$M_{\text{on}} = \sqrt{\frac{\hbar c}{G}} = 2.176 \times 10^{-8}\text{kg}, \tag{34}$$

and

$$M_{\text{off}} = \sqrt{\frac{\Lambda^2 \hbar^3 G}{c^5}} = 5.68 \times 10^{-130}\text{kg} \tag{35}$$

are obtained.

Formally, these vacua are irreducible representations of the entanglement degrees of freedom (the atoms of spacetime) stored by $SU(2)$. Since M_{off} is asymptotically close to zero, it exerts an enormous negative pressure ($w = -1$) proportional to the area density $\rho_{12} = c^2/l_p G = 10^{61}\text{kg/m}^2$ on $M_{\text{on}}(= M_p)$; which pushes back with an extremely small positive pressure ($w \approx 0$), proportional to $\rho_{21} = \Lambda\hbar/l_p c = 10^{-60}\text{kg/m}^2$. The ratio of the area densities of M_{on} and M_{off}

$$\frac{\rho_{21}}{\rho_{12}} = \Lambda l_p^2 \tag{36}$$

simultaneously provides a simple explanation for both the accelerating expansion of the universe and its missing mass. Since $\hat{E}$ evolves from a dense to a sparse state, our toy universe begins in an M_{on} dominated epoch proportional to the entanglement entropy

$$S_{\text{on}} = -\frac{k_B \Lambda}{4}\ln\left(\frac{G\hbar}{c^3}\right) \quad \text{at time} \quad t_{\text{on}} = \sqrt{\frac{G\hbar}{c^5}} \tag{37}$$

$$\text{with temperature} \quad T_{\text{on}} = \sqrt{\frac{\hbar c^5}{G k_B^2}} \tag{38}$$

and evolves towards an M_{off} dominated epoch proportional to the disentanglement entropy

$$S_{\text{off}} = -\frac{k_B c^3}{4G\hbar} \ln\left(\frac{G\hbar}{c^3}\right) \quad \text{halting at} \quad t_{\text{off}} = \sqrt{\frac{c}{\Lambda^2 G\hbar}} \tag{39}$$

$$\text{with temperature} \quad T_{\text{off}} = \sqrt{\frac{\Lambda^2 \hbar^3 G}{k_B^2 c}}. \tag{40}$$

As entropy production increases so too does production of M_{off}, predicting the accelerating expansion becomes more pronounced with the arrow of time.

This thermodynamic model of the vacuum tells us nothing about the initial configuration of $\hat{E}$, or for that matter any particular configuration of $\hat{E}$, only that it must evolve from a dense to a sparse state. If we recall, for the universe to be sufficiently approximated by a smooth manifold at large scales, then $\hat{E}$ must have evolved from a dense to a sparse state.

In the simplest inflationary models, the universe doubled in size at least 60 times between 10^{-36} and 10^{-33} seconds [62, 63]. In our toy model, the inflationary epoch corresponds to a quantum-classical phase transition, where 2^{60} entanglement degrees of freedom were turned off at the Planck time t_{on}. This Bang of disentanglement explains why GR works so well at large scales. In fact, GR is implicit in the architecture of this model. This is seen from the fact that the only quantities required to calculate curvature are distances, angles, and their rates. Since for any configuration of $\hat{E}$ we can calculate distances and angles with the metric eigenfunction, and since flipping entanglement on and off emerges the rate of time, gravitation manifests itself as the curvature of spacetime. This extended quantum theory, therefore, reproduces the central prediction of GR. Moreover, it predicts empty space is a dynamic medium that gravitates.

If we remove all matter and radiation in this toy universe, it undergoes a classical-quantum phase transition to $\mathcal{G}Cl(2)$. All configurations of $\hat{E}$ then have the zero-point energy

$$\begin{aligned} \hat{H}_{\text{vacc}} |g\rangle &= E_0 |g\rangle \\ &= 0 \end{aligned} \tag{41}$$

with the commutation relations

$$[\hat{x}_i, \hat{x}_j] = e_1 e_2 l_p^2 \hat{E}. \tag{42}$$

Some time after 2^{60} entanglement degrees of freedom are turned off, observers emerge as sub-configurations of $\hat{E}$; the system then undergoes a quantum-classical phase transition from $\mathcal{G}Cl(2)$ to $\mathcal{G}Cl(3)$. All configurations of $\hat{E}$ then have the vacuum energy

$$\hat{H}_{\text{vacc}} |g\rangle = E_{\text{vacc}} |g\rangle \tag{43}$$

where

$$\hat{H}_{\text{vacc}} = \hat{H}_{\text{on}} + \hat{H}_{\text{off}} \tag{44}$$

with the commutation relations

$$[\hat{x}_i, \hat{x}_j] = e_1 e_2 e_3 l_p^2 \hat{E}. \tag{45}$$

Since our Hilbert space is locally finite-dimensional, it's UV/IR finite. The reduced Compton wavelength of M_{off} and M_{on} provide the cutoffs

$$\lambda_{\text{on}} = l_p, \quad \lambda_{\text{off}} = \frac{1}{\Lambda l_p}. \tag{46}$$

