The Cosmic Geometry

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Abstract

The internal Schwarzschild solution is examined in the context of a cosmological model where the future vacuum is described by the internal metric. The anisotropy of the metric is discussed and it is shown by analyzing light signals on a Kruskal-Szekeres coordinate chart that since the anisotropy lies along the time dimension, we will still observe isotropic space in the internal metric. It is shown that the model predicts an accelerated expansion that agrees with current observations of the expansion history of our Universe, namely that the initial expansion is infinitely fast, and then the expansion slows for some time followed by an accelerated expansion. An examination of the Hubble parameter and redshift is made, and it is shown that the model agrees with cosmological data in predicting the transition redshift when the expansion of the Universe changes from deceleration to acceleration. Distance modulus is plotted against redshift and compared to cosmological data. The angular portion of the metric is interpreted, and it is discussed in terms of the celestial spheres of the CMB and Big Bang. In particular, it is shown that the internal Schwarzschild metric is able to describe the spherical nature of the Big Bang (the Big Bang must be a sphere behind the CMB since it occurred in the finite past and is fully contracted in the spatial dimension), while the FRW metric is unable to account for this.

Introduction

When looking out at the Universe, we see collections of stars and galaxies spread out across the sky at various distances from us in both space and time. The currently accepted metric describing this spacetime is the FRW metric. This is essentially the Minkowski metric scaled by a time-dependent scale factor that is determined based on the energy density of the Universe at a given time. It describes matter spherically distributed in space where time is simply a set of markers that determine the density of that distribution.

We will examine the possibility of using the internal Schwarzschild metric as an alternative metric for describing the cosmic geometry. If we neglect the angular part of this metric, we find that the metric can be put in a form identical to the FRW metric, with the only exception being that the scale factor is not dependent on the matter distribution, but is rather an energy-independent function of time (it will even be shown that the scale factor of the internal metric predicts an expansion that slows for some time and then accelerates and we show that this inflection occurs at a time consistent with current observations, all without resorting to a Cosmological Constant). But where this metric is truly distinct from the FRW metric is in the angular term. For the internal Schwarzschild metric, the radius of the angular component is just time, as opposed to the FRW metric where the radius is distance times the scale factor. This difference is key to justifying the use of the internal Schwarzschild metric to model the Universe and in exposing weaknesses in the FRW metric (note that the anisotropy of the Schwarzschild metric will be addressed in the 'Metric Anisotropy' section of this paper).

Consider the celestial spheres around an observer in the Universe. When we look out to distant events, we can use the redshift from these events to determine their distance from us. Events with the same distance from us can be thought of as residing on a celestial sphere, such that all these events are separated from us by the same magnitude of space and time. We can classify these spheres into three types:

1. Dynamic Spheres – These are the spheres that galaxies reside on. Objects on these spheres maintain a constant coordinate distance from us and move forward in time. We are able to move toward or away from objects on these spheres by moving through space. If we fix our sights on a particular galaxy, the light we see from that galaxy is being emitted later in time as we ourselves move through time.

2. Static Spheres – These are spheres fixed in time. The Cosmic Microwave Background is the most obvious example of these spheres. Light from the CMB sphere is always emitted from the same cosmological time, but as we ourselves move through time, we see light from that time emitted from farther and farther away from us in space, giving the impression that the CMB sphere is growing. We cannot move toward or away any objects on this sphere because it is frozen in time. Both metrics are able to capture this behaviour, but they do so in different ways.

3. The Dark Sphere – The Dark Sphere is where the internal Schwarzschild metric succeeds and the FRW metric fails. The Dark Sphere is the Big Bang and lies beyond the CMB. It is in principle unobservable for two reasons. First, the CMB is opaque so that any light from the Big Bang cannot penetrate it. Second, even if the CMB was not locking our view, any light from that sphere would be infinitely redshifted in the frame of all future observers since the scale factor on that sphere is zero. But though we cannot see the Dark Sphere, it must be there if the model of the Universe is consistent. Because the Big Bang happened a finite time in our past, it must in some sense be located a finite distance from us in the sky somewhere behind the CMB. Furthermore, it must be a sphere because there is no preferred direction in the Universe (if the cosmological principle holds). The Big Bang is sometimes pictured as an infinitely dense point containing all space. It is also sometimes described as an infinitely expansive 3D space with infinite density. The reality, however must be something in between. The Big Bang must be a 2D spherical surface with infinite density and finite area. The density is infinite because the scale factor is zero, and it must have a finite spherical surface area for the reasons given above. The FRW metric breaks down here and cannot account for the fact that based on our observations of the Universe, the Big Bang must have those properties. It is also unclear in the FRW metric what distance should be assigned to that sphere. The CMB appears to grow over time because we get light emitted at that time from greater and greater distances. But space is fully contracted at the Big Bang, so the sphere would not be able to appear to grow. How can the CMB sphere grow if the Big Bang sphere cannot? The internal

