11D04

The proof of the Collatz conjecture

Abstract:

The article provides with the evidence of the Collatz conjecture. It is proved that, the Collatz function is $C(n) = \begin{cases} n/2, & if \quad n - even, \\ 3n+1, & if \quad n - odd, \end{cases}$

equivalent to the function

$$C_k(n) = \begin{cases} n/2, \quad if \quad n - even, \\ 3n+1, \quad if \quad n - odd, \\ Start\ odd\ number\ 3k+1, \\ where\ k = 6m \pm 1, \ \ m - natural\ number \end{cases}$$

The only difference, between two functions $C_k(n)$ and C (n), is the starting number for function $C_k(n)$ must be an odd number having the form of $3k + 1$, where $k = 6m \pm 1$, m - is a natural number. It is proved that by repeating operations for the function $C_k(n)$ with any natural n, k completes with the unit (one), it follows that the Collatz Conjecture is true and it has been proved.

Keywords: Collatz conjecture, problem $3n + 1$, Syracuse problem, fractal, proof.

1. INTRODUCTION

The Collatz Conjecture also known as the problem of $3n + 1$ is one of the unsolved problems of mathematics. The following papers devoted to the problem 3 n + 1 [1, 2, 3, 4, 5] can be noted. In order to explain the essence of Collatz conjecture, we should take any natural n number, if n is even, divide it by 2 to get $n / 2$. If n is odd, multiply it by 3 and add 1 to obtain $3n + 1$. On obtained (deduced) number we perform same manipulations and so on. The conjecture is that no matter what number you start with, you will always eventually reach 1.

The Collatz function $C(n)$ is defined on the natural numbers as follows:

$$C(n) = \begin{cases} n/2, & if \quad n - even, \\ 3n+1, & if \quad n - odd. \end{cases} \qquad (1)$$

It is should be noted that, for analyzing the Collatz function it is enough to work only with odd numbers, because any even number with the process of dividing by 2 (one or several times) will become an odd number. Therefore, the main thing is to cover all odd numbers.

2. DEFINITIONS

It is known that, all odd numbers, except 1 and odd numbers multiply to 3, can be generated by the formulas $k^- = 6m - 1$ and $k^+ = 6m + 1$, where m – natural number. Thus, if we make a list of odd numbers generated by the formulas $6m - 1$; $6m + 1$ and $3m$, then we get a complete list of odd natural numbers, except for 1.

An odd numbers obtained by the formulas $k^- = 6m - 1$ and $k^+ = 6m + 1$, the author named "key-number". The top marks "-" and "+" in the notation of the key-numbers k^- and k^+ indicates the signs in appropriate formulas, further they will be called respectively "minus" and "plus" respectively of the key-numbers.

For further simplicity the procedure of multiplication and division of the specified number under the terms of the Collatz function (1) called "collatztization"

Firstly, let`s look at the collatztization of odd numbers multiples of 3 (factor of 3). For this procedure a table 1 is created, where the results of the multiplication of numbers multiples 3 by 3 and adding 1 (9n+1) is shown and division by 2 until the odd number or 1 is created, in case of creation an even number.

From the Table 1 it is to note that, if a natural numbers multiples by 3 is an even number , it will become an odd number by the process of the collatztization, which is $6m + 1$, the sequence of such numbers start with the number 19 and forms by the step 18. If a natural number multiple of 3 is an odd number, it will become an odd number by the process of the collatztization, which will have a form $6m - 1$or

$6m+1$or it will be an even number equal to the power of two. It follows that, the numbers resulted by the process of the collatztization of the numbers multiple 3, the sequence of the numbers is created which has a view $6m-1$ and $6m+1$ on the six columns, which can be calculated by the following formulas:

1) $k_m^+ = 19 + 18m$; 2) $k_m^- = 5 + 18m$; 3) $k_m^+ = 7 + 18m$;

4) $k_m^- = 17 + 18m$; 5) $k_m^+ = 13 + 18m$; 6) $k_m^- = 11 + 18m$.

By giving different values m we make sure that, the formulas above form infinite sequence of number with the view $6m-1$ and $6m+1$. For example, with m =0 we got a primary six numbers of the following sequence : 5,7,11, 13, 17,19, with m =1 results the second bunch of six numbers of the above-mentioned sequence: 23, 25, 29, 31, 35, 37 and so on. In other words, by the process of the collatztization of the sequence numbers multiple 3, we could get the sequence of the numbers with a view $6m-1$ and $6m+1$.

It is important to note that, if the process of the collatztization is used with a big numbers multiple 3 with a big values of the power of two, it could be seen that, repeatedly process of the collatztization of some number results with an earlier number with a view $6m-1$ and $6m+1$. For example, if the number 597 is collattztorized , which multiple 3, then it will become the number 7.

The pattern above could be shown by the following equality

$$(3m \cdot 3 + 1)/2^q = (3n \cdot 3 + 1)/2^{q+6a}, \qquad (2)$$

where $m, n = 1, 2, 3, \dots;\ q, a = 0, 1, 2, 3, \dots$.

For example $(9 \cdot 3 + 1)/2^2 = (597 \cdot 3 + 1)/2^{2+6};\ 28/4 = 1792/256;$ 7=7.

Table 1. The process of the collatztization of numbers multiple 3

n	3n	9n+1	(9n+1)/2	(9n+1)/4	(9n+1)/8	(9n+1)/16	(9n+1)/32	(9n+1)/64
1	3	10	5	2,5	1,25	0,625	0,3125	0,15625
2	6	19	9,5	4,75	2,375	1,1875	0,59375	0,296875
3	9	28	14	7	3,5	1,75	0,875	0,4375
4	12	37	18,5	9,25	4,625	2,3125	1,15625	0,578125
5	15	46	23	11,5	5,75	2,875	1,4375	0,71875
6	18	55	27,5	13,75	6,875	3,4375	1,71875	0,859375
7	21	64	32	16	8	4	2	1
8	24	73	36,5	18,25	9,125	4,5625	2,28125	1,140625
9	27	82	41	20,5	10,25	5,125	2,5625	1,28125
10	30	91	45,5	22,75	11,375	5,6875	2,84375	1,421875
11	33	100	50	25	12,5	6,25	3,125	1,5625
12	36	109	54,5	27,25	13,625	6,8125	3,40625	1,703125
13	39	118	59	29,5	14,75	7,375	3,6875	1,84375
14	42	127	63,5	31,75	15,875	7,9375	3,96875	1,984375
15	45	136	68	34	17	8,5	4,25	2,125
16	48	145	72,5	36,25	18,125	9,0625	4,53125	2,265625
17	51	154	77	38,5	19,25	9,625	4,8125	2,40625
18	54	163	81,5	40,75	20,375	10,1875	5,09375	2,546875
19	57	172	86	43	21,5	10,75	5,375	2,6875
20	60	181	90,5	45,25	22,625	11,3125	5,65625	2,828125
21	63	190	95	47,5	23,75	11,875	5,9375	2,96875
22	66	199	99,5	49,75	24,875	12,4375	6,21875	3,109375
23	69	208	104	52	26	13	6,5	3,25
24	72	217	108,5	54,25	27,125	13,5625	6,78125	3,390625
25	75	226	113	56,5	28,25	14,125	7,0625	3,53125
26	78	235	117,5	58,75	29,375	14,6875	7,34375	3,671875
27	81	244	122	61	30,5	15,25	7,625	3,8125
28	84	253	126,5	63,25	31,625	15,8125	7,90625	3,953125
29	87	262	131	65,5	32,75	16,375	8,1875	4,09375
30	90	271	135,5	67,75	33,875	16,9375	8,46875	4,234375
31	93	280	140	70	35	17,5	8,75	4,375
32	96	289	144,5	72,25	36,125	18,0625	9,03125	4,515625
33	99	298	149	74,5	37,25	18,625	9,3125	4,65625
34	102	307	153,5	76,75	38,375	19,1875	9,59375	4,796875
35	105	316	158	79	39,5	19,75	9,875	4,9375
36	108	325	162,5	81,25	40,625	20,3125	10,15625	5,078125
37	111	334	167	83,5	41,75	20,875	10,4375	5,21875
38	114	343	171,5	85,75	42,875	21,4375	10,71875	5,359375
39	117	352	176	88	44	22	11	5,5

Thus, one can state, that all natural numbers multiple 3, become an odd numbers with a view $k^{-} = 6m - 1$ and $k^{+} = 6m + 1$, through one operation $3n + 1$and division on the certain power of two, in the case of the creation an even number after primary actions. The exception are numbers, which after operation $3n + 1$will equal to the power of two. Let us formalize this fact in the form of a theorem.

Theorem 1. If any natural number multiple of 3 is multiplied by 3 and added 1, then the resulting even number is divided by a certain power of 2 until an odd number is obtained, then a number having the form 6n-1 or 6n + 1 is obtained.

It follows that, it is enough to work with the numbers $k^{-} = 6m - 1$ and $k^{+} = 6m + 1$ in order to solve the problem of $3n + 1$. Therefore, the next step is to work only with the key-numbers because, there is no sense to make unnecessary actions for even numbers and numbers multiple 3.

Theorem 1 can also be proved in another way, the following is another proof of Theorem 1.

It is known that all natural numbers, with the exception of 1, can be represented by the formulas *1) 3t; 2) 3t-1; 3) 3t + 1,* where *t = 1,2,3* ....

Obviously, as a result of the collateralization of any natural number, a number of the form *3t + 1* is formed. This implies the correctness of Theorem 1, since the numbers multiples of 3 have the form *3t*.

Let's answer the following question: *What is the reason of reiteration of the numbers with a view* $6m - 1$ *and* $6m + 1$*, during the process of collatztization of big numbers multiple 3, which earlier were obtained with small numbers?*

The reason of this reiteration of the numbers, with a view $6m - 1$ and $6m + 1$ by the process of the collatztization small and big numbers multiple, is that digital root of the numbers shown as the 2^{x} will be equal with the following exponent

$x = q + 6a$, where q - is a starting value; $q, a = 0,1,2,3$ .... It is known that, digital root of the number defines the division of the number by another number.

3. THE STRUCTURE AND THE PATTERNS OF THE NUMBERS

The proof of the Collatz conjecture, offered by the author, is based on the patterns of the connection between key-numbers $k_n^{\pm}$, obtained by the process of the collatztion of the other key-numbers $k_m^{\pm}$ i.e. number obtained after complex of actions, expressed by the formula $k_n^{\pm} = (k_m^{\pm} \cdot 3 + 1)/2^q$. In this connection, first of all we should analyze the collatztization of the key-numbers. The mechanism of the collatztization's influence on the key-numbers could be understood by the following formulas:

1) Collatztization of the "minus" key-numbers k^-:

$$3k^- + 1 = 3(6m - 1) + 1 = 18m - 2; \quad C(k^-) = 18m - 2. \qquad (3)$$

2) Collatztization of the "plus" key-numbers k^+:

$$3k^+ + 1 = (6m + 1) \cdot 3 + 1 = 18m + 4; \quad C(k^+) = 18m + 4. \qquad (4)$$

Further $C(k^-) = 18m - 2$ и $C(k^+) = 18m + 4$ one or several times divided by 2 until the integer odd number is created, which also will be a key-number or equal to the power of two or two.

Below the calculations by the formula (3) – table 2 and by the formula (4) – Table 3 are shown. The data from the Tables 2 and 3 show that, the collatztization of the key-number (of the odd numbers show as $6m - 1$; $6m + 1$), will result to the creation of a new key-numbers, which are formed in the dependence of the exponent power of two. The Tables 2 and 3 show that, the key-numbers located on the six columns complied with the certain value (indicators) power of two, amount an infinitive sequence of the key-numbers: 5, 7, 11, 13, 17,19, 23, 25, 29, 31, 35, 37 and so on.

Therefore, on the basis of the formulas (3) and (4) including data from the Table 2 and 3, it could be noted that, if by the process of the collatztization of the key-numbers, we will obtain a new key-numbers or even number which is equal to the power of two. This pattern is formed as the following theorem.

Theorem 2 . If any natural number 6m-1 or 6m+1 multiplied by 3 and add 1, then the number obtained divide by the certain degree 2 until an odd number is created, so it will result a number which has a view 6n-1 or 6n+1 or number 1. The following pairs of the key-numbers are unique.

Theorem 2, where it is asserted that any natural number of the form $6m-1$ or $6m + 1$ will turn into numbers of the form $6n-1$ and $6n + 1$ as a result of collateralization, is proved as follows.

It is clear that all numbers of the form $6m + 1$ and $6m-1$ are odd numbers of the form $3t + 1$ or $3t-1$, since they are not multiples of 3. If we multiply such numbers by 3 and add 1, then naturally we get numbers of the form $3s + 1$. And numbers of the form $3s + 1$ always correspond to numbers of the form $6m + 1$ or even numbers formed by their numbers of the form $6m-1$. Therefore, as a result of the collating of numbers of the form $6m-1$ or $6m + 1$, numbers of the form $6n-1$ and $6n + 1$ are always formed.

