# Special relativity in complex space-time. Part 2. Basic problems of electrodynamics.

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**Abstract**

*This article discusses an electric field in complex space-time. Using an orthogonal paravector transformation that preserves the invariance of the wave equation and does not belong to the Lorentz group, the Gaussian equation has been transformed to obtain the relationships corresponding to the Maxwell equations. These equations are analysed for compliance with classic electrodynamics. Although, the Lorenz gauge condition has been abandoned and two of the modified Maxwell's equations are different from the classical ones, the obtained results are not inconsistent with the experience because they preserve the classical laws of the theory of electricity and magnetism contained therein. In conjunction with the previous papers our purpose is to show that space-time of high velocity has a complex structure that differently orders the laws of classical physics but does not change them.*

Keywords: *Alternative special relativity, complex space-time, Maxwell equations, paravectors, wave equation*

## Introduction

The classical special theory of relativity (STR) assumes that space-time is a 4-dimensional real structure, and Lorentz transformation is its automorphism which preserves the invariance of the wave equation. In the paravector formalism, the Lorentz transformation has a form $X' = \Lambda X \Lambda^*$, where $\Lambda$ is a complex orthogonal paravector, and the asterisk means the conjugation [2]. The article [7] shows that transformation $X' = \Lambda X$ preserves the invariance of the wave equation and also states the hypothesis that space-time is a complex structure $C \times C^3$, and it is real only locally in the observer's rest frame. Our purpose is to show that if from the mathematical side a complex space-time is more natural for describing relativistic phenomena, then the space-time is complex. We want to show that what is directly observed by an observer is only a projection of more compound phenomena on to his local real space-time, and the fact that the carrier of information is a real energy also confirms to him that the space-time is real.

For the purpose of simplifying the formulas, an universal units *(NU)* system is chosen that entails no physical constants are present in the vacuum, and linear velocities are related to the speed of light or the module of light speed is equal to 1. Since the formalism used in this article is novel (albeit it is based on a matrix calculus), it is necessary for the reader to be acquainted with the articles [6] and [7].

In space-time the object is determined by its state, i.e. the coordinates of the position $(X)$ and parameters characterizing the movement, e.g. velocity $(V)$. Both these parameters for free movement describe a pair of coordinates $V^- X$ linked by the operation of multiplication. This pair is called **phase**. STR describes objects in motion at speeds comparable to the speed of light. Movement is a change of position over time, so the concept of phase has no physical significance, just like a moment (zero time interval). For dynamic phenomena it is important what happens in the interval of time. The same is with phase that correspond to time, so we will deal with the **phase interval**, which is defined as the phase difference.

$$\Delta\Theta = V^{-}\mathbb{X} = \Delta t^{0} \tag{1}$$

The phase interval is a non-negative real number and it is equivalent to the universal time interval $\Delta t^{0}$. For this reason, it is necessary to emphasize that the real and imaginary parts of coordinates of the described objects are not independent of each other. The coordinates *t, x, y, z* are mutually independent but, their real and imaginary components are not independent. You have to keep this in mind as it may interfere with understanding the interpretation of the calculations at the beginning. This note is general but it not relevant to the calculations in the current article, because to make the discussion clear, the electromagnetic field was researched at the boundary of real and complex space-time, i.e.

$$\Delta t \in R_{+}, \qquad V = \frac{1}{\sqrt{1-v^2}}\begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix} \text{ and } \mathbf{v} \in R^3.$$

This article is the fifth in the order of the publications that relating to a complex time-space, and the whole of our project is to show that it is possible to build a theory alternative to the valid special theory of relativity. So far, the following articles have been published:

1. *Algebra of paravectors* [6]

2. *Four-divergence as a paravector operator. Invariance of the wave equation under orthogonal paravector transformation.* [7]

3. *Does Thomas-Wigner rotation show the fallacy of „Lorentz rotation"?* [8]

4. *Special relativity in complex space-time. Part 1. A choice of the domain and transformation preserving the invariance of wave equation.* [9]

# 1 Wave

It is not difficult to verify[1] that the solution of the homogeneous wave equation $(\partial^2/\partial t^2 - \nabla^2)A(X) = 0$ is fulfilled by any function:

$$A(\begin{bmatrix} \alpha \\ -\boldsymbol{\beta} \end{bmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix}) \qquad \text{or} \qquad A(\begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} \begin{bmatrix} \alpha \\ -\boldsymbol{\beta} \end{bmatrix}) \tag{2}$$

such that $A$ is a paravector function and paravector $\begin{bmatrix} \alpha \\ \boldsymbol{\beta} \end{bmatrix}$ is a singular one.

The argument of wave function are the space-time coordinates combined in pairs with another paravector. This pair we call the **phase**. It is invariant under the orthogonal paravector transformation (complex relativistic transformation).

**Conclusion 1.1.** In the case when $\alpha = 1$, $\boldsymbol{\beta} = \mathbf{c} \in R^3$ and $|\mathbf{c}| = 1$, vector $\mathbf{c}$ is interpreted as the wave speed or the speed of light in real space-time.

If we talk about a **wavefront**, then this is equivalent to the condition of identity in phase:

$$\begin{bmatrix} 1 \\ -\mathbf{c} \end{bmatrix} \begin{pmatrix} t - t_0 \\ \mathbf{x} - \mathbf{x}_0 \end{pmatrix} = C^{-}\mathbb{X} = 0 \tag{3}$$

We talk about a **plane wave** if a vector $\mathbf{c}$ is given. If a point with the coordinates $\mathbf{x}_0$ is given and vector $\mathbf{c}$ has any direction, then we talk about the **spherical wave**.

By studying this class of problems, we can use the properties of the singular parallelism [6], which gives as follows:

**Conclusion 1.2.** For a periodic function $f(C^{-}\mathbb{X})$ the 4-vector $\mathbb{T}$ is a period of the function $f$ if and only if $\mathbb{T}$ is singularly parallel to the paravector $C$.

*Proof.*

A function is periodic with the period $\mathbb{T}$ if $f(C^{-}(\mathbb{X}+\mathbb{T})) = f(C^{-}\mathbb{X})$ which occurs when $\langle C, \mathbb{T} \rangle = 0$.

[1] Use the identities proven in the article [7]

In this case the order of the paravectors does not matter because if $(\mathbb{T}, C\rangle = 0$ then $\langle C, \mathbb{T}) = 0$ also.

$\square$

From the above conclusion we can deduce that if $\langle C, \mathbb{T}) = \begin{bmatrix} 1 \\ -\mathbf{c} \end{bmatrix} \begin{bmatrix} T \\ \mathbf{p} \end{bmatrix} = 0$ then

$$\begin{cases} T = \mathbf{c}\mathbf{p} \\ \mathbf{c} = \mathbf{p}/T \\ \mathbf{c} \times \mathbf{p} = \mathbf{0} \end{cases}$$

where $T$ is the period, and $\mathbf{p}$ is the wavelength. The last equations are obvious.