The cosmological constant thus becomes the fundamental UV/IR regime

$$\Lambda = \frac{1}{\lambda_{\text{on}} \lambda_{\text{off}}}. \tag{47}$$

If we were to vary the fundamental constants, we would slide $\lambda_{\text{on}} \leq \lambda \leq \lambda_{\text{off}}$ across the number line. Λ would therefore remain constant regardless of the values G, $\hbar$, or c takes.

Since entanglement is "mediated" by M_{on}, if we turn entanglement off between two qubits, energy must be conserved through the mass-energy equivalence principle. Since the atoms of our emergent spacetime are spin-1/2, M_{on} must decay into spin-1/2 particle-antiparticle pairs

$$M_{\text{on}} \longrightarrow \text{ fermions} + \text{anti-fermions} + M_{\text{off}}. \tag{48}$$

At first sight, the only way this is possible is if new degrees of freedom are created, which would signal a violation of unitary evolution. This, however, can be resolved by considering the union of $\hat{E}$ and a much larger empty graph $\hat{E}^*$. Energy can therefore be conserved in two ways: either M_{on} flows from $\hat{E}$ to $\hat{E}^*$, or M_{on} decays into particle-antiparticle pairs in $\hat{E}^*$. Computing the Schwarzschild radius

$$r_s = 2l_p \tag{49}$$

we see it is exactly twice the reduced Compton wavelength, implying M_{on} is the black hole quanta and M_{off} the mass gap [64–66].

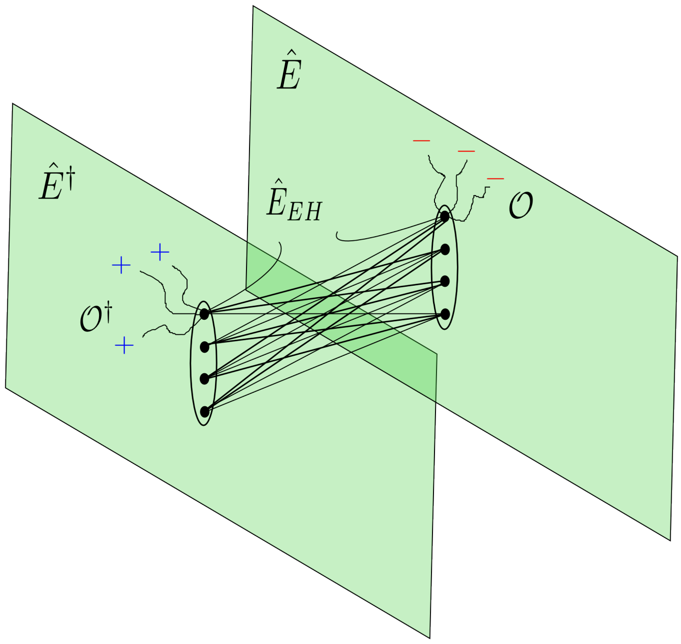

Fig. 3 A region of spacetime $\hat{E}$ with observers $\mathcal{O}$, entangled through the event horizon $\hat{E}_{\text{EH}}$, to its conjugate $\hat{E}^\dagger$ with observers $\mathcal{O}^\dagger$. When qubits entangled between conjugate regions of spacetime rupture, particle-antiparticle pairs are separated into $\hat{E}$ and $\hat{E}^\dagger$.

The above cartoon is illustrative of a region of spacetime entangled with its conjugate, possessing the commutation relations

$$[\hat{x}_i, \hat{x}_j] = i_3 l_p^2 \hat{E}, \quad [\hat{x}^\dagger, \hat{x}_j^\dagger] = i_3 l_p^2 \hat{E}^\dagger, \quad [\hat{x}_i, \hat{x}_j^\dagger] = [\hat{x}_i^\dagger, \hat{x}_j] = i_2 l_p^2 \hat{E}_{EH}. \tag{50}$$

Where the conjugated entanglement structure is defined as

$$\hat{E}^\dagger = \begin{cases} 0, \ \hat{x}_i^\dagger \text{ is adjacent to } \hat{x}_j^\dagger \\ 1, \qquad\quad \text{else.} \end{cases} \tag{51}$$

Demanding $\hat{E}^\dagger$ evolve from a dense to a sparse state is equivalent evolving $\hat{E}$ in reverse. From the frog's perspective, $\mathcal{O}$ and $\mathcal{O}^\dagger$ observe identical universes evolving forward in time. From the bird's perspective, the past, present, and future all exist simultaneously as a *reversible* quantum computer program – a kind of eternal and unchanging block universe, emulating dynamics only from the perspective of the observer.

