Special Relativity: its Inconsistency with the Standard Wave Equation

Stephen J. Crothers
thenarmis@yahoo.com
Tasmania, Australia
6th August, 2017

By means of the Lorentz Transformation, Einstein's Special Theory of Relativity purports invariance of the standard wave equation. Counter-examples, satisfying the Lorentz Transformation, and hence Lorentz Invariance, prove that the Lorentz Transformation does not in fact produce invariance of the standard wave equation. Systems of clock-synchronised stationary observers are Galilean and necessarily transform by the Galilean Transformation. Einstein's insistence that systems of Galilean observers transform, not by the Galilean Transformation, but by the non-Galilean Lorentz Transformation, is logically inconsistent. The Special Theory of Relativity is therefore logically inconsistent. Therefore, it is false. The Lorentz Transformation is meaningless.

1 Introduction

Engelhardt [1] recently proved that Einstein's method of clock-synchronisation is inconsistent with the Lorentz Transformation. I subsequently generalised his proof to all values of time $t \geq 0$ [2], in accordance with Einstein's time domain [3]. The æitology of this inconsistency is Einstein's tacit assumption that his systems of clock-synchronised stationary observers are consistent with the Lorentz Transformation. I recently proved that his assumption is false, by mathematically constructing counter-examples that satisfy the Lorentz Transformation yet are not systems of clock-synchronised stationary observers [4]. Einstein's 'system of clock-synchronised stationary observers' is actually the trivial case of a single observer, which Einstein erroneously allowed to speak for all observers (owing to his tacit false assumption), none of which are equivalent, mathematically proven in [4]. Although the counter-examples satisfy Lorentz Invariance, they do not satisfy the standard wave equation, except in one privileged case. This privileged case constitutes Einstein's 'system of clock-synchronised stationary observers', and being privileged, violates the fundamental tenet of Einstein's theory, that no observer is privileged.

It has been proven in [4] that a system of stationary observers satisfying the Lorentz Transformation cannot be clock-synchronised, and that a system of clock-synchronised observers satisfying the Lorentz Transformation cannot be stationary. In each case the set of observers is an infinite set. Only one element of each set has the appearance of being stationary and clock-synchronised. However, neither element (i.e. observer), being as it is singular and privileged, can synchronise its clock with anything, and cannot determine simultaneity with anything, owing to its singularity. Permitting any number of observers, as required by Einstein's theory, immediately reinstates the two inequivalent infinite sets of inequivalent observers. Thus, Special Relativity is logically inconsistent, and the Lorentz Transformation meaningless.

2 Lorentz Invariance

It has been shown in [4] that systems of stationary observers satisfying the Lorentz Transformation between a system K with coordinates x, y, z, t, and a system k with coordinates ξ, η, ζ, τ, respectively, has the form,

$$\begin{aligned}
&\tau = \beta\left(t_\sigma - \frac{v x_\sigma}{c^2}\right),\\
&x_\sigma = \sigma x_1,\\
&\xi_\sigma = \beta\left(x_\sigma - v t_\sigma\right) = \beta\left\{\left[\sigma\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2}{c^2}\right] x_1 - v t_1\right\},\\
&t_\sigma = t_1 + \frac{(\sigma - 1)\, v x_1}{c^2},\\
&\eta = y,\\
&\zeta = z,\\
&\beta = 1/\sqrt{1 - v^2/c^2},\\
&\sigma \in \Re,
\end{aligned} \tag{1}$$

where σ labels an observer located at the stationary position x_σ reading a clock time t_σ at that position, and $x_1 \neq 0$ is arbitrary. Setting $\sigma = 1$ yields Einstein's privileged observer, which he incorrectly allowed to speak for all observers. Interchanging the systems of coordinates and changing v to $-v$ gives the Inverse Stationary Lorentz Transformation. The system of stationary observers (1) is not clock-synchronised.