If (2) is a periodic function and if we take out $T$ from the phase then we get another phase

$$\Theta = \begin{bmatrix} 1/T \\ -\mathbf{c}/T \end{bmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} = \begin{bmatrix} \omega \\ -\mathbf{k} \end{bmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} \tag{4}$$

We transform this phase by relativistic transformation

$$\Theta = \begin{bmatrix} \omega \\ -\mathbf{k} \end{bmatrix} \begin{pmatrix} t \\ \mathbf{x} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} \omega \\ -\mathbf{k} \end{bmatrix} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} t' \\ \mathbf{x}' \end{pmatrix} = \begin{bmatrix} \omega' \\ -\mathbf{k}' \end{bmatrix} \begin{pmatrix} t' \\ \mathbf{x}' \end{pmatrix}, \tag{5}$$

where

$$\begin{bmatrix} \omega' \\ -\mathbf{k}' \end{bmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} \omega + \mathbf{v}\mathbf{k} \\ -(\mathbf{k} + \omega\mathbf{v} + i\mathbf{v} \times \mathbf{k}) \end{bmatrix} \tag{6}$$

We found **Doppler law** in a paravector notation.

## 1.1 Spherical wave

Let's get back to the equation of the wave-front (3) and assume that the coordinates of the 4-vectors $\mathbb{X}$ are real, and that the spatial coordinates of the common beginning point of these 4-vectors determine the particular point $\mathbf{x}_0$ in the rest frame. Now, we will examine how the image of the light speed paravector $C$ will be viewed from the frame moving at real velocity $-\mathbf{v}$ or the arguments of the field function entangled in the phase, are transformed according to the formula:

$$\begin{bmatrix}1\\-\mathbf{c}\end{bmatrix}\begin{pmatrix}\Delta t\\ \Delta\mathbf{x}\end{pmatrix}=0 \quad\rightarrow\quad \frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\-\mathbf{c}\end{bmatrix}\begin{bmatrix}1\\-\mathbf{v}\end{bmatrix}\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{pmatrix}\Delta t\\ \Delta\mathbf{x}\end{pmatrix}= \tag{7}$$

$$=\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1+\mathbf{vc}\\-\mathbf{v}-\mathbf{c}+i\mathbf{v}\times\mathbf{c}\end{bmatrix}\begin{pmatrix}\Delta t'\\ \Delta\mathbf{x}'+i\Delta\mathbf{y}'\end{pmatrix}=0$$

By extracting time and scalar $1+\mathbf{vc}$, we obtain the condition that must be satified by the complex speed of light in relation to the speed of light in the resting frame.

$$\begin{bmatrix}1\\-\frac{\mathbf{v}+\mathbf{c}-i\mathbf{v}\times\mathbf{c}}{1+\mathbf{vc}}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{v}'+i\mathbf{w}'\end{bmatrix}=0 \tag{8}$$

By the above we conclude that

$$\mathbf{c}'=\mathbf{v}'+i\mathbf{w}' \qquad \text{where} \qquad \mathbf{v}'=\frac{\mathbf{v}+\mathbf{c}}{1+\mathbf{vc}}, \qquad \mathbf{w}'=\frac{\mathbf{v}\times\mathbf{c}}{1+\mathbf{vc}} \qquad \text{and} \qquad c'^2=1 \tag{9}$$

The above solution of the equation (8) is trivial. Since the equation (8) is a system of dependent linear equations, we can guess that there is an infinite number of velocity vectors **c**' that satisfiy this equation. Another question is, which mathematical solution has a physical meaning? In the equation below we give another velocity vector **c**' which satisfies the equation (8) and also has a physical meaning, as we will see in the next publication.

$$\mathbf{c}'=\frac{(1+\mathbf{vc})(\mathbf{v}+\mathbf{c})-(\mathbf{v}+\mathbf{c})\times(\mathbf{v}\times\mathbf{c})}{(\mathbf{v}+\mathbf{c})^2} \tag{10}$$

*Proof.*

We assume that vector **c**' is real one. Let's rewrite the equation (8)

$$\begin{bmatrix}1\\-\frac{\mathbf{v}+\mathbf{c}}{1+\mathbf{vc}}+i\frac{\mathbf{v}\times\mathbf{c}}{1+\mathbf{vc}}\end{bmatrix}\begin{bmatrix}1\\ \mathbf{c}'\end{bmatrix}=0 \tag{11}$$

In order to simplify the calculation we introduce the auxiliary vectors **A** and **B**

$$\mathbf{A} = \frac{\mathbf{v}+\mathbf{c}}{1+\mathbf{vc}} \qquad \text{and} \qquad \mathbf{B} = \frac{\mathbf{v}\times\mathbf{c}}{1+\mathbf{vc}}. \tag{12}$$

The equation (11) has a form

$$\begin{bmatrix} 1 \\ -\mathbf{A}+i\mathbf{B} \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{c}' \end{bmatrix} = 0 \tag{13}$$

After the multiplication of singular paravectors, we get a set of four equations

$$1-\mathbf{A}\mathbf{c}' = 0 \tag{14}$$

$$i\mathbf{B}\mathbf{c}' = 0 \tag{15}$$

$$\mathbf{c}'-\mathbf{A}-\mathbf{B}\times\mathbf{c}' = \mathbf{0} \tag{16}$$

$$i\mathbf{B}-i\mathbf{A}\times\mathbf{c}' = \mathbf{0} \tag{17}$$

We group the equations (15) and (16) into the first pair, and (14) and (17) into second one and we write them in the form of the paravector equations

$$\begin{bmatrix} i\mathbf{B}\mathbf{c}' \\ \mathbf{c}'-\mathbf{B}\times\mathbf{c}' \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{A} \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 1 \\ i\mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{c}' \\ i\mathbf{A}\times\mathbf{c}' \end{bmatrix}, \tag{18}$$

which we write as the products

$$\begin{bmatrix} 1 \\ i\mathbf{B} \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{c}' \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{A} \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} 1 \\ i\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{A} \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{c}' \end{bmatrix}. \tag{19}$$

Since paravectors $\begin{bmatrix} 0 \\ \mathbf{A} \end{bmatrix}$ and $\begin{bmatrix} 1 \\ i\mathbf{B} \end{bmatrix}$ are non-singular ones and $A^2 = 1+B^2$ then from both equations we get the same result

$$\begin{bmatrix} 0 \\ \mathbf{c}' \end{bmatrix} = \frac{1}{A^2} \begin{bmatrix} 0 \\ \mathbf{A} \end{bmatrix} \begin{bmatrix} 1 \\ i\mathbf{B} \end{bmatrix} \tag{20}$$

hence

$$\mathbf{c}' = \frac{(1+\mathbf{vc})(\mathbf{v}+\mathbf{c})-(\mathbf{v}+\mathbf{c})\times(\mathbf{v}\times\mathbf{c})}{(\mathbf{v}+\mathbf{c})^2} \tag{21}$$

□

Let's go back to the formula (9). We note that although the real part of the light velocity vector $\mathbf{c}'$ may be greater than 1, the length of the complex vector is always equal to 1. There is no conflict with Michelson-Morley's experiment because it is done in the light source frame and it tells us only that there is no ether. This contradiction can be explained in the same way as it was done in the explanation of the experiment in article [9] because the formula (9) is analogous to the formula (37) [9]. So we can imagine an analogous experience but made with a lamp in a mirror lab. This explains the apparent deformation and the apparent superluminal speed.