Table 2. Calculation by the formula $C(k^{-}) = (18m - 2)/2^{q}$.

m	k^{-}	$3k+1$	q=1	q=2	q=3	q=4	q=5	q=6	q=7
1	**5**	16	8	4	2	1			
2	**11**	34	17	8,5	4,25	2,125	1,0625	0,53125	0,265625
3	**17**	52	26	13	6,5	3,25	1,625	0,8125	0,40625
4	**23**	70	35	17,5	8,75	4,375	2,1875	1,09375	0,546875
5	**29**	88	44	22	11	5,5	2,75	1,375	0,6875
6	**35**	106	53	26,5	13,25	6,625	3,3125	1,65625	0,828125
7	**41**	124	62	31	15,5	7,75	3,875	1,9375	0,96875
8	**47**	142	71	35,5	17,75	8,875	4,4375	2,21875	1,109375
9	**53**	160	80	40	20	10	5	2,5	1,25
10	**59**	178	89	44,5	22,25	11,125	5,5625	2,78125	1,390625
11	**65**	196	98	49	24,5	12,25	6,125	3,0625	1,53125
12	**71**	214	107	53,5	26,75	13,375	6,6875	3,34375	1,671875
13	**77**	232	116	58	29	14,5	7,25	3,625	1,8125
14	**83**	250	125	62,5	31,25	15,625	7,8125	3,90625	1,953125
15	**89**	268	134	67	33,5	16,75	8,375	4,1875	2,09375
16	**95**	286	143	71,5	35,75	17,875	8,9375	4,46875	2,234375
17	**101**	304	152	76	38	19	9,5	4,75	2,375
18	**107**	322	161	80,5	40,25	20,125	10,0625	5,03125	2,515625
19	**113**	340	170	85	42,5	21,25	10,625	5,3125	2,65625
20	**119**	358	179	89,5	44,75	22,375	11,1875	5,59375	2,796875
21	**125**	376	188	94	47	23,5	11,75	5,875	2,9375
22	**131**	394	197	98,5	49,25	24,625	12,3125	6,15625	3,078125
23	**137**	412	206	103	51,5	25,75	12,875	6,4375	3,21875
24	**143**	430	215	107,5	53,75	26,875	13,4375	6,71875	3,359375
25	**149**	448	224	112	56	28	14	7	3,5
26	**155**	466	233	116,5	58,25	29,125	14,5625	7,28125	3,640625
27	**161**	484	242	121	60,5	30,25	15,125	7,5625	3,78125
28	**167**	502	251	125,5	62,75	31,375	15,6875	7,84375	3,921875
29	**173**	520	260	130	65	32,5	16,25	8,125	4,0625
30	**179**	538	269	134,5	67,25	33,625	16,8125	8,40625	4,203125
31	**185**	556	278	139	69,5	34,75	17,375	8,6875	4,34375
32	**191**	574	287	143,5	71,75	35,875	17,9375	8,96875	4,484375
33	**197**	592	296	148	74	37	18,5	9,25	4,625
34	**203**	610	305	152,5	76,25	38,125	19,0625	9,53125	4,765625
35	**209**	628	314	157	78,5	39,25	19,625	9,8125	4,90625
36	**215**	646	323	161,5	80,75	40,375	20,1875	10,09375	5,046875
37	**221**	664	332	166	83	41,5	20,75	10,375	5,1875
38	**227**	682	341	170,5	85,25	42,625	21,3125	10,65625	5,328125
39	**233**	700	350	175	87,5	43,75	21,875	10,9375	5,46875
40	**239**	718	359	179,5	89,75	44,875	22,4375	11,21875	5,609375

Continuation of the Table 2.

41	**245**	736	368	184	92	46	23	11,5	5,75
42	**251**	754	377	188,5	94,25	47,125	23,5625	11,78125	5,890625
43	**257**	772	386	193	96,5	48,25	24,125	12,0625	6,03125
44	**263**	790	395	197,5	98,75	49,375	24,6875	12,34375	6,171875
45	**269**	808	404	202	101	50,5	25,25	12,625	6,3125
46	**275**	826	413	206,5	103,25	51,625	25,8125	12,90625	6,453125
47	**281**	844	422	211	105,5	52,75	26,375	13,1875	6,59375
48	**287**	862	431	215,5	107,75	53,875	26,9375	13,46875	6,734375
49	**293**	880	440	220	110	55	27,5	13,75	6,875
50	**299**	898	449	224,5	112,25	56,125	28,0625	14,03125	7,015625
51	**305**	916	458	229	114,5	57,25	28,625	14,3125	7,15625
52	**311**	934	467	233,5	116,75	58,375	29,1875	14,59375	7,296875
53	**317**	952	476	238	119	59,5	29,75	14,875	7,4375
54	**323**	970	485	242,5	121,25	60,625	30,3125	15,15625	7,578125
55	**329**	988	494	247	123,5	61,75	30,875	15,4375	7,71875
56	**335**	1006	503	251,5	125,75	62,875	31,4375	15,71875	7,859375
57	**341**	1024	512	256	128	64	32	16	8
58	**347**	1042	521	260,5	130,25	65,125	32,5625	16,28125	8,140625
59	**353**	1060	530	265	132,5	66,25	33,125	16,5625	8,28125
60	**359**	1078	539	269,5	134,75	67,375	33,6875	16,84375	8,421875
61	**365**	1096	548	274	137	68,5	34,25	17,125	8,5625
62	**371**	1114	557	278,5	139,25	69,625	34,8125	17,40625	8,703125
63	**377**	1132	566	283	141,5	70,75	35,375	17,6875	8,84375
64	**383**	1150	575	287,5	143,75	71,875	35,9375	17,96875	8,984375
65	**389**	1168	584	292	146	73	36,5	18,25	9,125
66	**395**	1186	593	296,5	148,25	74,125	37,0625	18,53125	9,265625
67	**401**	1204	602	301	150,5	75,25	37,625	18,8125	9,40625
68	**407**	1222	611	305,5	152,75	76,375	38,1875	19,09375	9,546875
69	**413**	1240	620	310	155	77,5	38,75	19,375	9,6875
70	**419**	1258	629	314,5	157,25	78,625	39,3125	19,65625	9,828125
71	**425**	1276	638	319	159,5	79,75	39,875	19,9375	9,96875
72	**431**	1294	647	323,5	161,75	80,875	40,4375	20,21875	10,10938
73	**437**	1312	656	328	164	82	41	20,5	10,25
74	**443**	1330	665	332,5	166,25	83,125	41,5625	20,78125	10,39063
75	**449**	1348	674	337	168,5	84,25	42,125	21,0625	10,53125
76	**455**	1366	683	341,5	170,75	85,375	42,6875	21,34375	10,67188
77	**461**	1384	692	346	173	86,5	43,25	21,625	10,8125
78	**467**	1402	701	350,5	175,25	87,625	43,8125	21,90625	10,95313
79	**473**	1420	710	355	177,5	88,75	44,375	22,1875	11,09375
80	**479**	1438	719	359,5	179,75	89,875	44,9375	22,46875	11,23438
81	**485**	1456	728	364	182	91	45,5	22,75	11,375

Table 3. Calculation by the formula $C(k^{+}) = (18m + 4)/2^{q}$.

m	k^{+}	$3k+1$	q=1	q=2	q=3	q=4	q=5	q=6	q=7
1	**7**	22	11	5,5	2,75	1,375	0,6875	0,34375	0,171875
2	**13**	40	20	10	5	2,5	1,25	0,625	0,3125
3	**19**	58	29	14,5	7,25	3,625	1,8125	0,90625	0,453125
4	**25**	76	38	19	9,5	4,75	2,375	1,1875	0,59375
5	**31**	94	47	23,5	11,75	5,875	2,9375	1,46875	0,734375
6	**37**	112	56	28	14	7	3,5	1,75	0,875
7	**43**	130	65	32,5	16,25	8,125	4,0625	2,03125	1,015625
8	**49**	148	74	37	18,5	9,25	4,625	2,3125	1,15625
9	**55**	166	83	41,5	20,75	10,375	5,1875	2,59375	1,296875
10	**61**	184	92	46	23	11,5	5,75	2,875	1,4375
11	**67**	202	101	50,5	25,25	12,625	6,3125	3,15625	1,578125
12	**73**	220	110	55	27,5	13,75	6,875	3,4375	1,71875
13	**79**	238	119	59,5	29,75	14,875	7,4375	3,71875	1,859375
14	**85**	256	128	64	32	16	8	4	2
15	**91**	274	137	68,5	34,25	17,125	8,5625	4,28125	2,140625
16	**97**	292	146	73	36,5	18,25	9,125	4,5625	2,28125
17	**103**	310	155	77,5	38,75	19,375	9,6875	4,84375	2,421875
18	**109**	328	164	82	41	20,5	10,25	5,125	2,5625
19	**115**	346	173	86,5	43,25	21,625	10,8125	5,40625	2,703125
20	**121**	364	182	91	45,5	22,75	11,375	5,6875	2,84375
21	**127**	382	191	95,5	47,75	23,875	11,9375	5,96875	2,984375
22	**133**	400	200	100	50	25	12,5	6,25	3,125
23	**139**	418	209	104,5	52,25	26,125	13,0625	6,53125	3,265625
24	**145**	436	218	109	54,5	27,25	13,625	6,8125	3,40625
25	**151**	454	227	113,5	56,75	28,375	14,1875	7,09375	3,546875
26	**157**	472	236	118	59	29,5	14,75	7,375	3,6875
27	**163**	490	245	122,5	61,25	30,625	15,3125	7,65625	3,828125
28	**169**	508	254	127	63,5	31,75	15,875	7,9375	3,96875
29	**175**	526	263	131,5	65,75	32,875	16,4375	8,21875	4,109375
30	**181**	544	272	136	68	34	17	8,5	4,25
31	**187**	562	281	140,5	70,25	35,125	17,5625	8,78125	4,390625
32	**193**	580	290	145	72,5	36,25	18,125	9,0625	4,53125
33	**199**	598	299	149,5	74,75	37,375	18,6875	9,34375	4,671875
34	**205**	616	308	154	77	38,5	19,25	9,625	4,8125
35	**211**	634	317	158,5	79,25	39,625	19,8125	9,90625	4,953125
36	**217**	652	326	163	81,5	40,75	20,375	10,1875	5,09375
37	**223**	670	335	167,5	83,75	41,875	20,9375	10,46875	5,234375
38	**229**	688	344	172	86	43	21,5	10,75	5,375
39	**235**	706	353	176,5	88,25	44,125	22,0625	11,03125	5,515625
40	**241**	724	362	181	90,5	45,25	22,625	11,3125	5,65625

Continuation of the Table 3.

41	**247**	742	371	185,5	92,75	46,375	23,1875	11,59375	5,796875
42	**253**	760	380	190	95	47,5	23,75	11,875	5,9375
43	**259**	778	389	194,5	97,25	48,625	24,3125	12,15625	6,078125
44	**265**	796	398	199	99,5	49,75	24,875	12,4375	6,21875
45	**271**	814	407	203,5	101,75	50,875	25,4375	12,71875	6,359375
46	**277**	832	416	208	104	52	26	13	6,5
47	**283**	850	425	212,5	106,25	53,125	26,5625	13,28125	6,640625
48	**289**	868	434	217	108,5	54,25	27,125	13,5625	6,78125
49	**295**	886	443	221,5	110,75	55,375	27,6875	13,84375	6,921875
50	**301**	904	452	226	113	56,5	28,25	14,125	7,0625
51	**307**	922	461	230,5	115,25	57,625	28,8125	14,40625	7,203125
52	**313**	940	470	235	117,5	58,75	29,375	14,6875	7,34375
53	**319**	958	479	239,5	119,75	59,875	29,9375	14,96875	7,484375
54	**325**	976	488	244	122	61	30,5	15,25	7,625
55	**331**	994	497	248,5	124,25	62,125	31,0625	15,53125	7,765625
56	**337**	1012	506	253	126,5	63,25	31,625	15,8125	7,90625
57	**343**	1030	515	257,5	128,75	64,375	32,1875	16,09375	8,046875
58	**349**	1048	524	262	131	65,5	32,75	16,375	8,1875
59	**355**	1066	533	266,5	133,25	66,625	33,3125	16,65625	8,328125
60	**361**	1084	542	271	135,5	67,75	33,875	16,9375	8,46875
61	**367**	1102	551	275,5	137,75	68,875	34,4375	17,21875	8,609375
62	**373**	1120	560	280	140	70	35	17,5	8,75
63	**379**	1138	569	284,5	142,25	71,125	35,5625	17,78125	8,890625
64	**385**	1156	578	289	144,5	72,25	36,125	18,0625	9,03125
65	**391**	1174	587	293,5	146,75	73,375	36,6875	18,34375	9,171875
66	**397**	1192	596	298	149	74,5	37,25	18,625	9,3125
67	**403**	1210	605	302,5	151,25	75,625	37,8125	18,90625	9,453125
68	**409**	1228	614	307	153,5	76,75	38,375	19,1875	9,59375
69	**415**	1246	623	311,5	155,75	77,875	38,9375	19,46875	9,734375
70	**421**	1264	632	316	158	79	39,5	19,75	9,875
71	**427**	1282	641	320,5	160,25	80,125	40,0625	20,03125	10,01563
72	**433**	1300	650	325	162,5	81,25	40,625	20,3125	10,15625
73	**439**	1318	659	329,5	164,75	82,375	41,1875	20,59375	10,29688
74	**445**	1336	668	334	167	83,5	41,75	20,875	10,4375
75	**451**	1354	677	338,5	169,25	84,625	42,3125	21,15625	10,57813
76	**457**	1372	686	343	171,5	85,75	42,875	21,4375	10,71875
77	**463**	1390	695	347,5	173,75	86,875	43,4375	21,71875	10,85938
78	**469**	1408	704	352	176	88	44	22	11
79	**475**	1426	713	356,5	178,25	89,125	44,5625	22,28125	11,14063
80	**481**	1444	722	361	180,5	90,25	45,125	22,5625	11,28125
81	**487**	1462	731	365,5	182,75	91,375	45,6875	22,84375	11,42188

The proof of the above theorem is the following dependency which is arithmetic progression:

The numbers obtained by the results of the collatztization of the “minus” key-numbers $k_m^- = 6m - 1$, i.e. the number obtained by the formula $C(k^-) = (18m - 2)/2^q$, create the sequence of the numbers with a view $k_n^- = 6n - 1$ и $k_n^+ = 6n + 1$ on the six columns (Table 2), complied with the indicators of two q: 1, 2, 3, 4, 5, 6, which could be found by the formulas:

1) $k_n^- = 17 + 18t$; 2) $k_n^+ = 13 + 18t$; 3) $k_n^- = 11 + 18t$;

4) $k_n^+ = 19 + 18t$; 5) $k_n^- = 5 + 18t$; 6) $k_n^+ = 7 + 18t$,

where t= 0, 1, 2, 3 … .