According to Special Relativity, the 'spacetime interval' is the same for all coordinate systems. Thus,

$$x^2 + y^2 + z^2 - c^2t^2 = \xi^2 + \eta^2 + \zeta^2 - c^2\tau^2. \tag{2}$$

By the Lorentz Transformation, $\eta = y$ and $\zeta = z$. Therefore,

$$x^2 - c^2t^2 = \xi^2 - c^2\tau^2. \tag{3}$$

Substituting into (3) the Stationary Lorentz Transformation

(1) yields,

$$\begin{aligned}
x_\sigma^2 - c^2 t_\sigma^2 &= \sigma^2 x_1^2 - c^2 \left[t_1 + \frac{(\sigma - 1)\, v x_1}{c^2} \right]^2 \\
&= \beta^2 \left\{ \left[\sigma \left(1 - \frac{v^2}{c^2} \right) + \frac{v^2}{c^2} \right] x_1 - v t_1 \right\}^2 - c^2 \beta^2 \left(t_1 - \frac{v x_1}{c^2} \right)^2 \\
&= \xi_\sigma^2 - c^2 \tau^2,
\end{aligned} \tag{4}$$

thus satisfying Lorentz Invariance.

It has been shown in [4] that a system of clock-synchronised observers satisfying the Lorentz Transformation between a system K with coordinates x, y, z, t, and a system k with coordinates ξ, η, ζ, τ, respectively, has the form,

$$\begin{aligned}
\tau_\sigma &= \beta \left(t - \frac{v x_\sigma}{c^2} \right) = \sigma \tau_1, \\
\xi_\sigma &= \beta \left(x_\sigma - v t \right), \\
x_\sigma &= \frac{(1 - \sigma)\, c^2 t}{v} + \sigma x_1, \\
\eta &= y, \\
\zeta &= z, \\
\beta &= 1 / \sqrt{1 - v^2/c^2}, \\
1 - \frac{v}{c} &< \sigma < 1 + \frac{v}{c}.
\end{aligned} \tag{5}$$

Interchanging the systems of coordinates and changing v to $-v$ therein gives the Inverse Clock-Synchronised Lorentz Transformation. The system of clock-synchronised observers (5) is not stationary. Setting $\sigma = 1$ yields Einstein's privileged observer, which he incorrectly allowed to speak for all observers.

Substituting into (3) the Clock-Synchronised Lorentz Transformation (5) yields,

$$\begin{aligned}
x_\sigma^2 - c^2 t^2 &= \left[\frac{(1 - \sigma)\, c^2 t}{v} + \sigma x_1 \right]^2 - c^2 t^2 \\
&= \beta^2 \left[\tfrac{(1-\sigma) c^2 t}{v} + \sigma x_1 - v t \right]^2 - c^2 \beta^2 \sigma^2 \left(t - \tfrac{v x_1}{c^2} \right)^2 \\
&= \xi_\sigma^2 - c^2 \tau_\sigma^2
\end{aligned} \tag{6}$$

thus satisfying Lorentz Invariance.

Equations (4) and (6) are identically equal only when $\sigma = 1$*:

$$\begin{aligned}
&\sigma^2 x_1^2 - c^2 \left[t_1 + \frac{(\sigma - 1)\, v x_1}{c^2} \right]^2 \\
&= \beta^2 \left[\frac{(1 - \sigma)\, c^2 t}{v} + \sigma x_1 - v t \right]^2 - c^2 \beta^2 \sigma^2 \left(t - v x_1 / c^2 \right)^2 .
\end{aligned} \tag{7}$$

*In (6), $t = t_\sigma \forall \sigma$; therefore $t = t_1$.

3 The Wave Equation

The 3-dimensional wave equation is,

$$\nabla^2 \Psi = \frac{1}{\rho^2} \frac{\partial^2 \Psi}{\partial t^2}, \tag{8}$$

where ρ is the speed of wave propagation. In the case of an electromagnetic wave $\rho = c$. For a sound wave $\rho =$ *the speed of sound* in the medium under consideration. The wave equation for an electromagnetic wave polarised in the y-direction and travelling in the x-direction with speed c is given by,

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}. \tag{9}$$

The Lorentz Transformation is purported to make the wave equation invariant for systems of clock-synchronised stationary observers in constant rectilinear relative motion:

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} \leftrightarrow \frac{\partial^2 \Psi}{\partial \xi^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial \tau^2}. \tag{10}$$

But systems of clock-synchronised stationary observers are inconsistent with the Lorentz Transformation. Consequently, the Lorentz Transformation does not make the wave equation invariant. This fact was also proven by Thornhill [5–7], from a different perspective†.