The problem is how to interpret the imaginary part of the velocity vector. It seems that on the physical side we should look at the real velocity vector as if it were rotating because the imaginary vector, perpendicular to the plane defined by the vectors $\mathbf{v}$ and $\mathbf{c}$, gives this feature to the real part of vector $\mathbf{c}$'. Although this real vector does not rotate with respect to the outer axis, the observer which is not collinear with it gets this impression. The product $\mathbf{v}\times\mathbf{c}$ shows that this is not just an impression. On the accounting side, the imaginary component of the vector is an element needed to properly balance the coordinates of the velocity vector so that its length is equal to 1.

The relativistic transformation considered by us is a four-dimensional orthogonal transformation of the four-vectors $\Lambda : \mathbb{X} \longrightarrow \mathbb{X}'$ that the integrated product (def 2.1 [6]) is invariant or $\langle\Gamma,\mathbb{X}\rangle = \langle\Gamma',\mathbb{X}'\rangle$. It means that the phase interval is invariant. For physical object paravectors $\Gamma$ and $\mathbb{X}$ are proper or singular ($\det\Gamma$ and $\det\mathbb{X} \in R_+$). Ideally, if all paravectors are real. Therefore, we will look for a physically interpretable transformation that would bring a complex orthogonal paravector $\Lambda$ to the form of the real paravector of velocity $V$ and the complex 4-vector $\mathbb{X}$ to the real form. On the other hand, we will look for interpretations of the imaginary coordinates. However, we should not deal with the coordinates of points in space-time. In physical phenomena the motion is essential i.e. the changes of coordinates ($\Delta\mathbf{x}$) in time ($\Delta t$) and valid are only these vectors. Please note that we have not calculated the coordinates of the points but their differences or the coordinates of vectors. Mathematically, this means that we are in the vector space, not in the affine one. The field disturbance moves in space at light velocity, otherwise: an argument of the field function is the phase interval which is the concept from vector space. Therefore, we pay attention that the whole time we are talking about frames, and not about Cartesian coordinate systems, which is not the same thing.

In order for the components of phase intervals to describe physical phenomena, it seems obvious that the paravectors representing them are proper (it means that they have modules). If there were superluminal speed, its paravector would have a negative determinant, so it would be improper and it could not be represented in the form of an orthogonal paravector unequivocally. We should stick to this direction because it gives us a mathematical confirmation of the empirical fact:

**Physical objects can not move at a speed faster than the speed of light**.

Since it is difficult for us to take up the complex vectors, for convenience we assume that we can find such a transformation of $\Lambda : \mathbb{X} \to \mathbb{X}'$ that $\mathbb{X}$ and $\mathbb{X}' \in R^{1+3}$. We will examine how the Maxwell's and SR theories will change as a result of the adoption of the relativistic transformation described by the real velocity paravector because such considerations will be understood. On the other hand, we will develop the knowledge of the transformations on complex paravectors, because they are complex by their nature. Later, we will try to reconcile the complex model of mathematical structure with the real physical phenomena.

We just wrote that $\mathbb{X} \in R^{1+3}$ instead of $R^4$. It is necessary to further separate the scalar part from the spatial one in the four-vector of position because the scalar part is time, and time does not run backwards. The time structure together with the + operation should be a monoid (half-group with zero). The spatial part of the structure is a 3-dimensional vector space. So we will write $\mathbb{X} \in R_+ \times R^3$ (or $C \times C^3$) and this space will be called space-time. This is not a strict definition of space-time yet, but only its important feature.

# 2 Electric field

The article [7] shows that the wave equation can be transformed in four basic ways:

Table 1

1. $\partial^{-}\partial A(X) = B(X) \quad\longrightarrow\quad \partial'^{-}\Lambda^{-}\Lambda\partial' A(X'\Lambda^{-}) = B(X'\Lambda^{-})$
2. $\partial\partial^{-} A(X) = B(X) \quad\longrightarrow\quad \partial'\partial'^{-}[\Lambda^{-} A(X'\Lambda^{-})] = \Lambda^{-} B(X'\Lambda^{-})$
3. $\partial^{-}\partial A(X) = B(X) \quad\longrightarrow\quad \partial'^{-}\partial'[\Lambda A(\Lambda^{-} X')] = \Lambda B(\Lambda^{-} X')$
4. $\partial\partial^{-} A(X) = B(X) \quad\longrightarrow\quad \partial'\Lambda\Lambda^{-}\partial'^{-} A(\Lambda^{-} X') = B(\Lambda^{-} X')$

This makes further considerations possible in different directions. However, we want to remain as close as possible to the formulas adopted in the theory of electric field, so we will deal with cases 1 and 4 because only in this case we obtain a magnetic field as a result of the transformation of the electric field.

In the paravector notation, the equations of the electrostatics have the form:

$$\partial\varphi(\mathrm{X} - X_0) = \begin{pmatrix} 0 \\ -\mathbf{E}(\mathrm{X} - X_0) \end{pmatrix} \qquad \text{and} \qquad \partial^{-}\begin{pmatrix} 0 \\ -\mathbf{E}(\mathrm{X} - X_0) \end{pmatrix} = \rho\,(X - \mathrm{X}_0) \tag{22}$$

or

$$\partial^{-}\varphi(\mathrm{X} - X_0) = \begin{pmatrix} 0 \\ \mathbf{E}(\mathrm{X} - X_0) \end{pmatrix} \qquad \text{and} \qquad \partial\begin{pmatrix} 0 \\ \mathbf{E}(\mathrm{X} - X_0) \end{pmatrix} = \rho\,(X - \mathrm{X}_0), \tag{23}$$

where $\mathrm{X}_0$ is the place in space-time at which the source of the field is located, and X is the place where the field has a specified function value. These coordinates are related to each other:

$$\mathbb{X} = X - X_0 = \begin{pmatrix} t - t_0 \\ \mathbf{x} - \mathbf{x}_0 \end{pmatrix} = \Delta t \begin{bmatrix} 1 \\ \mathbf{c} \end{bmatrix}, \qquad \text{where } |\mathbf{c}| = 1 \tag{24}$$

In other words: $\mathbb{X}$ is a singular four-vector. In the formulas (22) and (23) the straight letter denotes the coordinates of the point that directly affects the function value. In mathematical terms, it seems that only space-time distance is important, and therefore the difference of coordinates and the condition that it is a singular 4-vector. The above formulas should be understood: $\mathbf{E}(\mathrm{X} - X_0)$ is the strength of the field at the point $\mathbf{x}$ and at the moment $t$ from the charge $\rho\left(X-\mathbb{X}_0\right)$ located at the moment $t_0$ and at the point $\mathbf{x}_0$. The equivalence of reasoning constructed on the equations (22) and (23) is shown in the Appendix. Now we assume that the electric field is described by the equations (23).