The numbers obtained by the process of collatztization of the “plus” key-numbers $k_m^+ = 6m$ +1, i.e. the numbers obtained by the formula $C(k^-) = (18m + 4)/2^q$, create the sequence of the numbers with a view $k_n^- = 6n - 1$ и $k_n^+ = 6n + 1$ on six columns (Table 3), which are complied with indicators of two q: 1, 2, 3, 4, 5, 6, which could be calculated by the following formulas:

1) $k_n^- = 11 + 18t$; 2) $k_n^+ = 19 + 18t$; 3) $k_n^- = 5 + 18t$;

4) $k_n^+ = 7 + 18t$; 5) $k_n^- = 17 + 18t$; 6) $k_n^+ = 13 + 18t$,

where t = 0, 1, 2, 3…

Wherein, in the both cases the key-numbers k_n^-, which are located on the column complied with the first power of two, these numbers match with each second term of the “minus” sequence k_m^- (Table 2) and “plus” k_m^+ (Table 3) of the key-numbers. Other key numbers k_n^- , which are complied with the power of two from 2 to 6, are matched with the other terms of the sequence of the key-numbers k_m^- (Table 2) and k_m^+ (Table 3). In other words, each key-number is complied with another key-number with the exception of the key-numbers, which after collatztization became in the power of two.

From the data of Tables 2 and 3 and above patterns of the formation of the numbers with a view $k_n^- = 6n - 1$ and $k_n^+ = 6n + 1$ in the six columns, the following patterns are made:

- During the collatztization of the continuous sequence of the “minus” key-numbers, with the indicators of two q = 1, 2, 3, 4, 5, 6 the continuous sequence of the “minus” and “plus” key-numbers are created;
- During the collatztization of the continuous sequence of the “plus” key-numbers, with the indicators of two q = 1, 2, 3, 4, 5, 6 the continuous sequence of the “minus” and “plus” key-numbers are created;

The above patterns take the form of the following theorems:

Theorem 3.

3.1 If the complex of actions by the formula $(3k_m^- + 1)/2^q$ *are made with each term of the continuous sequence of the numbers with a view* $k_m^- = (6m - 1)$ *until an odd number is obtained and also limit the indicators of the power of two by numeric intervals with the length 6, starting with the first interval q=1-6, accordingly, on the each interval the continuous sequence of the numbers is created and each term of which alternately has a view* $k_n^- = (6n - 1)$ *and* $k_n^+ = (6n + 1)$*, where* $m, n, q = 1,2,3, \ldots$.

3.2 If the complex of actions by the formula $(3k_m^+ + 1)/2^q$ *are made with each term of the continuous sequence of the numbers with a view* $k_m^+ = (6m + 1)$*, until an odd number is obtained and also limit the indicators of the power of two by numeric intervals with the length 6, starting with the first interval q=1-6, accordingly, on the each interval the continuous sequence of the numbers is created and each term of which alternately has a view* $k_n^- = (6n - 1)$ *and* $k_n^+ = (6n + 1)$*, where* $m, n, q = 1,2,3, \ldots$.

It follows from Theorem 3 that, each key-number obtained by collatztization is certainly complied with two key-numbers, because two continuous sequences of the key-numbers are created by the process of the collatztization "minus" and "plus" of the key-numbers. Wherein, one out of two key-numbers will be "minus", and another one will be "plus" key-number ($k_i^-, k_j^+ \rightarrow k_n^-$ and $k_k^-, k_l^+ \rightarrow k_n^+$). Figuratively speaking, each key-number obtained by the process of the collatztization will have two parents, one of them is "minus" and another one is "plus" key-number.

This is important result. Therefore, the theorem should be formed.

Theorem 4. If the indicators of the power of two is changing by numeric intervals of the length 6 starting from q= 1-6, accordingly, on the each interval there are certainly three natural numbers with a view $v = 3t$, $k_m^- = (6m - 1)$ *and* $k_n^+ = (6n + 1)$, *which create the following equality* $(3k_m^- + 1)/2^q = (3k_n^+ + 1)/2^s = 3t/2^r$, *where* $m, n, t = 1, 2, 3, \dots;$ $q, s, r = 1, 2, 3, \dots$.

The Theorem 4 regarding the numbers $k_m^- = (6m - 1)$ u $k_n^+ = (6n + 1)$ is based on the Theorem 3, therefore, it's mainly proved. The clarification regarding the numbers multiple 3 ($v = 3t$) will be given later.

If we continue to increase the value of the key-numbers and the exponent of the power of two, then you will notice that, by the process of collatztization of some key numbers a new key-numbers are formed, which have already been obtained by collatztization of small key-numbers. For example, in a cell, which corresponds to line 78 and column 10 of Table 3, where the indicators of two is 7, the number 11 is located. The same number is located on Line 1 of this Table. The first number 11 is obtained by collatztization of number 7 (if q = 1), and the second number 11 obtained by collatztization of number 469 (when q = 7). This regularity is expressed by the following equations:

1) “Minus” key-numbers obtained by collatztization create the following equality

$(k_m^- \cdot 3+1)/2^q = (k_n^- \cdot 3+1)/2^{q+6a}$, где a=1, 2, 3, … (5)

2) “Plus” key-numbers obtained by collatztization create the following equality,

$(k_m^+ \cdot 3+1)/2^q = (k_n^+ \cdot 3+1)/2^{q+6a}$, где a=1, 2, 3, … (6)

where a =1, 2, 3, …

For example, $(7 \cdot 3+1)/2^1 = (469 \cdot 3+1)/2^{1+6}$; 22/2=1408/128; 11=11. It should be noted that, the equality (5) and (6) are calculated with any positive integer of q values. For example, let’s assume $q=2, a=1$, then if $k_2^- = 11$, $k_{121}^- = 725$ we have an equality

$(11 \cdot 3+1)/2^2 = (725 \cdot 3+1)/2^8$; 34/4=2176/256; 8,5=8,5.

From the above equality we could get a formula for calculating large-scale analog of specified key-number.

From the equality (5) follows that

$$k_n^- = [(k_m^- \cdot 3+1)2^{6a} - 1]/3. \qquad (7)$$

For example, $m=2, k_2^- = 11$ и $q=1, a=1$ is given, we should find k_n^- and n.

First of all, by the formula (7) we find k_n^-,

$$k_n^- = [(11 \cdot 3+1)2^6 - 1]/3 = 725.$$

Then calculate $n = (k_n^- + 1)/6$=(725+1)/6=121, т.е. $k_{121}^- = 725$.

The equality (6) follows that

$$k_n^+ = [(k_m^+ \cdot 3+1)2^{6a} - 1]/3. \qquad (8)$$

For example, $m=3, k_3^+ = 19$ и $q=1, a=2$ is given, we should find k_n^+ and n.

First of all, from the formula (8) we find k_n^+,

$k_n^+ = [(19 \cdot 3 + 1)2^{6 \cdot 2} - 1]/3$ = (237568-1)/3=79189.

Next calculate $n = (k_n^+ - 1)/6$ = (79189-1)/6= 13198, i.e. k_{13198}^+ =79189.

From the above formulas it follows that, equalities are formed on the basis of even numbers, so we emphasize this point. If even numbers obtained by multiplying initial key-numbers by 3 and adding 1, multiplied by 64, then divide them by complied with the power of two, increased by 6, then on each column forms the sequence of the key-number obtained by indicators of two q = 1-6 are formed. For example, let`s take the even numbers in Table 4, multiply them by 64, then on the indicators of two on the columns add 6 and make calculations. The calculation results - "scaled numbers" are listed in Table 4.

As seen in Table 4, all scaled key-numbers, formed after the collatztization of "minus" key-numbers, are located on the rows with odd-numbered when collatorized key-numbers with the indicators of the power of two q = 1-6 (first band) were located on the even-numbered rows. This means that, collatorized key-numbers of the subsequent bands of the indicators of two (q = 7-12; q = 13-18, etc.) will not be imposed upon the numbers that are located on the column with the corresponding figure of two q = 1. This pattern also applies to the collatztization the "plus" key-numbers.

It should be recalled that, earlier the pattern of the reiteration of numbers was shown by the process of collatztization of the numbers multiple 3. The reason of the reiteration of numbers with the view $6\mathrm{m} - 1$ and $6\mathrm{m} + 1$, during collatztization of small and big key-numbers as for numbers multiple 3, is the following: Digital root of the numbers shown as 2^x will be equal if indicator $x = q + 6a$, where q - initial (is starting) indicator ; $q, a = 0{,}1{,}2{,}3$ ..., and digital root of a number is determines its division on other number.

Table 4. Calculations with the indicators of two q=7-12

m	k^{-}	$3k+1$	q=7	q=8	q=9	q=10	q=11	q=12
57	**341**	1024	8	4	2	1	0,5	0,25
121	**725**	2176	17	8,5	4,25	2,125	1,0625	0,53125
185	**1109**	3328	26	13	6,5	3,25	1,625	0,8125
249	**1493**	4480	35	17,5	8,75	4,375	2,1875	1,09375
313	**1877**	5632	44	22	11	5,5	2,75	1,375
377	**2261**	6784	53	26,5	13,25	6,625	3,3125	1,65625
441	**2645**	7936	62	31	15,5	7,75	3,875	1,9375
505	**3029**	9088	71	35,5	17,75	8,875	4,4375	2,21875
569	**3413**	10240	80	40	20	10	5	2,5
633	**3797**	11392	89	44,5	22,25	11,125	5,5625	2,78125
697	**4181**	12544	98	49	24,5	12,25	6,125	3,0625
761	**4565**	13696	107	53,5	26,75	13,375	6,6875	3,34375
825	**4949**	14848	116	58	29	14,5	7,25	3,625
889	**5333**	16000	125	62,5	31,25	15,625	7,8125	3,90625
953	**5717**	17152	134	67	33,5	16,75	8,375	4,1875
1017	**6101**	18304	143	71,5	35,75	17,875	8,9375	4,46875
1081	**6485**	19456	152	76	38	19	9,5	4,75
1145	**6869**	20608	161	80,5	40,25	20,125	10,0625	5,03125
1209	**7253**	21760	170	85	42,5	21,25	10,625	5,3125
1273	**7637**	22912	179	89,5	44,75	22,375	11,1875	5,59375
1337	**8021**	24064	188	94	47	23,5	11,75	5,875
1401	**8405**	25216	197	98,5	49,25	24,625	12,3125	6,15625
1465	**8789**	26368	206	103	51,5	25,75	12,875	6,4375
1529	**9173**	27520	215	107,5	53,75	26,875	13,4375	6,71875
1593	**9557**	28672	224	112	56	28	14	7

3. THE GRAPH OF THE NUMBERS

Further, the formation of the chain of the numbers obtained by the process of the collatztization with a view $6m-1$ and $6m+1$ is explained on the basis of table 4 and 5. The chain of the numbers obtained by collatztization of the key-numbers on the basis of Tables 2 and 3 is made in the following way:

1) In the table 2 or 3, depending on the specified number, in the column k^{-} or k^{+} of the corresponding tables the specified number is defined. Let`s assume that, the odd number 29 is used, which is a key-number, it is required to build

the collatztization chain. The used number is “minus” key-number, therefore, it is located in the Table 2 (row 5, column k^{-});

2) Then, in this Table (row 5, column q =3) find key-number 11 obtained by the process of the collatztization of the number 29;
3) Further, in this Table (row 2, column q=1) we find key-number 17 obtained by the process of the collatztization of the number 11;
4) After these actions, look at the row 2, where the number 17 is located and find key-number 13 (row 3 , column q=2) obtained by the process of the collatztization of the number 17;
5) As the number 13 is a “plus” key-number, let’s move to Table 3, where follows that, by the process of collatztization of the number 13 transforms to number 5 (row 2, column q=3). Knowing that, number 5 is “minus” key-number, let’s return to the Table 4.
6) On the row of the key number 5 where the number 1 is located (row 1, column q=4), which means the end of the collatztization

The chain of numbers of the above-mentioned example looks like as:

$$29 \to 11 \to 17 \to 13 \to 5 \to 1.$$

This short chain of the collatztization was formed because; in given example the starting number is 29, which is a number with a view $6m - 1$ i.e. a key-number. If calculation will start with even number 618, then the chain of the collatztization complied with above example will have the following view, where there are even number and one number multiple 3 (309):

618 → 618/2=309 → 309·3+1=928 → 928/2=464 → 464/2=232 → 232/2=116 → 116/2=58 → 58/2=**29** →29·3+1=88 →88/2=44 → 44/2=22 → 22/2=**11** →11·3+1=34 → 34/2=**17** →17·3+1=52 → 52/2=26 → 26/2=**13** →13·3+1=40 →40/2=20 → 20/2=10 → 10/2=**5** →5·3+1=16 → 16/2= 8 → 8/2=4 →4/2=2 →2/2=**1**.