Applying the chain rule to equations (3), the differential operators are,

$$\begin{aligned}
\frac{\partial}{\partial x_1} &= \frac{\partial}{\partial \xi_\sigma} \frac{\partial \xi_\sigma}{\partial x_\sigma} \frac{\partial x_\sigma}{\partial x_1} + \frac{\partial}{\partial \tau} \frac{\partial \tau}{\partial x_\sigma} \frac{\partial x_\sigma}{\partial x_1} \\
&= \sigma \beta \left(\frac{\partial}{\partial \xi_\sigma} - \frac{v}{c^2} \frac{\partial}{\partial \tau} \right), \\
\frac{\partial^2}{\partial x_1^2} &= \sigma^2 \beta^2 \left(\frac{\partial^2}{\partial \xi_\sigma^2} - \frac{2v}{c^2} \frac{\partial^2}{\partial \xi_\sigma \partial \tau} + \frac{v^2}{c^4} \frac{\partial^2}{\partial \tau^2} \right), \\
\frac{\partial}{\partial t_1} &= \frac{\partial}{\partial \tau} \frac{\partial \tau}{\partial t_\sigma} \frac{\partial t_\sigma}{\partial t_1} + \frac{\partial}{\partial \xi_\sigma} \frac{\partial \xi_\sigma}{\partial t_\sigma} \frac{\partial t_\sigma}{\partial t_1} \\
&= \beta \left(-v \frac{\partial}{\partial \xi_\sigma} + \frac{\partial}{\partial \tau} \right), \\
\frac{\partial^2}{\partial t_1^2} &= \beta^2 \left(v^2 \frac{\partial^2}{\partial \xi_\sigma^2} - 2v \frac{\partial^2}{\partial \xi_\sigma \partial \tau} + \frac{\partial^2}{\partial \tau^2} \right).
\end{aligned} \tag{11}$$

Substituting (11) into (9) gives,

$$\left(\sigma^2 - \frac{v^2}{c^2} \right) \frac{\partial^2 \Psi}{\partial \xi_\sigma^2} - 2v \left(\sigma^2 - 1 \right) \frac{\partial^2 \Psi}{\partial \xi_\sigma \partial \tau} = \frac{1}{c^2} \left(1 - \frac{\sigma^2 v^2}{c^2} \right) \frac{\partial^2 \Psi}{\partial \tau^2}. \tag{12}$$

Only for $\sigma = 1$ is the wave equation invariant under the Stationary Lorentz Transformation: precisely Einstein's privileged observer.

†The theory of characteristics of linear partial differential equations.

Applying the chain rule to equations (5) the same differential operators (11) obtain, therefore leading again to (12). Thus, only for $\sigma = 1$ is the wave equation invariant under the Clock-Synchronised Lorentz Transformation: precisely Einstein's privileged observer.

4 Conclusions

Lorentz Invariance between stationary systems of observers and clock-synchronised systems of observers holds only for the trivial case of one privileged observer in each system. Systems of clock-synchronised stationary observers are inconsistent with the Lorentz Transformation.

The standard wave equation is not invariant under a Lorentz Transformation, except for one privileged observer.

Einstein's tacit assumption that systems of clock-synchronised stationary observers (i.e. Galilean observers) are consistent with the Lorentz Transformation is false. The Special Theory of Relativity is therefore logically inconsistent: It is therefore false.

References

[1] Engelhardt, W., Einstein's Third Postulate, *Physics Essays*, 29, 4 (2016).

[2] Crothers, S.J., Einstein's anomalous clock synchronisation, *Physics Essays*, Volume 30: Pages 246-247, 2017, https://physicsessays.org/browse-journal-2/product/1577-3-stephen-j-crothers-einstein-s-anomalous-clock-synchronization.html

[3] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905.

[4] Crothers, S.J., On the Logical Inconsistency of the Special Theory of Relativity, *American Journal of Modern Physics*. Vol. 6, No. 3, 2017, pp. 43-48. doi: 10.11648/j.ajmp.20170603.12, http://vixra.org/pdf/1703.0047v6.pdf

[5] Thornhill, C.K., Real and apparent invariants in the transformation of the equations governing wave-motion in the general flow of a general fluid, *Proc. R. Soc.* Lond. A (1993) 442, 495-504, www.etherphysics.net

[6] Thornhill, C.K., Real or Imaginary Space-Time? Reality or Relativity?, *Hadronic Journal Suppl.* 11, 3, (1996), 209-224, www.etherphysics.net

[7] Thornhill, C.K., The Foundations of Relativity, www.etherphysics.net