At this moment, for simplicity we assume that the speed is described by the real paravector which we call the velocity paravector

$$V=\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}.$$

According to the assumptions above for the field (23) we use the transformation $\mathbb{X}'=V\mathbb{X}$ by which we obtain the equivalent equation in a moving frame:

$$\begin{bmatrix}\frac{\partial}{\partial t}\\ \nabla\end{bmatrix}\begin{pmatrix}0\\ \mathbf{E}(\mathbb{X})\end{pmatrix}=\begin{pmatrix}\rho\left(\mathbb{X}\right)\\ 0\end{pmatrix} \qquad \Leftrightarrow \qquad \begin{bmatrix}\frac{\partial}{\partial t'}\\ \nabla'\end{bmatrix}\left(\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ \mathbf{E}(V^{-}\mathbb{X}')\end{pmatrix}\right)=\begin{pmatrix}\rho\left(V^{-}\mathbb{X}'\right)\\ 0\end{pmatrix}, \tag{25}$$

so

$$\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ \mathbf{E}\end{pmatrix}=\frac{1}{\sqrt{1-v^2}}\begin{pmatrix}\mathbf{vE}\\ \mathbf{E}+i\mathbf{v}\times\mathbf{E}\end{pmatrix}=\begin{pmatrix}e'\\ \mathbf{E}'+i\mathbf{B}'\end{pmatrix}, \tag{26}$$

hence Maxwell's equations (modified) have a form:

$$\begin{bmatrix}\frac{\partial}{\partial t}\\ \nabla\end{bmatrix}\begin{pmatrix}e\\ \mathbf{E}+i\mathbf{B}\end{pmatrix}=\begin{pmatrix}\rho\\ 0\end{pmatrix} \tag{27}$$

which gives

$$\frac{\partial e}{\partial t}+\nabla\mathbf{E}=\rho \qquad\qquad \nabla\mathbf{B}=0 \tag{28}$$

$$\nabla\times\mathbf{B}-\frac{\partial\mathbf{E}}{\partial t}=\nabla e \qquad\qquad \frac{\partial\mathbf{B}}{\partial t}+\nabla\times\mathbf{E}=0 \tag{29}$$

The size of $e$ we call the **scalar induction** (by analogy to the vector potential) and this is a something that does not exist in the obtained theory of electric field in accordance with the condition of the Lorenz gauge.

In the resting frame, the potential equations have the form:

$$\begin{bmatrix}\frac{\partial}{\partial t}\\ -\nabla\end{bmatrix}\begin{pmatrix}\varphi\left(\mathbb{X}\right)\\ 0\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{E}(\mathbb{X})\end{pmatrix}, \tag{30}$$

then based on the article [7] Theorem 2.3 identity 2, the formula above in the moving frame takes the form

$$\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ -\mathbf{v}\end{bmatrix}\begin{bmatrix}\frac{\partial}{\partial t'}\\ -\nabla'\end{bmatrix}\begin{pmatrix}\varphi(V^-\mathbb{X}')\\ 0\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{E}(V^-\mathbb{X}')\end{pmatrix}. \tag{31}$$

We obtain the same as above the 4-vector of strength-induction

$$\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ \mathbf{E}\end{pmatrix}=\frac{1}{\sqrt{1-v^2}}\begin{pmatrix}\mathbf{vE}\\ \mathbf{E}+i\mathbf{v}\times\mathbf{E}\end{pmatrix}=\begin{pmatrix}e'\\ \mathbf{E}'+i\mathbf{B}'\end{pmatrix}. \tag{32}$$

The potential, strength and induction of the electromagnetic field are interrelated

$$\frac{\partial\varphi'}{\partial t'}=e', \qquad \nabla\varphi'=-\mathbf{E}'-i\mathbf{B}' \tag{33}$$

In above case, the energy density of the field is

$$W'=\frac{1}{2}\mathbb{F}'\mathbb{F}'^*=\frac{1}{2}\begin{pmatrix}e'\\ \mathbf{E}'+i\mathbf{B}'\end{pmatrix}\begin{pmatrix}e'\\ \mathbf{E}'-i\mathbf{B}'\end{pmatrix}=\begin{bmatrix}\frac{e'^2+E'^2+B'^2}{2}\\ e'\mathbf{E}'+\mathbf{E}'\times\mathbf{B}'\end{bmatrix} \tag{34}$$

From the equation above it follows that the boost changes the energy according to the Lorentz transformation named by W.Baylis the *Lorentz rotation* [2].

$$W'=\frac{1}{2}\mathbb{F}'\mathbb{F}'^*=\frac{V\mathbb{F}(V\mathbb{F})^*}{2}=\frac{V\mathbb{F}\mathbb{F}^*V^*}{2}=VWV^* \tag{35}$$

The results above are summarized in the table below

Table 2

| | Rest frame | Moving frame |
|---|---|---|
| Charge density | $\rho(\mathbb{X})\in R$ | invariant |
| Field strength | $\mathbb{F}(\mathbb{X})\in\{0\}\times R^3$ | $\mathbb{F}'(V^-\mathbb{X}')=V\mathbb{F}(V^-\mathbb{X}')\in R\times C^3$ |
| Potencial | $\varphi(\mathbb{X})\in R$ | invariant |
| Potential energy density | $\frac{\varphi(\mathbb{X})\rho(\mathbb{X})}{2}\in R_+$ | invariant |
| Field energy density | $\frac{\mathbb{F}(\mathbb{X})\mathbb{F}^*(\mathbb{X})}{2}\in R_+$ | $\frac{\mathbb{F}'(V^-\mathbb{X}')\mathbb{F}'^*(V^-\mathbb{X}')}{2}=\frac{V\mathbb{F}(V^-\mathbb{X}')\mathbb{F}^*(V^-\mathbb{X}')V}{2}\in R_+\times R^3$ |

Please note the choice of the field equation (23) and the transformation because the Maxwell equations (modified!) are not always obtained. In the reasoning above the scalar induction is a real field but, the value of this function can be complex numbers when boosts are composed.

Looking at the formulas in Table 2 we can see that they are not completely consistent with the classical theory because if the equation of the form 1 or 4 is transformed then values of potential and charge density are invariant. For these cases it is possible to interpret the magnetic field and there is formed a scalar size which in the classic theory of electric field, according to the Lorenz gauge condition, is absent.

In cases 2 and 3 the strength of electric field is invariant (the magnetic field does not have a mathematical explanation!), but we obtain: the current density, vector potential and justification of the Lorenz's gauge condition.