When entanglement is ruptured between two qubits, M_{on} instantaneously decays into causally disconnected particle-antiparticle pairs and remnant masses M_{off}. The event horizon is therefore identified as the entanglement structure $\hat{E}_{EH}$ (the boundary between $\hat{E}$ and $\hat{E}^{\dagger}$). What lies beneath the horizon is neither a singularity nor an interior, only a 2-dimensional boundary between conjugated regions of spacetime. If an observer were to fall through the horizon, all of the bits that encode their 3-dimensional structure would be compressed to qubits and stored on its surface. Formally, the observer is mapped to their inverse $\mathcal{O} \longrightarrow \mathcal{O}^{-1}$. Since $\hat{E}_{EH}$ evolves from a dense to a sparse state, the horizon will evaporate into a cloud of M_{off} and causally disconnected particle-antiparticle pairs, thereby resolving both the black hole information paradox [67–77] and baryon asymmetry problems [78–80].

10 Experimental Design

Extraordinary claims require extraordinary evidence. While our claims could be considered speculative, they are unavoidable consequences of the postulates put forth. Nonetheless, the oracle of truth is not theory (no matter how rigorous or beautiful) but experiment. Since M_{on} is the Planck mass, its gravitational effects will become the dominant interaction in particular laboratory frames. Indeed, there's compelling evidence it's already been detected.

Our central prediction: the entanglement mass M_{on} sources gravitation and the disentanglement mass M_{off} sources anti-gravitation. Fluctuations in the gravitational field correspond to *surface* density fluctuations of M_{on} and M_{off}. Since the earth's rate of rotation (length of day) oscillates with a period of 5.9 years, so too does the density of M_{on}. Since this mass is present in all experiments, it will interfere with measurements of the gravitational constant G. In 2015, Anderson et al. looked at all measurements of G spanning from 1962 to 2014. They concluded that the value of G oscillates with a period of 5.9 years and an amplitude of $(1.619 \pm 0.103) \times 10^{-14}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$, resulting in a G/LOD correlation with a statistical significance of 0.99764 [81].

The G/LOD correlation isn't the only phenomenon to be expected. If black hole quanta are the constituents of spacetime, momentum can be exchanged with them. In 2016, White et. al at NASA developed an electromagnetic resonant cavity thruster that produced a consistent thrust-to-power ratio of 1.2 ± 0.1 mN/kW in vacuum [82]. They argue from Pilot-Wave theory that the vacuum is an immutable medium, capable of supporting acoustic vibrations for the emdrive to push off of. Presumably, the electromagnetic field inside the device couples to spacetime outside of it. Thrust can, therefore, be generated by disentangling spacetime in front of the device and entangling spacetime behind it, effectively pushing off of a surface of M_{on}. Therefore, the emdrive can be used to design a direct detection experiment.

As a thought experiment suppose a ball is dropped passed a speaker and into a cup. When the speaker is on, acoustic vibrations transfer momentum to the surrounding air, colliding with the ball and nudging it slightly to the right.

By turning the speaker on and off, the ball's rate of free fall is perturbed in the z direction. Replacing the speaker with the emdrive, the ball with an ensemble of atoms, the cup with an atom interferometer (Mach-Zehnder or gravimeter type), and the air molecules with vacuum, M_{on} can be directly detected.

When the emdrive is off the ensemble feels earth's gravitational pull. When the emdrive is on the following momenta is transferred to the ensemble along the x axis

$$\mathbf{p}_{\text{on}} = M_{\text{on}}\mathbf{v} \tag{52}$$

resulting in a perturbation of the rate of free fall

$$g' = \left(\frac{M_{\text{test}} - M_{\text{on}}}{M_{\text{test}}}\right)g. \tag{53}$$

If M_{test} is an order of magnitude larger than M_{on}, then the rate of free fall will decrease by an order of magnitude. Since this effect is so large, it's possible to conduct this as a relatively inexpensive table top experiment.