As can be seen, the last chain significantly longer than the first, at the same time, all key-numbers, i.e. with a view $k^{-} = 6m - 1$ и $k^{+} = 6m + 1,$ in the both cases are the same.

If we combine all possible chains of the numbers obtained by collatztization, then we get the directed graph. The process of the building of the graph will be made in reverse direction to the process of collatztization, namely, starting from the last odd number, which has common edge with the number 1, to the side of the expansion of the graph. Starting from final vertex, which complied with the final odd number, each vertex will split by two, this process will continue infinitely. The calculation will also be made vice versa. For determining the numbers which are complied with two vertexes, which are connected with considered vertex, the calculations will be the following:

1) The number of considered vertex is multiplied by specified power of two until the first integer number equal to $N = 3k_1 + 1$ is created, then the number $k_1 = (N - 1)/3$ is calculated, which will corresponds to the first vertex.

2) The number of considered vertex is multiplied by other power of two until the second integer number equal to $N = 3k_2 + 1$ is created, then the number $k_2 = (N - 1)/3$ is calculated, which will corresponds to the second vertex.

3) Further, the same action are made with the found numbers, which correspond to the both of the vertexes, then the same operations are repeated with the numbers following vertexes until the specified end number is reached.

It is vital to note that, the numbers k_1, k_2 are the key-numbers i.e. numbers which has a view $k^{-} = 6m - 1$ и $k^{+} = 6m + 1,$ moreover, according to the theorem 3, one of them is “minus” and another one is “plus” key-number.

The Figure 1 shows the fragment of the initial part of the graph of the key-numbers, which were obtained by the process of the collatztization of the other key-numbers on the index interval of the power of two *q*=1-6. As seen on the Figure 1 on the first band, where *q*=1-6, orientable graph is forming which is ending on the final vertex, which is connected with number 1. Each vertex of the graph has two incoming edges and one outgoing edge, if look through only one band, i.e. one numerical interval with the length of 6 with power of two. If we take into account band which corresponds to the interval with a big exponents *q*=7-12, *q*=13-18 and so on, then each vertex will has infinitely amount incoming edges, but only one outgoing edge. All edges directed to the side of the final vertex including edges from other bands.

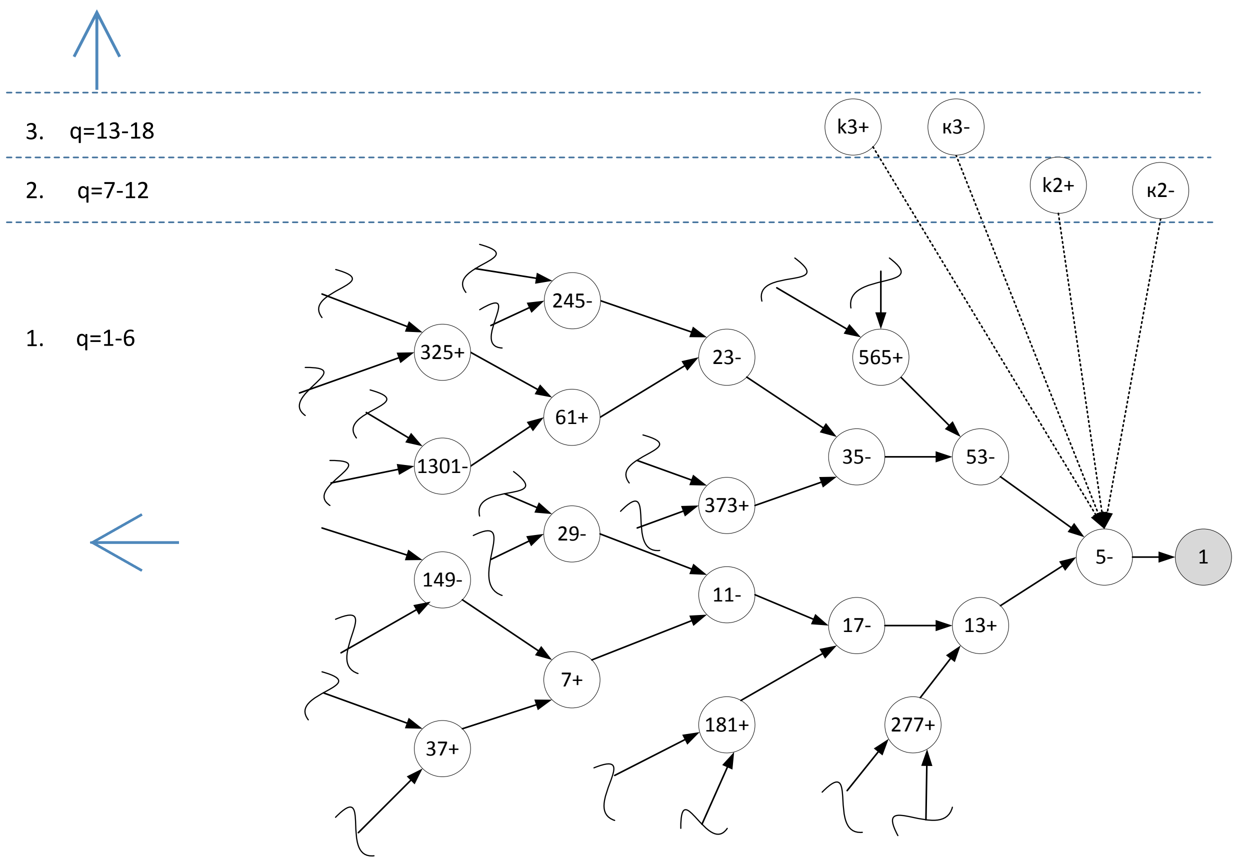

Figure 1. The fragment of the initial part of the graph of the key-numbers with the ultimate vertex 5, which are formed as a result of the collatztization of other key-numbers in the interval of the power of two *q* = 1-6.

On the Figure 1 only for the vertex 5 an incoming edges of the vertexes, which are located on the other band, are show by dotted lines. Actually, all vertexes are connected with specified vertex of other bands, the amount of the band are infinitely. The signs of the numbers on the vertexes of the graph correspond to signs of the formulas $k_m^- = (6m-1)$ and $k_n^+ = (6n+1)$.

The numbers, which by the process of the collatztization transforms to the power of two, are infinitely many, however, they are met very rare comparing with other numbers. Each of this number forms their own graph, for example, if we take such number as 85 and 341, then by the process of the collatztization of these numbers the power of two is created, it means that, they are directly connected with the number 1. On the Figures 2 and 3 the fragments of initial part of the graphs are shown, which begin with the number 85 and 341 on the index segment of the power of two *q*=1-6.

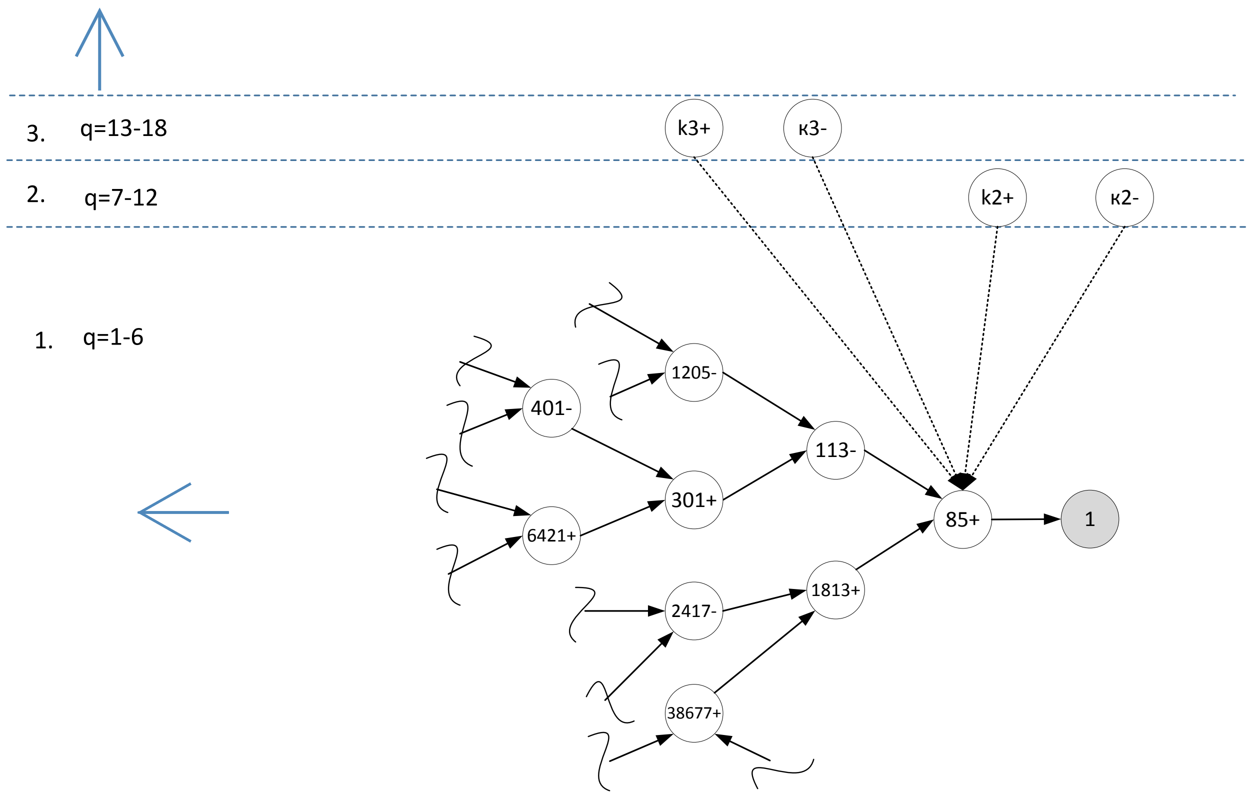

Figure 2. The fragment of the initial part of the graph of the key-numbers with a final vertex 85, which are obtained by the collatztization of the other key-numbers on the index segment power of two *q*=1-6.