To obtain all the classical components of the field and to satisfy the invariance condition of the wave equation, we would have to multiply the equation (25) on the right side by $\Lambda$, but that would be an artificial solution, because we would get an absurd Maxwell equation. Moreover, we could do it with any orthogonal paravector and we would get a correct mathematical expression but with no physical meaning. For example:

$$\partial(VF) = \rho \qquad / \cdot V_1 \qquad \text{hence} \qquad \partial(VFV_1) = \rho V_1$$

Conclusion: **In the complex space-time with the assumption that the relativistic boost has the form of $\mathbb{X}' = \Lambda\mathbb{X}$ (or $\mathbb{X}' = \mathbb{X}\Lambda$), the concepts of the magnetic field and current density can not exist together in Maxwell's equations!**

Although, the obtained results are a little different from the textbook formulas, in practice they do not change anything instead they are differently ordered according to the laws of Ampere, Gauss, and Faraday in the wave equations. Do not be discouraged! In practice, we are only conducting mathematical considerations. If scientists contemplate an existence of the magnetic charges (Dirac) or superluminal speeds then why are we not be able to try to modify some classic theories, but sticking to their classical principles. In this situation, **we are abandoning the Lorenz gauge condition**. The consequences of this step are:

- Removal of current density from Maxwell's equations,
- Resignation of the vector potential,
- Introduction of the scalar element into induction 4-vector.

Although the change is revolutionary, it greatly simplifies and reorganizes the theory. For the sake of justification let us note that the proposed changes have historical, intuitive and practical justification:

1. The vector potential was introduced by scientists that had looked for the general solutions of the wave equation who had not know in what mathematical structure the equation is, and that it is a purely mathematical rather than in a physical size [5]chap. 6.5. In our work it is assumed that this structure is not known either, but we will define it in the future.
2. The Lorenz gauge condition is a purely theoretical assumption for the purpose of refining the abstract concept of the electric field potential so that the field is described by a wave equation. Since we are coming out of the wave equation, and the Lorenz gauge bothers us, then we have a legitimate right to ignore it.
3. We chose variants at which the field equations have no vector potential and no current density either. Looking at the transformation formulas known from STR we come to the conclusion that it would not be wrong if that was so, since the density of the charge should be as invariant as the charge, because the shape would be invariant.
4. Experimentally, we have an access only to sizes associated with the transferred energy because energy is the information carrier. All other sizes are abstract and they are used to build a clear mathematical model.

## 2.1 A field of the point charge

Let $\mathbb{X}$ be a four-vector describing the coordinates of an inertial moving object or it satisfies the equation

$$\mathbb{X}^0 = V^- \mathbb{X} \qquad \text{or} \qquad \Delta t^0 = V^-(X - X_0).$$

The potential of an electric field with spherical symmetry around a stationary charge placed at point $X_0$ describes the function

$$\varphi(X - X_0) = \frac{1}{r} q(C^-(X - X_0)) \tag{36}$$

where $C$ is a singular paravector e.g. $C = \begin{bmatrix} 1 \\ \mathbf{c} \end{bmatrix}$. For wave-front the argument is the same phase $|\mathbb{X}| = 0$, or

$$\mathbb{X} = X - X_0 = \begin{pmatrix} t - t_0 \\ \mathbf{x} - \mathbf{x}_0 \end{pmatrix} = \begin{pmatrix} r \\ \mathbf{r} \end{pmatrix} \qquad \text{, where } r = |\mathbf{r}|.$$

The function $\varphi(C^-(X - X_0)) = r^{-1} q(C^-(X - X_0))$ is a scalar function with a value of $q/r$ spread over the coordinates involved in the phase $C^-(X - X_0)$. The point $X_0$ is interpreted as time and space location of the charge $q$ being the source of the field. Same, from the above we have $\mathbf{r} = \mathbf{x} - \mathbf{x}_0 = \mathbf{c}(t - t_0) = \mathbf{c}\Delta t$.

For these deliberations we have selected the transformation and the wave equation which make sure that the potential is an invariant scalar function with respect to this orthogonal transformation. An argument of the potential function is the space-time distance from the charge ($\mathbb{X}$). The denominator is $r$, but keep in mind that it is the length of the vector $\mathbf{r}$ only in the resting frame of the charge. In every other frame $r$ is not the spatial distance, because the vector $\mathbf{r}$ is not invariant. Only an observer stationary to the charge can treat the value $r$ in the denominator as a distance. In order to have no doubt, it is better to interpret $r$ as the phase distance $r = \Delta t$ and the vector $\mathbf{c} = \mathbf{r}/r$ as the phase direction.

The field strength is not invariant. When transforming a formula (26) describing the field strength, the value of $r$ remains invariant as the formula of the potential because it is the phase distance equal to the distance from the source but, only if the source is at rest (or it is moving at non-relativistic speed). Otherwise, $r$ is only a factor influencing the value of the field inversely proportionally than the charge.

For an observer passing to a frame moving at a speed **-v** the strength is transformed

$$\partial^{-}\begin{pmatrix}\varphi(C^{-}(X-X_0))\\ 0\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{E}(C^{-}(X-X_0))\end{pmatrix} \qquad \longrightarrow \tag{37}$$

$$\longrightarrow \qquad V^{-}\partial'^{-}\begin{pmatrix}\varphi\left((VC)^{-}(X'-X_0')\right)\\ 0\end{pmatrix}=\begin{pmatrix}0\\ \mathbf{E}\left((VC)^{-}(X'-X_0')\right)\end{pmatrix}.$$

In the frame of charge, the electric field at the point $X$ from the charge located at the point $X_0$ is described by the formula:

$$\mathbf{E}=-\nabla\frac{1}{r}q\left(C^{-}(X-X_0)\right)=\frac{\mathbf{r}}{r^3}q\left(C^{-}(X-X_0)\right)=\frac{\mathbf{c}}{r^2}q\left(C^{-}(X-X_0)\right) \tag{38}$$

From the equation above and based on the relationship (37) the 4-vector of the field strength in a frame moving at a speed **-v** becomes:

$$\begin{pmatrix}e\\ \mathbf{E}'\end{pmatrix}=\frac{1}{\sqrt{1-v^2}}\begin{bmatrix}1\\ \mathbf{v}\end{bmatrix}\begin{pmatrix}0\\ \frac{\mathbf{c}}{r^2}q\left(C'^{-}\mathbb{X}'\right)\end{pmatrix}, \tag{39}$$

where $\mathbb{X}'$ defines a new space-time distance from the source of the field, and the product $VC$ is replaced by the paravector $C'$.

From the last formula anyone can see that:

- The scalar induction, introduced as a result of the rejection of the Lorenz gauge condition, is equal to

$$e\left(\mathbb{X}'\right)=\frac{\mathbf{vc}}{r^2\sqrt{1-v^2}}q(C'^{-}\mathbb{X}') \tag{40}$$

- The electric field strength is

$$\mathbf{E}\left(\mathbb{X}'\right)=\frac{\mathbf{c}}{r^2\sqrt{1-v^2}}q(C'^{-}\mathbb{X}') \tag{41}$$

- magnetic induction

$$\mathbf{B}(\mathbb{X}') = \frac{\mathbf{v}\times\mathbf{c}}{r^2\sqrt{1-v^2}} q(C'^{-}\mathbb{X}') \tag{42}$$

where $r$ is the phase distance and $\mathbf{c}$ is the phase direction (unit vector) transferred from the old system [2]. The obtained equations describe the field of a moving point charge.