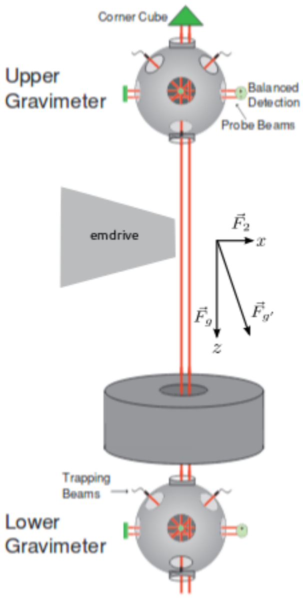

Fig. 4 Cesium interferometer based gravity gradiometer. Hyperfine splitting of cesium in its ground state acts as the ruler of the apparatus which employs a $\pi/2 - \pi - \pi/2$ light pulse sequence on two identical ensembles of cesium atoms. This apparatus achieved a differential acceleration sensitivity of $4 \times 10^{-9} g/\sqrt{Hz}$ and an accuracy of $\leq 10^{-9}$g [83].

11 Conclusion

Through the introduction of the Principle of General Invariance, three additional quantum mechanical postulates have been put forth. Postulates I & II propose entanglement and holography arise from an underlying noncommutative structure between spatial positions and directions. Postulate III proposes an observationally motivated field equation governing these noncommutative degrees of freedom. The solution to this equation yields a coupled system of equations of state, describing the thermodynamics of the vacuum. To great surprise, these equations predict the existence of the vacua M_{on} and M_{off}. These so-called atoms of spacetime reproduce the observations of the ΛCDM model of cosmology – bringing dark energy, dark matter, inflation, and gravitation into a single unified framework. Finally, (and most importantly) we've designed a relatively simple and inexpensive table-top experiment to falsify such extraordinary claims.

12 Acknowledgements

The author would like to thank David Peak for many helpful and enlightening discussions.

13 Appendix

Theorem 1 *Let G be a non-empty graph with a traceless adjacency matrix. The power-law,*

$$\mathcal{P}(r) = Ar^{\alpha} \tag{54}$$

with boundary condition,

$$\mathcal{P}(0) = 0 \tag{55}$$

as a function of the adjacency matrix $\boldsymbol{A}(G)$, maps the graph G to an eigenfunction g, belonging to the complex L^2-space of functions.

Proof Let us denote $\mathbf{A} = \mathbf{A}(G)$. It's rather trivial to show that $\mathcal{P}(\mathbf{A})$ is complex. Since $\mathbf{A}$ is symmetric, it is always diagonalizable and can always be written as $\mathcal{P}(\mathbf{A}) = \mathbf{O}\mathcal{P}(\mathbf{\Lambda})\mathbf{O}^{\top}$. $\mathbf{\Lambda}$ is the diagonal matrix of the eigenvalues of $\mathbf{A}$ and $\mathbf{O}$ is an orthogonal matrix belonging to the Orthogonal group $O(n)$, whose columns are the eigenvectors of $\mathbf{A}$. Since $\mathbf{A}$ is traceless, the sum of the eigenvalues is always zero. We are therefore guaranteed at least one negative eigenvalue $(-\lambda)^{\alpha} = e^{i\pi\alpha}\lambda^{\alpha}$. In a stepwise manner,

1. $\mathbf{A} = \mathbf{A}^\top \Rightarrow \mathbf{A}$ is always diagonalizable.

2. Since $\mathbf{A}$ is always diagonalizable, $\mathcal{P}(\mathbf{A})$ is a function of *only* the eigenvalues of $\mathbf{A}$.

3. Since $\operatorname{tr}\mathbf{A} = \sum_i \lambda_i = 0$, we are guaranteed at least one negative eigenvalue.

4. $\mathcal{P}(-\lambda) = A(-\lambda)^\alpha = Ae^{i\pi\alpha}\lambda^\alpha$.

Let us rewrite $\lambda^\alpha = e^{\alpha \ln \lambda}$ and $\theta = \alpha\pi$ and compute the spectral decomposition

$$
\begin{aligned}
\mathcal{P}[\mathbf{A}] &= \mathbf{O}\mathcal{P}[\mathbf{\Lambda}(\theta)]\mathbf{O}^\top \\
&= \mathbf{O}\mathcal{P}(\mathbf{\Lambda}_+)\mathbf{O}^\top + \mathbf{O}\mathcal{P}[\mathbf{\Lambda}_-(\theta)]\mathbf{O}^\top \\
&= \sum_{n=1}^{N} O_{nm}\mathcal{P}(\lambda_n^+) + \sum_{m=1}^{M} O_{nm}e^{i\theta}\mathcal{P}(\lambda_m^-) \\
&= \sum_{n=1}^{N} O_{nm}\exp\left\{\frac{\theta}{\pi}\ln(\lambda_n^+)\right\} + \sum_{m=1}^{M} O_{nm}\exp\left\{\frac{\theta}{\pi}\big(\ln(\lambda_m^-) + i\pi\big)\right\}.
\end{aligned}
\tag{56}
$$