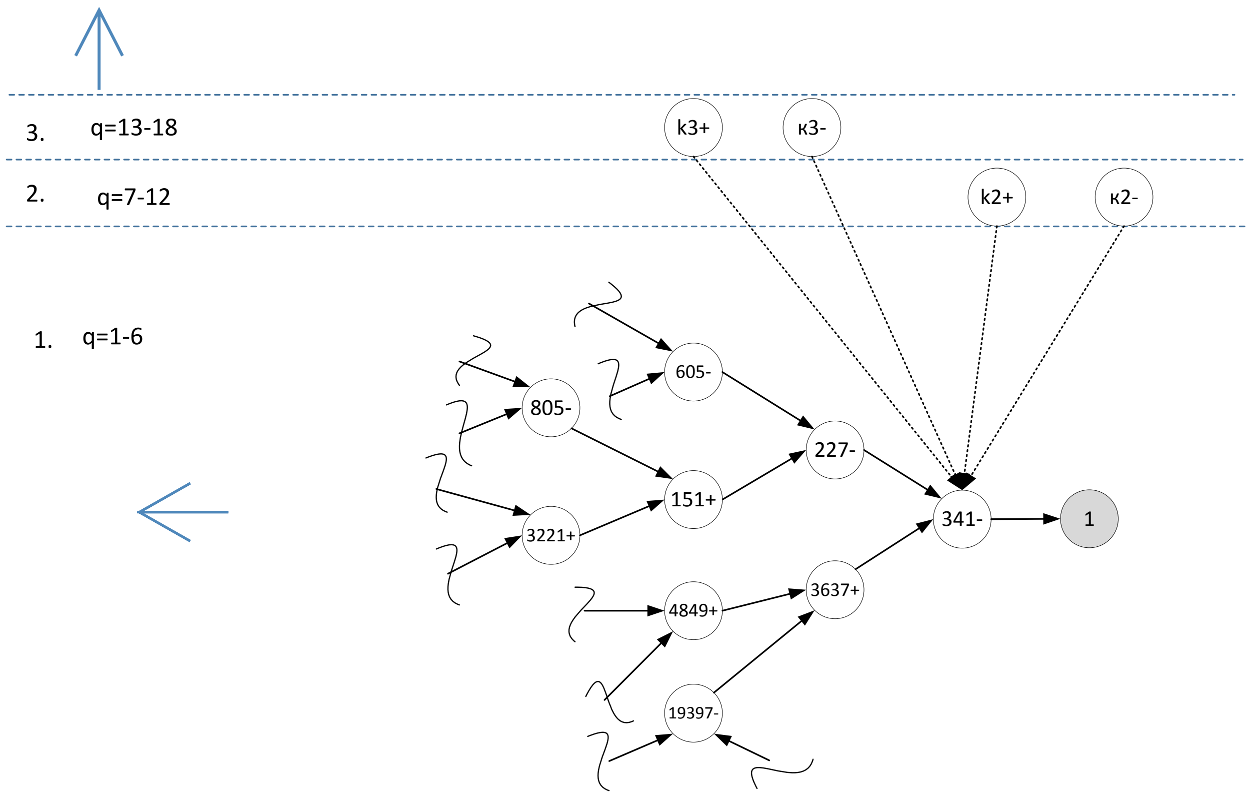

Figure 3. The fragment of the initial part of the graph of the key-numbers with a final vertex 341, which are obtained by the collatztization of the other key-numbers on the index segment power of two *q*=1-6.

On Figures 2 and 3, for the final vertex 85 and 341 an incoming edges of the vertex place on the other bands, are marked by dotted line. As in the case of number 5 all vertex of the graphs 85 and 341 are connected with specified vertexes of the other bands.

Thus, each key-number which by the process of the collatztization creates the number equal to the power of two, its graph of the key-numbers is formed. Wherein, the graphs are not cross, i.e. it do not have common vertex, however, they are place on the same bands, which are correspond to the index segment to the power of two with the length of 6.

From the above patterns, the creation of the numbers and relations between numbers, it should be understandably the features of the collatztization process.

Nevertheless, in order to learn it by heart, let's look at the process, which is reverse collatztization. Firstly, by the below formulas (9) and (10) make calculations, which are reverse collatztization. The result are show in the Table 5,

$$k_n^{\pm} = (k_m^{-} \cdot 2^q - 1)/3; \quad (9)$$

$$k_n^{\pm} = (k_m^{+} \cdot 2^q - 1)/3\,, \text{ где } q = 0{,}1{,}2{,}3\ldots. \quad (10)$$

The Table 5 shows that, all integer numbers obtained by the reverse collatztization are formed with a certain dependence. In the Table 5 cells, in which key-number "minus" and "plus" are located and marked by yellow and blue colors respectively and cells, which includes number multiple by 3 are located, are marked by green color.

At the same time it is easy to note that, each key-number, which could consists one out of two types k_m^{-} or k_m^{+} , by using formulas (9) and (10), on the index segment of two with the length 6 are complied with three numbers: k_n^{-}, k_n^{+} и $3t$, i.e. "minus" and "plus" key-numbers and also number multiple 3. Therefore, in Theorem 3 three numbers are discussed, including the number multiple 3. This fact supports the assertion made earlier that, by the process of the collatztization all numbers multiple 3 transform into key-numbers. The Table 5 is one the evidence of the correctness of the Theorem 3.

Unlike Tables 2 and 3, in Table 5 the numbers located in the column $k_m^{\pm}$ corresponds to the main vertex in the graph and key-numbers, which is located on same row as specified number (on two out of six columns of the block exponent of two), are vertexes on which the main vertex will be split.

Table 5. The calculations by the formula (9) and (10)

№№	$k_m^{\pm}$	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8	q=9	q=10	q=11	q=12
1	5	3	6,333333	13	26,33333	53	106,3333	213	426,3333	853	1706,333	3413	6826,333
2	7	4,333333	9	18,33333	37	74,33333	149	298,3333	597	1194,333	2389	4778,333	9557
3	11	7	14,33333	29	58,33333	117	234,3333	469	938,3333	1877	3754,333	7509	15018,33
4	13	8,333333	17	34,33333	69	138,3333	277	554,3333	1109	2218,333	4437	8874,333	17749
5	17	11	22,33333	45	90,33333	181	362,3333	725	1450,333	2901	5802,333	11605	23210,33
6	19	12,33333	25	50,33333	101	202,3333	405	810,3333	1621	3242,333	6485	12970,33	25941
7	23	15	30,33333	61	122,3333	245	490,3333	981	1962,333	3925	7850,333	15701	31402,33
8	25	16,33333	33	66,33333	133	266,3333	533	1066,333	2133	4266,333	8533	17066,33	34133
9	29	19	38,33333	77	154,3333	309	618,3333	1237	2474,333	4949	9898,333	19797	39594,33
10	31	20,33333	41	82,33333	165	330,3333	661	1322,333	2645	5290,333	10581	21162,33	42325
11	35	23	46,33333	93	186,3333	373	746,3333	1493	2986,333	5973	11946,33	23893	47786,33
12	37	24,33333	49	98,33333	197	394,3333	789	1578,333	3157	6314,333	12629	25258,33	50517
13	41	27	54,33333	109	218,3333	437	874,3333	1749	3498,333	6997	13994,33	27989	55978,33
14	43	28,33333	57	114,3333	229	458,3333	917	1834,333	3669	7338,333	14677	29354,33	58709
15	47	31	62,33333	125	250,3333	501	1002,333	2005	4010,333	8021	16042,33	32085	64170,33
16	49	32,33333	65	130,3333	261	522,3333	1045	2090,333	4181	8362,333	16725	33450,33	66901
17	53	35	70,33333	141	282,3333	565	1130,333	2261	4522,333	9045	18090,33	36181	72362,33
18	55	36,33333	73	146,3333	293	586,3333	1173	2346,333	4693	9386,333	18773	37546,33	75093
19	59	39	78,33333	157	314,3333	629	1258,333	2517	5034,333	10069	20138,33	40277	80554,33
20	61	40,33333	81	162,3333	325	650,3333	1301	2602,333	5205	10410,33	20821	41642,33	83285
21	65	43	86,33333	173	346,3333	693	1386,333	2773	5546,333	11093	22186,33	44373	88746,33
22	67	44,33333	89	178,3333	357	714,3333	1429	2858,333	5717	11434,33	22869	45738,33	91477
23	71	47	94,33333	189	378,3333	757	1514,333	3029	6058,333	12117	24234,33	48469	96938,33
24	73	48,33333	97	194,3333	389	778,3333	1557	3114,333	6229	12458,33	24917	49834,33	99669
25	77	51	102,3333	205	410,3333	821	1642,333	3285	6570,333	13141	26282,33	52565	105130,3
26	79	52,33333	105	210,3333	421	842,3333	1685	3370,333	6741	13482,33	26965	53930,33	107861

The continuation of the Table 5.

27	**83**	55	110,3333	221	442,3333	885	1770,333	3541	7082,333	14165	28330,33	56661	113322,3
28	**85**	56,33333	113	226,3333	453	906,3333	1813	3626,333	7253	14506,33	29013	58026,33	116053
29	**89**	59	118,3333	237	474,3333	949	1898,333	3797	7594,333	15189	30378,33	60757	121514,3
30	**91**	60,33333	121	242,3333	485	970,3333	1941	3882,333	7765	15530,33	31061	62122,33	124245
31	**95**	63	126,3333	253	506,3333	1013	2026,333	4053	8106,333	16213	32426,33	64853	129706,3
32	**97**	64,33333	129	258,3333	517	1034,333	2069	4138,333	8277	16554,33	33109	66218,33	132437
33	**101**	67	134,3333	269	538,3333	1077	2154,333	4309	8618,333	17237	34474,33	68949	137898,3
34	**103**	68,33333	137	274,3333	549	1098,333	2197	4394,333	8789	17578,33	35157	70314,33	140629
35	**107**	71	142,3333	285	570,3333	1141	2282,333	4565	9130,333	18261	36522,33	73045	146090,3
36	**109**	72,33333	145	290,3333	581	1162,333	2325	4650,333	9301	18602,33	37205	74410,33	148821
37	**113**	75	150,3333	301	602,3333	1205	2410,333	4821	9642,333	19285	38570,33	77141	154282,3
38	**115**	76,33333	153	306,3333	613	1226,333	2453	4906,333	9813	19626,33	39253	78506,33	157013
39	**119**	79	158,3333	317	634,3333	1269	2538,333	5077	10154,33	20309	40618,33	81237	162474,3
40	**121**	80,33333	161	322,3333	645	1290,333	2581	5162,333	10325	20650,33	41301	82602,33	165205
41	**125**	83	166,3333	333	666,3333	1333	2666,333	5333	10666,33	21333	42666,33	85333	170666,3
42	**127**	84,33333	169	338,3333	677	1354,333	2709	5418,333	10837	21674,33	43349	86698,33	173397
43	**131**	87	174,3333	349	698,3333	1397	2794,333	5589	11178,33	22357	44714,33	89429	178858,3
44	**133**	88,33333	177	354,3333	709	1418,333	2837	5674,333	11349	22698,33	45397	90794,33	181589
45	**137**	91	182,3333	365	730,3333	1461	2922,333	5845	11690,33	23381	46762,33	93525	187050,3
46	**139**	92,33333	185	370,3333	741	1482,333	2965	5930,333	11861	23722,33	47445	94890,33	189781
47	**143**	95	190,3333	381	762,3333	1525	3050,333	6101	12202,33	24405	48810,33	97621	195242,3
48	**145**	96,33333	193	386,3333	773	1546,333	3093	6186,333	12373	24746,33	49493	98986,33	197973
49	**149**	99	198,3333	397	794,3333	1589	3178,333	6357	12714,33	25429	50858,33	101717	203434,3
50	**151**	100,3333	201	402,3333	805	1610,333	3221	6442,333	12885	25770,33	51541	103082,3	206165
51	**155**	103	206,3333	413	826,3333	1653	3306,333	6613	13226,33	26453	52906,33	105813	211626,3
52	**157**	104,3333	209	418,3333	837	1674,333	3349	6698,333	13397	26794,33	53589	107178,3	214357

An important moment in the Table 5 is that, all key-numbers, which by the process of the collatztization form the power of two, do not exist in the columns with corresponding power of two. For example, in such columns number 5 and 85 are missed, which by the process of the collatztization give the power of two.

As mentioned, the rotation of different integer numbers in the Table 5 has a pattern on vertical and horizontal axes. These patterns are shown below, exactly the creation of the "minus" and "plus" key-numbers on the columns corresponding to the power of two, on the basis of formulas.

1) Patterns of formation of the "minus" and "plus" key-numbers on the columns with q=1-6 and t =0, 1, 2, 3 … :

$$q = 1; \quad k_n^+ = 7 + 12t; \quad k_n^- = 11 + 12t; \tag{11}$$

$$q = 2; \quad k_n^+ = 25 + 24t; \quad k_n^- = 17 + 24t; \tag{12}$$

$$q = 3; \quad k_n^+ = 13 + 48t; \quad k_n^- = 29 + 48t; \tag{13}$$

$$q = 4; \quad k_n^+ = 37 + 96t; \quad k_n^- = 101 + 96t; \tag{14}$$

$$q = 5; \quad k_n^+ = 181 + 192t; \quad k_n^- = 53 + 192t; \tag{15}$$

$$q = 6; \quad k_n^+ = 277 + 384t; \quad k_n^- = 149 + 384t, \tag{16}$$

2) Patterns of formation of the "minus" and "plus" key-numbers on the rows.

If any block of the matrix (like Table 5), which combine 6 columns and corresponding to the exponents power of two with the length of 6, marked by *b* and cells of given block with key numbers as *ij (b)*, then key-numbers of the same cells of the subsequent block with the serial number *b+1* calculated by formulas,

$$k_{ij(b+1)}^- = k_{ij(b)}^- \cdot 64 + 21; \tag{17}$$

$$k^{+}_{ij(b+1)} = k^{+}_{ij(b)} \cdot 64 + 21, \tag{18}$$

where $k^{-}_{ij(b)}$; $k^{+}_{ij(b)}$ "minus" and "plus" key-numbers respectively, located on the specified cells *ij* of the block *b* of the matrix;

$k^{-}_{ij(b+1)}$; $k^{+}_{ij(b+1)}$ "minus" and "plus" key-numbers respectively, located on the same cells *ij* of the block *b+1* of the matrix.