Let's now look at the relation between the above equations and Maxwell's equations. In complex model, the equations of electrostatics $\partial^{-}\mathbb{F} = \rho$ transformed by relativistic transformation have the form:

$$\frac{\partial \mathbf{vE}}{\partial t} + \nabla\mathbf{E} = \rho\sqrt{1-v^2} \tag{43}$$

$$\nabla(\mathbf{v}\times\mathbf{E}) = 0 \tag{44}$$

$$\frac{\partial(\mathbf{v}\times\mathbf{E})}{\partial t} = -\nabla\times\mathbf{E} \tag{45}$$

$$\frac{\partial \mathbf{E}}{\partial t} + \nabla(\mathbf{vE}) = \nabla\times(\mathbf{v}\times\mathbf{E}) \tag{46}$$

By introducing $\mathbf{B} = \mathbf{v}\times\mathbf{E}$ we can see that the equations (44) and (45) are not different from the known Maxwell equations. For velocity $v \ll c$ the equation (43) also takes a familiar form. However, the formula (46) is not consistent with the obvious theory. Let's discuss this problem.

## 2.2 The field in the environment of the wire in which direct current flows

As the first step we calculate the value of the product **vE** at any point of the space for the case of field resulting from non-relativistic current flowing in a loop, because with these currents we have to deal in practice. The consequence of assuming non-relativistic current velocity is that in an infinitesimal interval of time we can assume that the current in the circuit is constant and the same in every circuit segment.

[2] in the new frame the phase direction is a complex vector and its imaginary part is a vector $\mathbf{v}\times\mathbf{c}$ because the resulting strength vector is in form $\mathbf{E}'(\mathbb{X}') = \mathbf{E}(\mathbb{X}') + i\mathbf{B}(\mathbb{X}')$

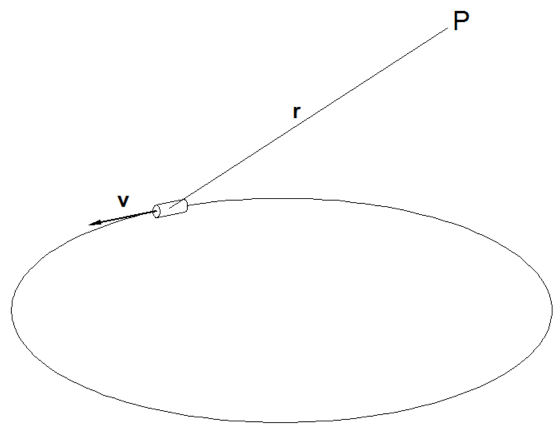


Figure 1:

Scalar induction from a particular section is

$$de(P) = \mathbf{v}d\mathbf{E}(P) = \rho\left(\mathbf{vr}/r^3\right)(\mathbf{s}d\mathbf{l}), \tag{47}$$

which gives

$$de(P) = \rho\left(\mathbf{vs}/r^3\right)(\mathbf{r}d\mathbf{l}), \tag{48}$$

because $\mathbf{v} \parallel d\mathbf{l}$ or cosinus of an angle between vectors $\mathbf{s}$ and $d\mathbf{l}$ is the same as between vectors $\mathbf{s}$ and $\mathbf{v}$. Same with the vector $\mathbf{r}$.

Since $\mathbf{sv}\rho = J$ is a current flowing in the circuit, the scalar induction $e$(P) at any point P from the circuit is:

$$e(P) = J\oint \frac{\mathbf{r}}{r^3} d\mathbf{l} = J\iint \nabla \times \frac{\mathbf{r}}{r^3} d\mathbf{s} = 0 \tag{49}$$

**Conclusion 2.1.**

**Conclusion 2.2.** The field formed by the current flowing in the loop has no scalar induction.

The above result is important because it shows and confirms that in practice we do not see scalar induction. In macroscopic systems we always deal with closed current circuits.

Equations (43) - (46) take the form

$$\nabla\mathbf{E} = \rho\sqrt{1-v^2} \tag{50}$$

$$\nabla\mathbf{B} = 0 \tag{51}$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \tag{52}$$

$$\frac{\partial\mathbf{E}}{\partial t} = \nabla\times\mathbf{B} \tag{53}$$

**Biot-Savart's law.**

We assume that the wire has a constant cross-section and that the charge density is constant along the wire.

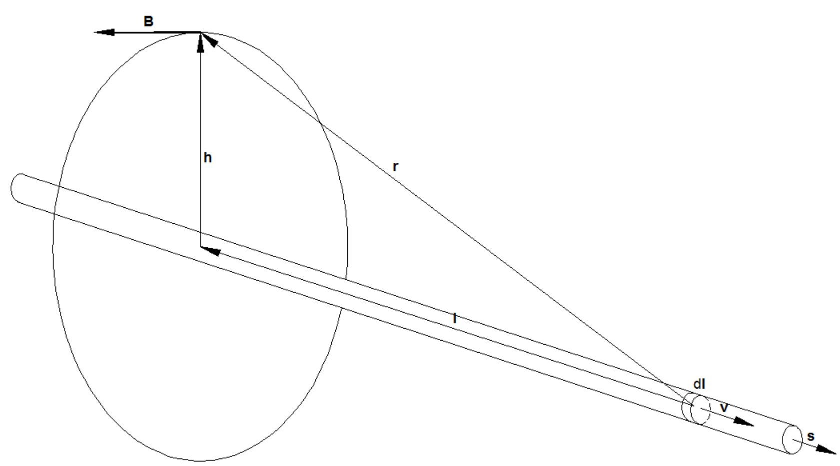


Figure 2:

Let's integrate at any point the field from the charges flowing at the relativistic velocity **v** in the wire (Fig. 2).

$$\begin{pmatrix} e \\ \mathbf{E}+i\mathbf{B} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}}\int_{-\infty}^{+\infty}\begin{bmatrix} 1 \\ \mathbf{v} \end{bmatrix}\begin{pmatrix} 0 \\ \frac{(\mathbf{l}+\mathbf{h})}{r^3}\rho\,(\mathbf{s}d\mathbf{l}) \end{pmatrix} \tag{54}$$

We get three integrals:

1. $e = \frac{\rho}{\sqrt{1-v^2}}\int_{-\infty}^{+\infty}\frac{\mathbf{lv}}{r^3}\mathbf{s}d\mathbf{l}$

2. $\mathbf{E} = \frac{\rho}{\sqrt{1-v^2}}\int_{-\infty}^{+\infty}\frac{\mathbf{r}}{r^3}\mathbf{s}d\mathbf{l}$

3. $\mathbf{B} = \frac{\rho}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{\mathbf{v}\times\mathbf{h}}{r^3} \mathbf{s} d\mathbf{l}$