We've separated the spectrum $\mathbf{\Lambda}$ into its non-negative part $\mathbf{\Lambda}_+$ and its negative part $\mathbf{\Lambda}_-$,

$$
\mathbf{\Lambda}_+ = \begin{pmatrix} \lambda_1^+ & & & & \\ & \ddots & & & \\ & & \lambda_n^+ & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}, \quad \mathbf{\Lambda}_- = e^{i\theta}\begin{pmatrix} 0 & & & & \\ & \ddots & & & \\ & & \lambda_{n+1}^- & & \\ & & & \ddots & \\ & & & & \lambda_m^- \end{pmatrix}. \tag{57}
$$

The eigenfunctions of the respective spectrums are

$$
|g_+\rangle := \exp\left\{\frac{\theta}{\pi}\ln(\lambda_n^+)\right\}, \quad |g_-\rangle := \exp\left\{\frac{\theta}{\pi}\big(\ln(\lambda_m^-) + i\pi\big)\right\}. \tag{58}
$$

Defining the linear operators

$$
\mathbf{L}^+ := \sum_n O_{nm}, \quad \mathbf{L}^- := \sum_m O_{nm} \tag{59}
$$

we express the power-spectrum of the adjacency matrix as

$$
|G\rangle \equiv \mathbf{L}^+ |g_+\rangle + \mathbf{L}^- |g_-\rangle \tag{60}
$$

where the plus and minus sub and superscripts are representative of summation over the negative and non-negative eigenvalues. Taking the real and imaginary parts we have

$$\operatorname{Re} g = \sum_{n=1}^{N} \exp\left\{\frac{\theta}{\pi}\ln(\lambda_n^+)\right\} + \sum_{m=1}^{M}\cos\theta\exp\left\{\frac{\theta}{\pi}\ln(\lambda_m^-)\right\} \tag{61}$$

$$\operatorname{Im} g = \sum_{m=1}^{M} i\sin\theta\exp\left\{\frac{\theta}{\pi}\ln(\lambda_m^-)\right\}. \tag{62}$$

We immediately see that $\operatorname{Im} g = 0$ when $\alpha = \pm 1, \pm 2, \pm 3, ...$ but is non-zero when $\alpha = \pm(1/2), \pm(2/3), \pm(3/4), ...$ or more generally when α is any real number. If we ask how many possible ways there are to move from one vertex to the next for a non-integral number of steps, a complex phase is assigned returning a complex number. The power-spectrum of a traceless adjacency matrix is thus the analytic continuation of the graph-theoretic walk. An immediate property of the *complex walk* is that it's not possible to enumerate any of its elements. These sets act as perfect black boxes, preventing us from keeping track of any of the vertices.

We are now in the position to sum over all walks in a continuous interval, thereby reconstructing the path integral as an inner-product of the above eigenfunction. As an example, let's calculate all walks in the $(0, 2\pi)$ interval for some arbitrary graph G.

$$\langle g|g\rangle = \langle g_+|g_+\rangle + \langle g_+|g_-\rangle + \langle g_-|g_+\rangle + \langle g_-|g_-\rangle$$

$$= \frac{1}{\pi}\int_0^{2\pi} d\theta \sum_n \left[e^{2\theta\pi^{-1}\ln(\lambda_n^+)} + 2\cos\theta e^{\theta\pi^{-1}\ln(\lambda_n^+\lambda_n^-)} + e^{2\theta\pi^{-1}\ln(\lambda_n^-)}\right]$$

$$= \frac{1}{\pi}\sum_n \left[\frac{\pi e^{2\theta\pi^{-1}\ln(\lambda_n^+)}}{2\ln\lambda_n^+} + \frac{e^{\theta\pi^{-1}\ln(\lambda_n^-\lambda_n^+)}(\cos\theta\ln(\lambda_n^-\lambda_n^+) + \pi\sin\theta)}{2\ln(\lambda_n^-)\ln(\lambda_n^+) + \ln^2(\lambda_n^-\lambda_n^+) + \pi^2} + \frac{\pi e^{2\theta\pi^{-1}\ln(\lambda_n^-)}}{2\ln\lambda_n^-}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\sum_n \left(\frac{(e^{2\ln(\lambda_n^-\lambda_n^+)} - 1)\ln(\lambda_n^-\lambda_n^+)}{2\ln(\lambda_n^-)\ln(\lambda_n^+) + \ln^2(\lambda_n^-\lambda_n^+) + \pi^2} + \frac{\pi}{2}\frac{e^{4\ln(\lambda_n^+)} - 1}{\ln\lambda_n^+} + \frac{\pi}{2}\frac{e^{4\ln(\lambda_n^-)} - 1}{\ln\lambda_n^-}\right)$$