For example, let`s take two numbers $k^{-}_{ij(1)} = 29$ and $k^{+}_{ij(1)} = 37$ placed on the first block (*q=1-6*) of the matrix (Table 5) and calculate key-numbers, which are located in the same cell of the second (*q=7-12*) and the third block *(q=13-18)* of the matrix,

$$k^{-}_{ij(2)} = k^{-}_{ij(1)} \cdot 64 + 21 = 29 \cdot 64 + 21 = 1877;$$

$$k^{+}_{ij(2)} = k^{+}_{ij(1)} \cdot 64 + 21 = 37 \cdot 64 + 21 = 2389;$$

$$k^{-}_{ij(3)} = k^{-}_{ij(2)} \cdot 64 + 21 = 1877 \cdot 64 + 21 = 120149;$$

$$k^{+}_{ij(3)} = k^{+}_{ij(2)} \cdot 64 + 21 = 2389 \cdot 64 + 21 = 152917.$$

With a help of formulas (17) and (18) we could calculate starting key-numbers on each column of the subsequent block of the columns. Then, by adding appropriate step, we could calculate all key-numbers of the chosen column. However, by calculating in a that way you should take into account that, an increase in a serial number of the block results in an increase of steps of the formation of the key-numbers on the columns of a block. For example, the "minus" key-numbers of the column *q=3* of the first block, starting from 29, are formed with a step *48t*, while the "minus" key-numbers of the same columns *q=9* of the second block, starting from 1877, are formed with a step $3072t$ $(48 \cdot 64 = 3072)$. It means that, the formation step of the key-number of the subsequent bloc 64 times greater than formation stem of the key-number of previous block.

It should be noted that, number multiple 3, also formed on the basis of the above pattern. However, the numbers multiple 3 do not influence on the process of the collatztization, the formulas above are shown for them.

1) The patterns of the number creation which is multiple 3 on the columns, if *q=1-6:*

$q = 1;\ \ 3n = 3 + 12t;$ (19) $\quad q = 2;\ \ 3n = 9 + 24t;$ (20)

$q = 3;\ \ 3n = 45 + 48t;$ (21) $\quad q = 4;\ \ 3n = 69 + 96t;$ (22)

$q = 5;\ \ 3n = 117 + 192t;$ (23) $\quad q = 6;\ \ 3n = 405 + 384t;$ (24)

where $t = 0,1,2,3 \ldots.$

2) The patterns of the number creation which is multiple 3 on the rows.

$$3n_{ij(b+1)} = 3n_{ij(b)} \cdot 64 + 21. \qquad (25)$$

For illustration the dependency from their location in the table, all key-number of Table 5 should be replaced by serial number of the rows of the corresponding key-numbers, then we will get a Table 6.

Formulas for calculating a serial number of the "minus" and "plus" key-numbers are located on the columns, if *q=1-6*, where *t=0,1,2,3 …*,

$q = 1;\quad №(k_n^+) = 2 + 4t;\quad №(k_n^-) = 3 + 4t;$ (26)

$q = 2;\quad №(k_n^+) = 8 + 8t;\quad №(k_n^-) = 5 + 8t;$ (27)

$q = 3;\quad №(k_n^+) = 4 + 16t;\quad №(k_n^-) = 9 + 16t;$ (28)

$q = 4;\quad №(k_n^+) = 12 + 32t;\quad №(k_n^-) = 33 + 32t;$ (29)

$q = 5;\quad №(k_n^+) = 60 + 64t;\quad №(k_n^-) = 17 + 64t;$ (30)

$q = 6;\quad №(k_n^+) = 92 + 128t;\quad №(k_n^-) = 49 + 128t,$ (31)

Table 6. Serial numbers of rows corresponding to the key-numbers.

$k_m^{\pm}$	№№	q=1	q=2	q=3	q=4	q=5	q=6	q=7	q=8	q=9	q=10	q=11	q=12
5	**1**			4		17				284		1137	
7	**2**				12		49				796		3185
11	**3**	2		9				156		625			
13	**4**		5				92		369				5916
17	**5**	3				60		241				3868	
19	**6**		8		33				540		2161		
23	**7**			20		81				1308		5233	
25	**8**				44		177				2844		11377
29	**9**	6		25				412		1649			
31	**10**		13				220		881				14108
35	**11**	7				124		497				7964	
37	**12**		16		65				1052		4209		
41	**13**			36		145				2332		9329	
43	**14**				76		305				4892		19569
47	**15**	10		41				668		2673			
49	**16**		21				348		1393				22300
53	**17**	11				188		753				12060	
55	**18**		24		97				1564		6257		
59	**19**			52		209				3356		13425	
61	**20**				108		433				6940		27761
65	**21**	14		57				924		3697			
67	**22**		29				476		1905				30492
71	**23**	15				252		1009				16156	
73	**24**		32		129				2076		8305		
77	**25**			68		273				4380		17521	
79	**26**				140		561				8988		35953

By comparing formulas (11-16) with formulas (26-31) it is easy to note the following dependence:

1) To obtain the formula for “plus” key-numbers on the basis of formulas of the serial numbers, the first number must be multiplied by 3 and add 1, while the second number multiplied by 3;
2) To obtain the formula for “minus” key-numbers on the basis of formulas of the serial numbers, the first number must be multiplied by 3 and add 2, while the second number multiplied by 3;

For example, for q=1, from formulas (26) we obtained formula (11),

$$k_n^+ = 2 \cdot 3 + 1 + 3 \cdot 4t = 7 + 12t; \qquad k_n^- = 3 \cdot 3 + 2 + 3 \cdot 4t = 11 + 12t.$$

It is easy no note an unusual pattern in the Table 5 – all non-integer number have identical fraction part which is equal to 1/3. It means that, there are numbers in such cells, which are resulted by the division “plus” of the key-numbers by 3. Once again this fact proves the close connection between key-numbers and the power of two.

Another important moment from the Tables 5 and 6 is that, the interpositions of the marked cells, which are set at the beginning, do not change with a process of the tables extension, i.e. there is no shifts of the interpositions of the cells. This fact proves the correctness of the Theorem 4, i.e. each key-number, with in the range of two exponent with a length 6, corresponds one “minus” and one “plus” of the key-numbers. If the interpositions of the cell have been changed relatively to each other, then there would be a probability of the violation of the pattern described in the Theorem 4.

We note that the formulas showing the linear dependence of the regularities on the basis of which Theorems 1, 2, 3 and 4 are formulated imply the proof of all theorems using the methods of mathematical induction of mathematical logic.

4. THE PROOF OF THE COLLATZ CONJECTURE

Now, when the mechanism, structure of the Collatz function and its connections between numbers involved in the process of the collatztization are clear, we can move to the proof of the Collatz conjecture.

An important moment, which proves the correctness of the Collatz conjecture, is tree directed graph showed on the Figure 4, which is also the example of classic fractal. Such fractal graph is formed by the combination of the numbers chain obtained by the process of the collatztization of key-numbers. The exception of such graph is that, each its vertex splits by two starting from final vertex. Accordingly, this form of the graph guarantees docking of all possible ways on the final vertex, which directly connected with a number 1. Therefore, in order to prove the correctness of the Collatz conjecture it is enough to prove the following:

1) Each key-number by the process of the collatztization, which creates the number equal to the power of two and being a final vertex, forms an directed graph, of exactly the same form as shown on the Figure 2;

2) Each vertex of any graph, which is formed by the process of the collatztization of the key-numbers, corresponds only one number which is a key-number and is not repeated neither on the described graph nor in other similar graphs. If vertex of the graph corresponds to the number which doesn't have view $6m-1$ or $6m+1$, then it means a fallibility of the model based on the key-numbers.

3) None of the graphs, which correspond to the Collatz function, could have universal vertex and none of them could create closed path. If a closed path is formed within the graph, then by the process of the collatztization it would be impossible to achieve number 1, because the calculations will be endlessly repeated.

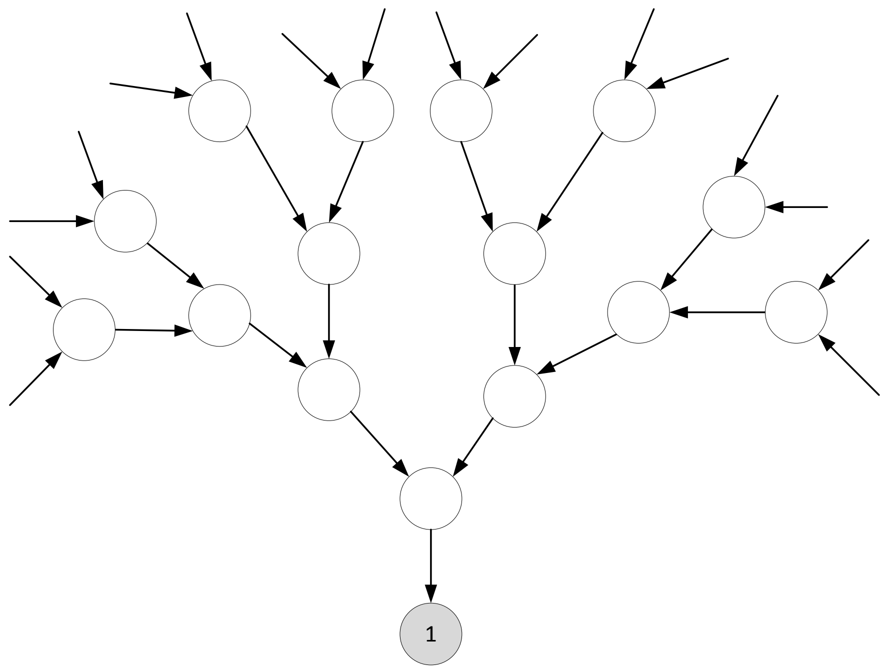

Figure 4. The tree-like directed fractal graph

Why each key -number, by the process of the collatztization of which the number equal the power of two is created, forms the directed fractal graph?

According to the Theorem 4, if the exponent of two is changing by numerical segments with a length 6 starting from segment *q=1-6*, then on each its segment only two natural numbers with a views $k_m^- = (6m - 1)$ и $k_n^+ = (6n + 1)$ will be complied with each key-number.

It means that, each key-number, by the process of the reverse collatztization, will be split into two key-numbers, moreover, one of them has a “minus” status and the other one “plus”. It follows that, key-number, which has direct connection with number 1, splits into two key-numbers. Then, each of two key-numbers also is split by two key-numbers and this process repeat endlessly. Thus, key-number, by the process of the collatztization of which the number equal to the power of two is created, is the base for graph – forms the directed graph, exactly the of same form as shown on the Figure 4.

Why each vertex of any graph, which is formed by the process of the collatization of the key-numbers, corresponds only one key-number, which is not repeated neither on the considered graph nor on the other similar graphs?

The answer for this question is the Theorem 2. The Theorem 2 describes that, by the process of the collatztization of the key-numbers only key-number are created, i.e. there could not be other numbers in the graph. Uniqueness of such numbers in the graph is described in Theorem 4, which is proved by data from the Tables 2, 3, 5, 7 and formulas in which key numbers on each column shows disjoint of subsets of the key-numbers formed on the basis of the arithmetic progression.

Why none of the graph, which is correspond to the Collatz function, could not have a common vertex and in any of them looped path could not be created?

The answer on the question above is a Theorem 3. According to this theorem on each exponent of two with the length 6, by the process of the collatztization of any key-number, only one key-number or number equal to the power of two is created, i.e. of any vertex only one edge could be created. If assume that, different graphs have a common vertex, then taking into consideration that, different graphs have the different final vertex, then, such vertex should have more than one outgoing edge, which contradicts to the Theorem 3. For the creation of the looped way within one graph, there should at least one outgoing edge which is directed not to the side of the final vertex. For example, let`s assume that, on the graph with the final vertex 5 there is a looped way: (35-) - (565+) - (53-) – (35-), as shown on the Figure 5. In this case, the vertex 35- split by three vertexes: (23-), (373+) и (53-), the vertex 565+ also split by three vertex: (35-), (3013+) and (12053-), while the vertex (53-) will not have the connection with the final vertex (5-).

It contradicts with Theorem 4. According Theorem 3, each key-number, which is located within the segment of the indicator of the power of two with the length 6, corresponds only two key-numbers.