Since $\mathbf{s} \parallel \mathbf{v} \parallel \mathbf{l} \perp \mathbf{h}$ we conclude that

**Conclusion 2.3.** .

1. $e = \frac{\rho}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{\mathbf{lv}}{r^3} \mathbf{s} d\mathbf{l} = \frac{\rho}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{lv}{r^3} s\, dl = \frac{J}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{l}{\left(\sqrt{l^2+h^2}\right)^3} dl = 0$
2. In the conductor there are as many negative charges as positive ones, only negative ones move and thei field is equal to $\mathbf{E}_- = \frac{\rho_-}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{\mathbf{r}}{r^3} \mathbf{s} d\mathbf{l}$. The field from dissimilar charges differ in dilatation coefficient $\mathbf{E}_+ = \rho_+ \int\limits_{-\infty}^{+\infty} \frac{\mathbf{r}}{r^3} \mathbf{s} d\mathbf{l}$. In practice, the resultant electric field around the conductor is zero, because the electrons in the wires move at non-relativistic speed.
3. $\mathbf{B} = \frac{\rho}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{\mathbf{v}\times\mathbf{h}}{r^3} \mathbf{s} d\mathbf{l} = \frac{\mathbf{j}\times\mathbf{h}}{\sqrt{1-v^2}} \int\limits_{-\infty}^{+\infty} \frac{1}{\left(\sqrt{l^2+h^2}\right)^3} dl = \frac{2\mathbf{j}\times\mathbf{h}}{h^2\sqrt{1-v^2}} \lim_{l\to\infty} \frac{l}{\sqrt{l^2+h^2}} = \frac{2\mathbf{j}\times\mathbf{h}}{h^2\sqrt{1-v^2}}$

   The last equation is Biot-Savart's law.

## 2.3 Integral field equations

Starting from the known Gaussian, Stokes and Maxwell integral equations (without the currents, since they should be consistent with (50) - (53)), we can derive their complex integral counterparts.

Gauss equation: $$\oiint (\mathbf{f} + i\mathbf{g})\, d\mathbf{s} = \iiint \nabla(\mathbf{f} + i\mathbf{g})\, d\Omega$$

Stokes law: $$\oint (\mathbf{f} + i\mathbf{g})\, d\mathbf{l} = \iint \nabla \times (\mathbf{f} + i\mathbf{g})\, d\mathbf{s}$$

Maxwell's equations (ME): $$\oiint (\mathbf{E} + i\mathbf{B})\, d\mathbf{s} = \iiint \rho\, d\Omega$$

(ME without current): $$\oint (\mathbf{E} + i\mathbf{B})\, d\mathbf{l} = \iint \left(\frac{\partial \mathbf{E}}{\partial t} + i\frac{\partial \mathbf{B}}{\partial t}\right) i\, d\mathbf{s}$$

From the equations above we get dependencies:

$$\iint \left[\frac{\partial}{\partial t}(\mathbf{E}+i\mathbf{B})+i\nabla\times(\mathbf{E}+i\mathbf{B})\right] i\,d\mathbf{s}=0 \tag{55}$$

$$\iiint \left[\nabla(\mathbf{E}+i\mathbf{B})-\rho\right] d\Omega=0 \tag{56}$$

In order to be compatible with our equations, the equation (55) requires the scalar field gradient $\nabla e$ is required, and the equation (56) requires the differential of this field over time

$$\iint \left[\frac{\partial}{\partial t}(\mathbf{E}+i\mathbf{B})+\nabla e+i\nabla\times(\mathbf{E}+i\mathbf{B})\right] i\,d\mathbf{s}=0 \tag{57}$$

$$\iiint \left[\nabla(\mathbf{E}+i\mathbf{B})+\frac{\partial e}{\partial t}-\rho\right] d\Omega=0 \tag{58}$$

Certainly, the conditions

$$\iint \nabla e\,ds=0 \qquad \text{and} \qquad \iiint \frac{\partial e}{\partial t}d\Omega$$

are met for the fields around stationary charges as well as currents flowing in the loop, because scalar induction does not occur in these cases. Based on these results, we can conclude that Gauss and Stokes theorems are also invariant when integrations take place on a surface or contour independent of time.

## 2.4 Ampere's law

Let's go back to formula (42). The magnetic field at the point $X'$ from the charges distributed throughout the space can be described by the equation

$$\mathbf{B}\left(X'-X_0'\right)=\int \frac{\mathbf{v}\times\mathbf{c}}{|V|\,r^2}q\left(\left(VC\right)^{-}\left(X'-X_0'\right)\right)d^3x_0'$$

and since $\frac{\mathbf{v}}{|V|}q\left(C'^{-}\left(X'-X_0'\right)\right)=\mathbf{j}\left(C'^{-}\left(X'-X_0'\right)\right)$, which the current density that flowed in point $\mathbf{x}_0$ at moment $t_0$, then we get the dependence

$$\mathbf{B}\left(X'-X_0'\right)=\int \frac{\mathbf{j}\left(C'^{-}\left(X'-X_0'\right)\right)\times\mathbf{c}}{r^2}d^3x_0'$$

Since the volume integral is invariant, as it was mentioned above, then having the formula above we can derive Ampere's law (Jackson [5] (sec. 5.3))

$$\nabla \times \mathbf{B} = \mathbf{j}$$

So we see that Ampere's law does not have to (and should not!) explicitly appear in Maxwell's equations. Using complex relativistic transformations we can derive them from complex equations of on electric field.

## 2.5 Potential energy

Since in our model the charge density and the potential are always real scalars (they have no vector component), then potential energy density is also the invariant real scalar field:

$$w(\mathbb{X}) = \frac{1}{2}\rho(\mathbb{X})\varphi(\mathbb{X}), \tag{59}$$

where $\mathbb{X} \neq 0$ or $X \neq X_0$, and $\varphi(\mathbb{X}) = \varphi(X - X_0)$ is the charge density at point $X_0$ that is distanced from the point $X$ at which the field is described. We remind that our model is constructed in vector space rather than affine, and therefore we do not place objects in a coordinate system, but we do consider their relative positions ($\mathbb{X}$).

## 2.6 Summary

In summary of the results above, we find that using the paravector calculus in complex space-time we can create such a theory of electricity and magnetism by starting with the equations of electrostatics

$$\begin{bmatrix} \frac{\partial}{\partial t} \\ \nabla \end{bmatrix} \begin{pmatrix} 0 \\ \mathbf{E} \end{pmatrix} = \begin{pmatrix} \rho \\ 0 \end{pmatrix} \tag{60}$$

and converting them according to the principles of complex relativistic transformations, we obtain the laws of electrodynamics (Maxwell equations) without the current density component which does not require the condition of Lorenz gauge.

$$
\begin{aligned}
\frac{\partial e}{\partial t} + \nabla \mathbf{E} &= \rho & (61)\\
\nabla \mathbf{B} &= 0 & (62)\\
\frac{\partial \mathbf{B}}{\partial t} &= -\nabla \times \mathbf{E} & (63)\\
\frac{\partial \mathbf{E}}{\partial t} + \nabla e &= \nabla \times \mathbf{B}, & (64)
\end{aligned}
$$

where $e$ is the scalar induction of electric field introduced by us.