We observe that the last two terms are indeterminates of the form $0/0$ when λ_n^+ and $\lambda_n^- \to 1$. Upon a change of variable the limit becomes,

$$\lim_{x\to 1}\frac{\pi}{2}\frac{x^4 - 1}{\ln x} = 2\pi \tag{63}$$

and we have our desired result

$$\langle g|g\rangle < \infty \quad \therefore g \in L^2(0, 2\pi). \tag{64}$$

It's rather straightforward to demonstrate that the above is true for any bounded interval. We need not concern ourselves with the unbounded case since we'd be integrating over an infinite number of backtracked walks. Furthermore, to capture all non-redundant walks from the graph, we need only integrate over its longest path. This ensures no divergences of any kind can appear within the theory. The proof is thus completed.

13.1 Identities

1. $\langle\delta(\theta-\pi)|G\rangle = \mathbf{A}(G)$
2. $\langle\delta(\theta+\pi)|G\rangle = \mathbf{A}^{+}(G)$ (Moore-Penrose Psuedo Inverse)
3. $\langle\delta(\theta)|G\rangle = \mathbf{L}$

13.2 Metric

We identify the eigenfunction $|g\rangle = |g_{+}\rangle + |g_{-}\rangle$ as a metric on a state space of graphs $\Omega = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, ..., \mathcal{G}_i\}$

$$d(\mathcal{G}_i, \mathcal{G}_j) = ||g_i - g_j|| = \sqrt{\langle g_j - g_i|g_j - g_i\rangle} \tag{65}$$

Theorem 2 *There exists a graph $\mathcal{G}_{\hat{\mathbf{q}}}$ and $\mathcal{G}_{\hat{\mathbf{p}}}$ with the self-edges $\mathcal{E}_{self} = (\hat{x}, \hat{y}, \hat{z})$ and $\mathcal{E}_{self} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$; with the vertices $\mathcal{V} = (e_1, e_2, e_3)$ such that the morphisms $\mathcal{G}_{\hat{\mathbf{q}}} \longrightarrow \mathcal{G}'_{\hat{\mathbf{q}}}$ and $\mathcal{G}_{\hat{\mathbf{p}}} \longrightarrow \mathcal{G}'_{\hat{\mathbf{p}}}$ give a graph with the reduced vertex set $\mathcal{V} = (e_1, e_2)$ such that* $\operatorname{tr}\mathbf{A}(\mathcal{G}') = 0$.

Proof Let us consider the 2nd-rank symmetric tensor

$$\hat{\mathbf{q}} \otimes \hat{\mathbf{q}} = \begin{bmatrix} \hat{x}^2 & \hat{x}\hat{y} & \hat{x}\hat{z} \\ \hat{y}\hat{x} & \hat{y}^2 & \hat{y}\hat{z} \\ \hat{z}\hat{x} & \hat{z}\hat{y} & \hat{z}^2 \end{bmatrix}. \tag{66}$$

We seek a background independent operator that stores the operator-valued coordinates $(\hat{x}, \hat{y}, \hat{z})$ and the orthonormal basis (e_1, e_2, e_3). We construct this operator such that its trace is $\hat{\mathbf{q}} = \hat{x}e_1 + \hat{y}e_2 + \hat{z}e_3$ and such that we can define a product of these operators which gives us $\hat{\mathbf{q}} \otimes \hat{\mathbf{q}}$. If we define the following "square root vector"

$$\sqrt{\hat{\mathbf{q}} \cdot \mathbf{e}} := (\sqrt{\hat{x}e_1}, \sqrt{\hat{y}e_2}, \sqrt{\hat{z}e_3}), \tag{67}$$

we can construct a background independent operator with the following outer-product

$$\pm(\sqrt{\hat{\mathbf{q}}\cdot\mathbf{e}})\cdot\mathbf{e}\otimes\pm\sqrt{\hat{\mathbf{q}}\cdot\mathbf{e}} = \begin{bmatrix} \hat{x}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{y}e_2}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{z}e_3}e_1 \\ \sqrt{\hat{y}e_2}\sqrt{\hat{x}e_1}e_2 & \hat{y}e_2 & \sqrt{\hat{y}e_2}\sqrt{\hat{z}e_3}e_2 \\ \sqrt{\hat{z}e_3}\sqrt{\hat{x}e_1}e_3 & \sqrt{\hat{z}e_3}\sqrt{\hat{y}e_2}e_3 & \hat{z}e_3 \end{bmatrix}. \quad (68)$$