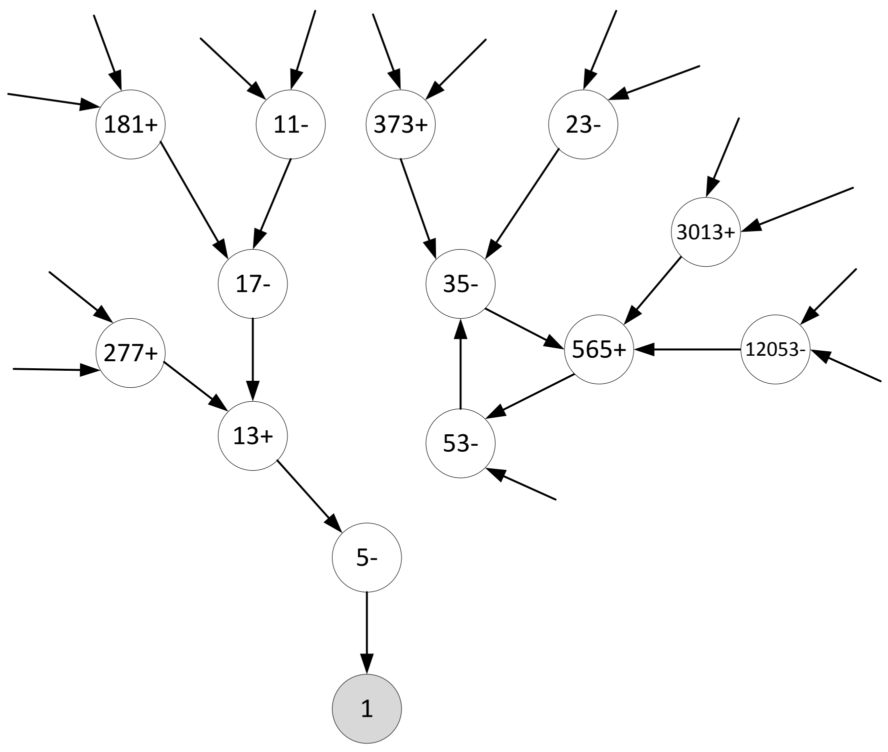

Figure 5. The example, which shows the impossibility of the creation of looped parts in the graph.

In this case, it is necessary to give the explanation of the Ttheorem 4 again. According to this theorem, if the exponent of two is changing on the numerical segments with the length 6, starting from q=1-6, then on the each of the following segment there are only three natural numbers $v = 3t$, $k_m^- = (6m-1)$ and $k_n^+ = (6n+1)$, by the process of the collatztization of these numbers only one number is obtained. It follows that, on the graph, except numbers with a view $k_m^- = (6m - 1)$ and $k_n^+ = (6n + 1)$, should has numbers with a view $v = 3t$, i.e. the numbers multiple by 3.

In fact, the graph, which shows the chains obtained by the process of the collatztization of the odd numbers, has a view as shown on the Figure 6.

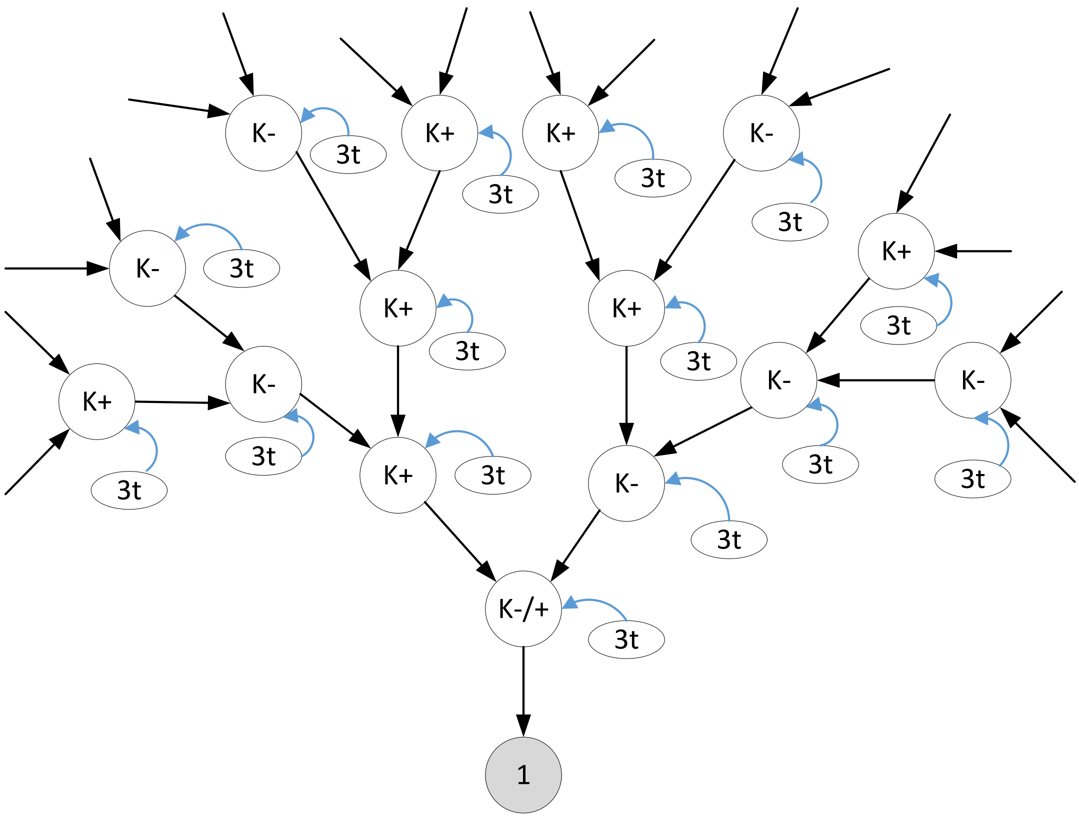

Figure 6. Tree-like directed graph with numbers multiple 3.

From the graph, which shown of the Figure 6, follows that, each vertex has an own number multiple 3, which corresponds to the Theorem 4. At the same time, the numbers multiple 3, as shown on the figure 6, do not influence on the formation of the graph structure. If we start the collatztization from the numbers multiple by 3, then the way is combined with the vertex, which corresponds to key-number, and then the way will be continued by the graph structure. The following structure corresponds to the assertion about that, by the process of the collatztization, the number multiple by 3 transform to the key-numbers.

Thus, on the basis of the above answers to the questions, we could state that, the Collatz conjecture is correct and proved. Anyway, for excluding further misunderstanding of the above proof of the correctness of the Collatz conjecture, we should answer the following questions:

1) *Why by the process of the collatztization of some number, the numbers obtained are increasing wavy and then decreasing in the same way to the number 1?*

2) *How the reliability of the stop mechanism of the progressive-return process, which is observed with Collatz function, could be explained mathematically?*

Practically, these two questions are same. Therefore, the below explanation will be related to the both questions. For answering on the above questions, the key-number used in the Tables 2 and 3 should be replace with its factor (multiplier), and then we will get the Tables 7 and 8.

In the Tables 7 and 8 the cells with the multipliers of the "minus" key-numbers are marked by yellow color, while the cells with the multipliers of the "plus" key-numbers are marked by blue color. For example, in the Table 2 "minus" of the key-number 11, which has multiplier 2, corresponds to "minus" key-number 17 with a multiplier 3. In the Table 7, the multipliers 2 (column 1) corresponds to multiplier 3 (column 2). And for "minus" key-number 17 in the Table 2 corresponds to the "plus" key-number 13. In the Table 7, the multiplier 3 (column 1) corresponds to multiplier 2 on the blue cell (column 3). The collatztization the "plus" collatztization of the key-numbers (Table 3) are also changed by multipliers of the key-numbers. At the same time, it should be noted that, the serial number of the key-numbers equal to its multipliers, i.e. #(k-) = m_k^- and #(k+) = m_k^+.

Table 7. The presentation of collatztization of the “minus” key-numbers on the basis of multipliers of key-numbers.

m_k^-	q=1	q=2	q=3	q=4	q=5	q=6
1-						
2-	3-					
3-		2+				
4-	6-					
5-			2-			
6-	9-					
7-		5+				
8-	12-					
9-					1-	
10-	15-					
11-		8+				
12-	18-					
13-			5-			
14-	21-					
15-		11+				
16-	24-					
17-				3+		
18-	27-					
19-		14+				
20-	30-					
21-			8-			
22-	33-					
23-		17+				
24-	36-					
25-						1+
26-	39-					
27-		20+				
28-	42-					
29-			11-			
30-	45-					
31-		23+				
32-	48-					
33-				6+		
34-	51-					
35-		26+				
36-	54-					
37-			14-			
38-	57-					
39-		29+				
40-	60-					

Table 8. The presentation of the collatztization of the “plus” key-numbers on the basis of multipliers of key-numbers.

m_k^+	q=1	q=2	q=3	q=4	q=5	q=6
1+	2-					
2+			1-			
3+	5-					
4+		3+				
5+	8-					
6+				1+		
7+	11-					
8+		6+				
9+	14-					
10+			4-			
11+	17-					
12+		9+				
13+	20-					
14+						
15+	23-					
16+		12+				
17+	26-					
18+			7-			
19+	29-					
20+		15+				
21+	32-					
22+				4+		
23+	35-					
24+		18+				
25+	38-					
26+			10-			
27+	41-					
28+		21+				
29+	44-					
30+					3-	
31+	47-					
32+		24+				
33+	50-					
34+			13-			
35+	53-					
36+		27+				
37+	56-					
38+				7+		
39+	59-					
40+		30+				

Further, as the result of the analysis of the interposition of cells, including the numbers with the “minus” and “plus” status (of yellow and blue cell) of the Tables 7 and 8, following pattern was found out:

The multipliers (serial numbers) of the collatorized key-numbers $m_c^{\pm}$ depend on the exponent of the power of two q and its multiplier of the collatorzing “minus” key-number m_k^- and also its multiplier of the collatorzing “plus” key-number m_k^+ i.e. $m_c^{\pm} = f(q, m_k^-)$ and $m_c^{\pm} = f(q, m_k^+)$. Defined patterns are shown in tables 9 and 10, where t = 0,1,2,…..

Before explaining the mechanism of hard sinuous increase of numbers by the process of the collatztization and its obligatory stop, we will show the scheme of creation of the chain of collatorized numbers on the basis of multipliers of key-numbers shown in Tables 7 or 8:

1) Depending on status of specified key-number from Tables 7 or 8, multiplier, corresponding to the specified number, is defined.

For example, let`s assume that “minus” key-number 23 is defined, it is necessary to build the chain of numbers on the basis of multipliers, which is created by the process of the collatztization of numbers starting from the number 23. At the beginning we should define the multiplier of the number 23, m_{23}^-=(23+1)/6=4. Further, in the Table 7 on the column m_k^- we find number 4;

2) Then, define the multiplier which is corresponds to the specified number. For doing it, find the numbers which is located at the same line as specified multiplier. For our case, the number 4 placed at the same row as 6 (column q=1);

3) Further, on the column m_k^- of the Table 7 we find number 6, as in paragraph 2 the number 4 corresponds to the number 6;

4) Repeat manipulation, described in paragraph 2, taking into account paragraph 3. In our case define the multiplier which corresponds to multiplier 6. For this, find

the number, placed on the same row with the multiplier 6. Number 9 is located at the same row as number 6 (column q=1);

5) Taking into account paragraph 4, repeat the manipulation described in the paragraph 3. For considered example, let's define the multiplier, which corresponds to the multiplier 9. For that, we find the number placed on the same row as multiplier 9. The number 18 is located on the same row as number 1 (column q=5);

6) Further, on the column m_k^- (Table 7) find the number 1, as in 5th paragraph the number 9 corresponds to the number 1;

7) The multiplier $m_k^- = 1$ corresponds to "minus" key-number 5, which is connected with number 1, which means the end of the process.

Thus, the number chain, which is consisted of the multipliers of key-numbers, will has a vies as: $(4-)$ → $(6-)$ → $(9-)$ → $(1-)$ → finish. And if instead of using multipliers, we will use key-number, then the chain will be the following: $(23-)$ → $(35-)$ → $(53-)$ → $(5-)$ →1.

From the Tables 7 and 8 it is easy to note that, only numbers located on the columns, where the exponent is *q*=1, increase the number located on the first columns and other numbers decrease. Then it follows that, if by the process of the collatztization the connection between numbers of the column m_k^- (or k_m^-) and *q=1* will be infinitely created and then the numbers obtained by the process of the collatztization will be infinitely raise. However, as known, the raise of the numbers sooner or later will stop and start reducing. In the above example, the connection between numbers of the column m_k^- and *q=1* happened two times: $(4-)$ → $(6-)$ and → $(6-)$ → $(9-)$, therefore, the raise of the number happened two times. Now we explain, why there could not be an infinite raise of the number obtained by the process of the collatztization.

Tables 3 and 8, where "plus" key-numbers and its multipliers are shown, perform subsidiary role in relation to the Tables 2 and 7. It is explained by the fact that, Tables 3 and 8 after processing the number return it to Tables 2 and 7. Therefore, the main thing is to prove the reliability of the stop mechanism of number's raise in the table 2 and 7, where "minus" key-numbers and key-numbers obtained by the process of the collatztization of the "minus" key-number and multipliers of the "minus" key-numbers and multipliers of the key-numbers obtained by the process of the collatztization are shown.

It is seen from the Table 7 that "minus" key-numbers that have multipliers (serial numbers) are even numbers, which means that for each second "minus" key-number ($\mathrm{m_k^-} = 2 + 2\mathrm{t}$) a key-number that is resulting from collatztization of first which has multiplier equal to $\mathrm{m_c^-} = 3 + 3\mathrm{t}$ is assigned.