The equations above in the paravector notation together with the equations of potentials:

$$
\partial \begin{pmatrix} e \\ \mathbf{E} + i\mathbf{B} \end{pmatrix} = \begin{pmatrix} \rho \\ 0 \end{pmatrix} \quad \text{i} \quad \partial^{-} \begin{pmatrix} \varphi \\ 0 \end{pmatrix} = \begin{pmatrix} e \\ \mathbf{E} + i\mathbf{B} \end{pmatrix} \tag{65}
$$

It should be noted that although Maxwell's paravector equations do not have Ampere's law, they are not contradictory to the classical theory - they just order it differently. We also notice the aesthetic side of the modified equations. Although in the real form the equations (61) - (64) are not fully symmetrical but in the paravector form (65) there is no doubt about their elegance. It is evident that the subject of transformation is a strength of electric field only. The charge density and potential are invariant in complex space-time. Please note that in the general case, the speed paravector is complex. This means that the scalar induction introduced by us may be a complex number as well.

On the grounds of such a theory, it would lose the basic premise of the search for a magnetic monopole and it would be clear why magnetic charge has not been discovered, and if our reasoning is correct, then it cannot be discovered.

Above are formulas derived of the electric field for a particular case in which velocity is represented by a real paravector. This was done on purpose, to show that the search at this direction makes sense, because the calculations are not complicated, and the results are easy to interpret and most importantly, do not conflict with experimental knowledge. In the current and in the previous article [9] we have shown that by accepting the real boost paravector the obtained imaginary vectors can be interpreted, but still we do not know how to interpret the results after composed boosts. We come to the basic question:

*How to reconcile the theory perfectly white developing complex space-time with real time which is obvious for every observer?*

The hypothesis on this subject will be presented in the next article, where we will discuss the transformation representing the projection of the complex space-time phenomena on the local real space-time of the observer.

# Appendix

Here again we will show the equivalence of the reasoning based on the transformation $\mathbb{X}' = \mathbb{X}V$ invariant for potential and charge density in the wave equation $\partial^{-}\partial\varphi(\mathrm{X}-X_0)=\rho(X-\mathrm{X}_0)$ and reasoning based no the transformation $\mathbb{X}' = V\mathbb{X}$ for the wave equation $\partial\partial^{-}\varphi(\mathrm{X}-X_0)=\rho(X-\mathrm{X}_0)$.

Starting from identity 4 in Table 1, the field described by the formulas (22) according to the transformation $\mathbb{X}'=\mathbb{X}V$ is transformed:

Table 3

| | Rest frame | Moving frame |
|---|---|---|
| Charge density | $\rho(\mathbb{X})\in R$ | invariant |
| Field strength | $\mathbb{F}(\mathbb{X})\in\{0\}\times R^3$ | $\mathbb{F}'(\mathbb{X}'V^{-})=V^{-}\mathbb{F}(\mathbb{X}'V^{-})\in R\times C^3$ |
| Potential | $\varphi(\mathbb{X})\in R$ | invariant |
| Potential energy density | $\frac{\varphi(\mathbb{X})\rho(\mathbb{X})}{2}\in R_+$ | invariant |
| Field energy density | $\frac{\mathbb{F}^*(\mathbb{X})\mathbb{F}(\mathbb{X})}{2}\in R_+$ | $\frac{\mathbb{F}'^*(\mathbb{X}'V^{-})\mathbb{F}'(\mathbb{X}'V^{-})}{2}=\frac{\mathbb{F}^*(\mathbb{X}'V^{-})V^{-}V^{-}\mathbb{F}(\mathbb{X}'V^{-})}{2}\in R_+\times R^3$ |

The field strength from the stationary charge is described by the formula (22)

$$\partial^- \begin{pmatrix} 0 \\ -\mathbf{E}(\mathbb{X}'V^-) \end{pmatrix} = \begin{pmatrix} \rho(\mathbb{X}'V^-) \\ 0 \end{pmatrix}$$

After changing the frame to a moving one, based on [7] Theorem 2.4 it transforms to

$$\begin{bmatrix} \frac{\partial}{\partial t'} \\ -\nabla' \end{bmatrix} \left( \frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ -\mathbf{E}(\mathbb{X}'V^-) \end{pmatrix} \right) = \begin{pmatrix} \rho(\mathbb{X}'V^-) \\ 0 \end{pmatrix},$$

where

$$\frac{1}{\sqrt{1-v^2}} \begin{bmatrix} 1 \\ -\mathbf{v} \end{bmatrix} \begin{pmatrix} 0 \\ -\mathbf{E} \end{pmatrix} = \frac{1}{\sqrt{1-v^2}} \begin{pmatrix} \mathbf{vE} \\ -\mathbf{E} + i\mathbf{v}\times\mathbf{E} \end{pmatrix} = \begin{pmatrix} e' \\ -\mathbf{E}' + i\mathbf{B}' \end{pmatrix}.$$

In the paravector form modified Maxwell's equations would look

$$\begin{bmatrix} \frac{\partial}{\partial t} \\ -\nabla \end{bmatrix} \begin{pmatrix} e \\ -\mathbf{E} + i\mathbf{B} \end{pmatrix} = \begin{pmatrix} \rho \\ 0 \end{pmatrix}, \tag{66}$$

after splitting into components it gives the same formulas as before

$$\frac{\partial e}{\partial t} + \nabla\mathbf{E} = \rho \quad , \quad \nabla\mathbf{B} = 0$$
$$\nabla \times B - \frac{\partial \mathbf{E}}{\partial t} = \nabla e \quad , \quad \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

and the energy density

$$W' = \frac{1}{2}\mathbb{F}^*\mathbb{F} = \frac{1}{2} \begin{pmatrix} e' \\ -\mathbf{E}' - i\mathbf{B}' \end{pmatrix} \begin{pmatrix} e' \\ -\mathbf{E}' + i\mathbf{B}' \end{pmatrix} = \begin{bmatrix} \frac{e'^2 + E'^2 + B'^2}{2} \\ -e'\mathbf{E}' + \mathbf{E}' \times \mathbf{B}' \end{bmatrix}$$

In view of the above, it has been confirmed once again that it does not matter whether in complex time-space the theory of SR is constructed on the transformation of $\mathbb{X}' = \mathbb{X}V$ or of $\mathbb{X}' = \mathbb{X}V$, because the results from both theories will be equivalent. It is only important that the wave equation and transformation be compatible, that is

- for the equation $\partial\partial^-\varphi(\mathbb{X}) = \rho(\mathbb{X})$ must be the transformation $\mathbb{X}' = \Lambda\mathbb{X}$,
- for the equation $\partial^-\partial\varphi(\mathbb{X}) = \rho(\mathbb{X})$ must be the transformation $\mathbb{X}' = \mathbb{X}\Lambda$.

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