We then solve for $\mathcal{G}$ by identifying the algebraic constants under the radical give us an adjacency relation, where the algebraic constants outside of the radical give us the orientation of this relation. Therefore $\mathcal{G} = \mathcal{K}^3$, a complete directed graph on 3 vertices. Let us denote the adjacency relation as $e_i \sim e_j \equiv e_{ij}$, then

$$\mathcal{K}^3 = (\mathcal{V}, \mathcal{E}) = \big(\{e_1, e_2, e_3\}, \{e_{ij}, e_{ii}\}\big). \quad (69)$$

The ± 1 pre-factor in eq (68) indicates that it transforms as a spinor. We can re-write it as

$$\hat{\mathbf{Q}}(\mathcal{K}^3) := e^{i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^s\cdot\mathbf{e} \;\otimes\; e^{i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^s \quad s = (1/2)$$

where the transpose flips the orientation of the edges, sending $s \mapsto -s$

$$\hat{\mathbf{Q}}^\top(\mathcal{K}^3) := e^{-i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^{-s}\otimes e^{-i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^{-s}\cdot\mathbf{e} \quad s = (-1/2).$$

We recover our 2nd-rank tensor through the schur-product (element-wise product)

$$\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3) = \hat{\mathbf{q}}\otimes\hat{\mathbf{q}}. \quad (70)$$

Given N independent quantum harmonic oscillators in 3-dimensions

$$\hat{H} = \sum \frac{1}{2m}\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} + \frac{1}{2}m\omega^2\hat{\mathbf{q}}\cdot\hat{\mathbf{q}} \quad (71)$$

we send the vertices of $\hat{E}$ to

$$\begin{aligned}\hat{\mathrm{G}}(\mathcal{K}^3) &:= \sum \frac{1}{2m}\hat{\mathbf{P}}(\mathcal{K}^3)\circ\hat{\mathbf{P}}(\mathcal{K}^3) + \frac{1}{2}m\omega^2\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3) \\ &= \sum \frac{1}{2m}\begin{bmatrix}\hat{p}_x^2 & \hat{p}_x\hat{p}_y & \hat{p}_x\hat{p}_z \\ \hat{p}_y\hat{p}_x & \hat{p}_y^2 & \hat{p}_y\hat{p}_z \\ \hat{p}_z\hat{p}_x & \hat{p}_z\hat{p}_y & \hat{p}_z^2\end{bmatrix} + \frac{1}{2}m\omega^2\begin{bmatrix}\hat{x}^2 & \hat{x}\hat{y} & \hat{x}\hat{z} \\ \hat{y}\hat{x} & \hat{y}^2 & \hat{y}\hat{z} \\ \hat{z}\hat{x} & \hat{z}\hat{y} & \hat{z}^2\end{bmatrix}\end{aligned} \quad (72)$$

where

$$\begin{aligned} \hat{p}_x &= -i_3\hbar\frac{\partial}{\partial x} = -e_1e_2e_3\hbar\frac{\partial}{\partial x} \\ \hat{x} &= x \\ \hat{H} &= \operatorname{tr}\hat{\mathrm{G}}(\mathcal{K}^3) \\ &= \operatorname{tr}\hat{\mathrm{G}}. \end{aligned} \tag{73}$$

Since $\operatorname{tr}\hat{\mathbf{A}}(\mathcal{G}') = 0$ the morphism $\mathcal{G} \longrightarrow \mathcal{G}'$ minimizes the Hamiltonian. To solve for $\mathcal{G}'$ we demand

$$\operatorname{tr}\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3) = \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 0 \quad\text{and}\quad \operatorname{tr}\hat{\mathbf{P}}(\mathcal{K}^3)\circ\hat{\mathbf{P}}(\mathcal{K}^3) = \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 0 \tag{74}$$

which forms a 2-dimensional operator-valued isotropic Hilbert space, a sufficient condition for the construction of spinors. We can write down a matrix $\hat{\mathbf{Q}}(\mathcal{G}')$ that represents $\hat{\mathbf{q}}$ in complex 3-space. This matrix admits a factorization as an outer-product

$$\hat{\mathbf{Q}}(\mathcal{G}') = 2\begin{bmatrix}\hat{\zeta}_0\\ \hat{\zeta}_1\end{bmatrix}\begin{bmatrix}\hat{\zeta}_0 & \hat{\zeta}_1\end{bmatrix}, \tag{75}$$

yielding the overdetermined system of equations

$$\begin{aligned} \hat{\zeta}_0^2 - \hat{\zeta}_1^2 &= \hat{x} \\ i_2(\hat{\zeta}_0^2 + \hat{\zeta}_1^2) &= \hat{y} \\ -2\hat{\zeta}_0\hat{\zeta}_1 &= \hat{z} \end{aligned} \tag{76}$$

with the solutions

$$\hat{\zeta}_0 = \pm\sqrt{\frac{\hat{x} - i_2\hat{y}}{2}}, \quad \hat{\zeta}_1 = \pm\sqrt{\frac{-\hat{x} - i_2\hat{y}}{2}}. \tag{77}$$