From the given dependences, for each second "minus" key-number, after its collatztization "minus" key-number that is bigger than first in 1.5 times should be assigned. For example, let $\mathrm{m_k^-} = 4,\ \mathrm{t} = 1,$ then $\mathrm{m_c^-} = 3 + 3 \cdot 1 = 6,\ \mathrm{m_c^-}/\mathrm{m_k^-} = 6/4 = 1{,}5.$ After taking into account collatztization scheme, let $\mathrm{m_k^-} = 6,\ \mathrm{t} = 2,$ then $\mathrm{m_c^-} = 3 + 3 \cdot 2 = 9,\ \mathrm{m_c^-}/\mathrm{m_k^-} = 9/6 = 1{,}5.$

Then, the connection between numbers of the column m_k^- and *q=1* of the Table 9 could be shown in this way,

$$m_c^- = m_k^- \cdot (3/2)^x, \qquad (32)$$

where $x = 1,2,3, \dots .$

Table 9. Dependencies between multipliers of the “minus” key-numbers and collatorized key-numbers.

$\boldsymbol{k_m^-}$	$\boldsymbol{q=1}$	$\boldsymbol{q=2}$	$\boldsymbol{q=3}$	$\boldsymbol{q=4}$	$\boldsymbol{q=5}$	$\boldsymbol{q=6}$
$m_k^- = 1$	8					
$m_k^- = 2+2t$	$m_c^- = 3+3t;$					
$m_k^- = 3+4t$		$m_c^+ = 2+3t;$				
$m_k^- = 5+8t$			$m_c^- = 2+3t$			
$m_k^- = 17+16t$				$m_c^+ = 3+3t$		
$m_k^- = 9+32t$					$m_c^- = 1+3t$	
$m_k^- = 25+64t$						$m_c^+ = 1+3t$

Table 10. Dependencies between multipliers of the “plus” key-numbers and collatorized key-numbers.

$\boldsymbol{k_m^+}$	$\boldsymbol{q=1}$	$\boldsymbol{q=2}$	$\boldsymbol{q=3}$	$\boldsymbol{q=4}$	$\boldsymbol{q=5}$	$\boldsymbol{q=6}$
$m_k^+ = 1+2t$	$m_c^- = 2+3t$					
$m_k^+ = 4+4t$		$m_c^+ = 3+3t$				
$m_k^+ = 2+8t$			$m_c^- = 1+3t$			
$m_k^+ = 6+16t$				$m_c^+ = 1+3t$		
$m_k^+ = 30+32t$					$m_c^- = 3+3t$	
$m_k^+ = 46+64t$						$m_c^+ = 2+3t$

From the formula (32) follows that, by multiplying number m_k^- by the power of 1,5 an even number could be create only by a certain exponent *x*. In the considered case, the exponent *x* shows the amount of raising step. For example, in the above example, an odd number $m_c^- = 4 \cdot 1{,}5^2 = 9$ has been already created with an exponent *x*=2, which means the raise of the number stop after two step. Thus, formula (32) shows the stop mechanism of the raise of a number and proves its infallibility.

The dependency between multipliers of the "minus" and "plus" key-numbers and collattztorized key-numbers consequently shown in the Tables 9 and 10.

If the multiplier of the "minus" key-number m_k^- will be an odd number then, it will correspond to the decreasing number, which will be located on one of the column *q=2-6* (Table 7).

In case with "plus" key-number vise a versa, if multiplier "plus" of the key-number m_k^+ will be an even number then, it will correspond to decreasing number, which will be located on one of the columns *q=2-6* (Table 8).

At the same time, the more exponent 2 of the column is, the more decreasing coefficient placed within numbers. Let`s define the decreasing coefficient over columns and match Table 11 on the basis of the Tables 9 and 10.

Table 11. The increasing and decreasing coefficient of the numbers of tables 9 and 10.

q=1	***q=2***	***q=3***	***q=4***	***q=5***	***q=6***
3/2	3/4	3/8	3/16	3/32	3/64

Wherein, as was said above, only in the column q=1 the coefficient is heightening and on the other column coefficients are reducing.

The dependences, presented in the Tables 9 and 10 between multipliers and coefficients presented in the table 11 are evidence that the stop mechanism of the progressive return process will work without a hitch.

The proof that, during the process of collatztization, there could not be created closed endless cycles, by the process of the key-number only key-numbers could be obtained, which is not repeated, i.e. each key-number by collatztization transforms to other key-number. Such pairs are unique. For the creation of the endless cycles, there should be at least two or more numbers, which create similar numbers, by the process of the collatztization. Impossibility of the repetition of the numbers is proved by Theorems 1, 2 and 3 and presented in the tables 4, 5, 9 and 10. Herein, is should be noted that, by analyzing the connection of the number we should always take into account the statuses of the key-numbers.

The uniqueness of any pair of the key-numbers, obtained from two key-numbers, used in the process of the collatztization and obtained by it respectively, is the evidence that, the amount of the collatztization steps at the beginning of any number will always be endless. Due to the fact that, the uniqueness of pairs of the key-numbers guarantee that, in the process of the cycle raise and drop of the numbers and because of the impossibility of the repetition of steps and raising of the number, the pairs of the key-numbers are joining the process, having high decreasing coefficients which sharply reduce numbers.

Thus, on the basis of the above proof stages of a logical sequence of Collatz conjecture could be finalized in the following way:

1) By proving the Collatz conjecture only two numbers with a view 6m-1 and 6m+1 are considered, which named “minus” and “plus” key-numbers respectively. It is proved that, if the complex of the actions, in accordance with the conditions of the Collatz conjecture over even and multiple 3 numbers, are performed, then the numbers with a views 6m-1, 6m+1 and number 1 will be obtained (Theorem 1);

2) It is proved that, if any key-number, i.e. natural number with a view $k_m^- = 6m - 1$ or $k_m^+ = 6m + 1$ is multiplied by 3 and add 1, then obtained even number divided by the power of two until an odd number obtained, then we will get the number with a view $k_n^- = 6n - 1$ or $k_n^+ = 6n + 1$ or number 1 (Theorem 2);

3) It is proved that, if the power of two will be increased by the process of the collatztization , then the following equality will be created

$$(k_m^- \cdot 3 + 1)/2^q = (k_n^- \cdot 3 + 1)/2^{q+6a}, \text{ где } \quad a=1, 2, 3, \dots ,$$

$$(k_m^+ \cdot 3 + 1)/2^q = (k_n^+ \cdot 3 + 1)/2^{q+6a}, \text{ где } \quad a=1, 2, 3, \dots$$

Wherein, the repetition of the number with increasing exponent of two with the step 6 is explained by the following, the digital roots of numbers presented as 2^x will be equal $x=q+6a$, where q - starting index; $q, a = 0,1,2,3 \dots$, and the digital root of a number defines its division on another number.

Taking into account the above pattern, the analysis of number’s pattern was performed on the numerical segments with the length 6, corresponding to the exponent of two q, starting from segment *q=1-6*. In this connection, the below paragraphs, where the formulas with the exponent of two are shown, it is necessary to consider that, everywhere the exponent of two q is changing by numerical segments with the length 6;

4) If the complex of actions is performed by the formula $(3(6m-1)+1)/2^q$, over each component of the sequence of the "minus" key-numbers, then, the odd numbers obtained, each of which corresponds only to one component of the sequence of the "minus" key-numbers, create the sequence of the "minus" and "plus" key numbers. (Theorem 3.1);

5) If the complex of actions is performed by the formula $(3(6m+1)+1)/2^q$, over each component of the sequence of the "plus" key-numbers, then, the odd numbers obtained, each of which corresponds only to one component of the sequence of the "plus" key-numbers, create the sequence of the "minus" and "plus" key-numbers. (Theorem 3.2);

6) Paragraphs 3 and 4 show that, each component of the sequence of the key-numbers correspond to two key-numbers, one of which is "minus" (paragraph 3), and another one is "plus" (paragraph 4), which are not repeated. The soleness of these two key-numbers is proved by created sequence of the "minus" and "plus" key-numbers, as a result of collatztization, each component of which corresponds only to one key-number;

7) Further, it is proved (Theorem 4) that, if the complex of actions is performed by the formulas $(k_i^- \cdot 2^q - 1)/3$ and $(k_i^+ \cdot 2^q - 1)/3$ over each component of the sequence of the key-number, which is reverse collatztization, then each component of the sequence certainly will corresponds to three odd numbers with a view $v = 3t, k_m^- = (6m-1)$ and $k_n^+ = (6n+1)$.

It means that, three different odd numbers with a view $v = 3t,\ k_m^- = (6m-1)$ and $k_n^+ = (6n+1)$, which by the process of the collatztization gives only one key-number, certainly exist on the segment of the exponent 2 with length 6;

8) The 7th paragraph shows that, if the number multiple 3 are not taken into account, as they are transformed into key-numbers, then each key-number by using

formula $(k_i^- \cdot 2^q - 1)/3$ and $(k_i^+ \cdot 2^q - 1)/3$ is split into the "minus" and "plus" key-numbers. If one looks on this operation from the graph's point of view, then one vertex creates two other vertexes. Further, if we continue consistent creation of the vertexes and connect them by its edges, then the dendritic fractal graph, the vertexes of which correspond to its key-numbers, is formed;

9) Each key-number by the process of the collatztization transforms into even number, which is equal to the power of two, is the final vertex of fractal graph, because it has direct connection with number 1. It means that, each such number creates its own directed graph, as all edges outgoing from each vertex are directed to the side of the final vertex. It means that, the numbers corresponding to vertex of each graph are unique, i.e. the numbers, which correspond to its vertexes of the graph, are not repeated.

10) On the basis of the analysis of the key-number's multipliers, obtained by the process of the collatztization, it is proved that, the number's increase happens only if "minus" key-number is collattztorized, which has even multiplier number. If "minus" key-number has odd multiplier, then, the number is decreasing by the process of the collatztization.

11) It is found that, by the process of the collatztization of any "plus" key-number, after certain cycles, the process is redirected to the collatztization of "minus" key-numbers. It means that, the process of the collatztization of "minus" key-numbers is the determinant.

12) It is found that, with number increase, the multiplier of key-number, which is obtained by the process of the collatztization, is defined by its dependency

$m_c^- = m_k^- \cdot (3/2)^x$,

where m_k^- - is a multiplier of collattztorized key-number – an even number; *x*-exponent, which also shows the amount of the increasing steps. From the considered dependency it follows that, by multiplying even number m_k^- by the exponent of the number 1,5 an even number could be created only by certain exponent x.

This shows the work of stop mechanism of the number increase and is an evidence of its infallibility.

13) By the process of the collatztization of the key-numbers only key-numbers can be obtained. Also such pairs are unique (Theorem 2). It proves that, the amount of the steps of the collatztization always will be finite and by calculating (collatztization) there could not be infinite cycles, because by the collatztization of two or more numbers an identical numbers must be obtained. Also proved that, there could not be looped cycles on the graph, which proves impossibility of the creation of endless cycles during calculations.

14) From all vertexes of the fractal graph there is a way to the final vertex, which is connected with number 1. The creation of such graph, by the process of the collatztization of the key-numbers, is the proof of the Collatz conjecture.

According to the above patterns, the proof of the Collatz conjecture shortly could be formulated in the following way:

It is proved that, the Collatz function $C(n) = \begin{cases} n/2, & If \quad n - even, \\ 3n+1, & if \quad n - odd, \end{cases}$

which is defined on the basis of natural numbers, is equivalent to the function

$$C_k(n) = \begin{cases} n/2, \quad if\ n - even, \\ 3n+1, \quad If\ \ n - odd, \\ start\ odd\ number\ 3k+1, \\ where\ k = 6m \pm 1,\ \ m - natural\ number \end{cases}$$

The function $C_k(n)$ *differ from C (n) only that the starting number for the function* $C_k(n)$ *should be an odd with the view 3k+1, where* $k = 6m \pm 1$, *m- natural number. Further, it is proved that, the function* $C_k(n)$ *by repeating operation of any natural n, k is finished with 1. Therefore, the Collatz conjecture is correct and proved.*

As a result of analysis of the connection of odd numbers, calculated by formula 2 and 3, the following function is defined by the author, which by a cyclic execution of the operation also is finished with 1, i.e. is an analog of Collatz function.

$$K(n) = \begin{cases} n/2, \quad n - for\ even\ numbers \\ \\ n+9\ - for\ odd\ number, \\ start\ odd\ number \quad n = 6m \pm 1 \\ where\ , \quad m - natural\ number \end{cases} \tag{33}$$

The function $K(n)$ differ from *C (n)* only that the starting number for the function $K(n)$ should be an odd with the view n=$6m-1$ or n=$6m+1$. If you start key-number by using function *K(n)*, you will always eventually reach 1. The proof of the correctness of the function $K(n)$ much easier, at the same time, similar to the proof of the function $C_k(n)$, so it is not given.

I want to note that in the preparation of this article, the function $3x-1$ was also investigated and the reason for the impossibility of achieving 1 on the basis of $3x-1$ was established. Due to the fact that the volume of the article became too large, the results of the $3x-1$ study were not included in this article.

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