We can solve eq. (75) by taking the Pauli vector

$$\boldsymbol{\sigma} = \sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3 \tag{78}$$

and dotting it with $\hat{\mathbf{q}}$,

$$\hat{\mathbf{Q}}(\mathcal{K}^2) = \hat{\mathbf{q}}\cdot\boldsymbol{\sigma} = \begin{bmatrix} -\hat{z} & \hat{x} - i_2\hat{y} \\ \hat{x} + i_2\hat{y} & \hat{z} \end{bmatrix}. \tag{79}$$

Thus, under the following map,

$$\hat{\mathbf{Q}}(\mathcal{K}^3) \longrightarrow \hat{\mathbf{Q}}(\mathcal{K}^2), \quad \hat{\mathbf{P}}(\mathcal{K}^3) \longrightarrow \hat{\mathbf{P}}(\mathcal{K}^2) \tag{80}$$

we get

$$\begin{aligned}
\hat{\mathrm{G}} &= \sum \frac{1}{2m}\hat{\mathbf{p}}\cdot\boldsymbol{\sigma} + \frac{1}{2}m\omega^2\hat{\mathbf{q}}\cdot\boldsymbol{\sigma}, \\
&= \sum \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x - i_2\hat{p}_y \\ \hat{p}_x + i_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega^2\begin{bmatrix} -\hat{z} & \hat{x} - i_2\hat{y} \\ \hat{x} + i_2\hat{y} & \hat{z} \end{bmatrix} \\
&= \sum \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x - e_1e_2\hat{p}_y \\ \hat{p}_x + e_1e_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega^2\begin{bmatrix} -\hat{z} & \hat{x} - e_1e_2\hat{y} \\ \hat{x} + e_1e_2\hat{y} & \hat{z} \end{bmatrix} \\
&= \sum \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x + e_2e_1\hat{p}_y \\ \hat{p}_x + e_1e_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega^2\begin{bmatrix} -\hat{z} & \hat{x} + e_2e_1\hat{y} \\ \hat{x} + e_1e_2\hat{y} & \hat{z} \end{bmatrix}
\end{aligned} \tag{81}$$

where

$$\begin{aligned}
\hat{p} &= -i_2\hbar\frac{\partial}{\partial x} = -e_1e_2\hbar\frac{\partial}{\partial x} \\
\hat{x} &= x \\
\hat{H}_{grav} &= \operatorname{tr}\hat{\mathrm{G}}(\mathcal{K}^2). \\
&= \operatorname{tr}\hat{\mathrm{G}}' \\
&= 0
\end{aligned} \tag{82}$$

and

$$\mathcal{K}^2 = \big(\{e_1, e_2\}, \{e_2, e_1\}, \{e_{11}, e_{22}\}\big). \tag{83}$$

Theorem 3 *The solution to the entanglement field equation at the Planck scale is* $g(\delta) = l_p^{(\delta/2\pi)}(1 + e^{i_2\delta/2})$.

Proof From Theorem 1 and 2

$$\mathcal{P}(\mathbf{q}\cdot\sigma) = (\mathbf{q}\cdot\sigma)^{\theta/\pi} = (\mathbf{q}\cdot\sigma)^{\delta s/\pi} = (\mathbf{q}\cdot\sigma)^{\delta/2\pi} \tag{84}$$

which is re-written as the entanglement field equation and solved for the metric eigenfunction by the spectral decomposition

$$\begin{aligned}
|\mathcal{G}\rangle &= \mathbf{L}^+|g_+\rangle + \mathbf{L}^-|g_-\rangle \\
&= \sum_i U_{ij}|g_i\rangle + \sum_j U_{ij}|g_j\rangle \\
&= \sum_i U_{ij}\exp\left\{\frac{\delta}{2\pi}\ln\sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}\right\} \\
&\quad + \sum_j U_{ij}\exp\left\{\frac{\delta}{2\pi}\Big(\ln\big(\sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}\big) + i_2\pi\Big)\right\}
\end{aligned} \tag{85}$$

yielding

$$|g\rangle = |g_+\rangle + |g_-\rangle = e^{(\delta/2\pi)\ln\sqrt{\hat{x}^2+\hat{y}^2+\hat{z}^2}}(1 + e^{i_2\delta/2}). \quad (86)$$

Substituting $l_p = \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}$ completes the proof.

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