Schwarzschild metric, handles these conditions effortlessly thanks to the structure of the angular component of the metric.

These spheres are shown in terms of the internal Schwarzschild metric in Figure 1. Figure 1 shows the Schwarzschild coordinates of the internal metric plotted on the Kruskal-Szekeres coordinate plane. In these coordinates, space is the 't' coordinate emanating from the center of the diagram (Big Bang) and time is the 'r' coordinate depicted as hyperbolas (time is flowing forward as r goes toward zero). The upper right quadrant of this diagram represents a single fixed direction ($\theta = const, \phi = const$). So each bold line representing a sphere would be a point on each sphere over time. Note that light on this diagram travels on 45-degree lines.

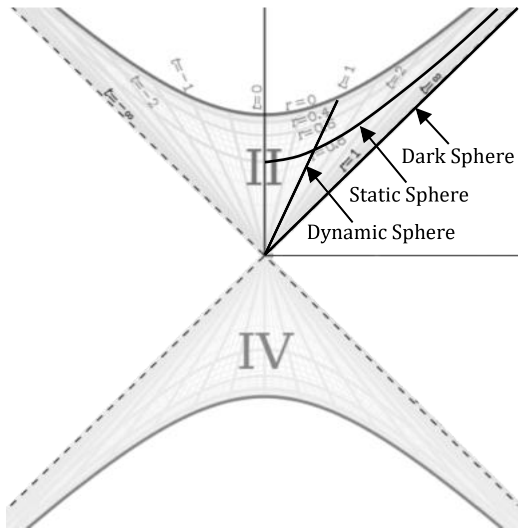

Figure 1 – Celestial Sphere Types on Kruskal-Szekeres Coordinate Chart[1]

In this paper, we will first examine the space time of the internal metric from the perspective of the inertial observer and compare results to experimental data. We will then examine the angular part of the metric more closely. Finally, we will examine at the classical theory of Black Holes in this context.

The Schwarzschild Metric

The Schwarzschild metric is the simplest solution to Einstein's field equations. It is a vacuum solution for the spacetime around a spherically-symmetric distribution of energy. The general form of the metric can be expressed as:

$$d\tau^2 = \frac{r}{u-r}dr^2 - \frac{u-r}{r}dt^2 - r^2 d\Omega^2 \quad (1)$$

Depending on the ratio $\frac{u}{r}$, we get three distinct descriptions of spacetime:

[1] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

1. $u = 0$: This gives us the flat Minkowski metric of Special Relativity.

2. $\frac{u}{r} < 1$: This describes the external metric for the spacetime surrounding a spherically-symmetric energy distribution occupying a finite amount of space for an infinite amount of time
3. $\frac{u}{r} \geq 1$: This describes the internal metric for the spacetime surrounding a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time

The internal metric is typically considered an extension of the external metric, giving rise to the theory of Black Holes where an observer can cross from the external to the internal metric by freefalling toward an infinitely dense mass. In this paper, we will be examining the internal metric completely independently of the external metric such that we do not need to consider the pre-existence of a gravitationally collapsed mass from the external metric in order for the spacetime of the internal metric to exist. We simply hypothesize that the internal metric is the metric of the Universe and so will not view it in the context of Black Holes (though this context will be addressed later in the paper).

An important observation is that the internal metric describes a vacuum solution to the field equations. But the Universe is clearly filled with energy, so how can this solution apply? In order to satisfy the requirements of the metric, the Universe must be "*a spherically-symmetric energy distribution occupying an infinite amount of space for a finite amount of time*". For this metric to be a cosmological description, it must be that Universe only truly exists in the present and in a very real sense moves into the future. The surrounding vacuum is the future, and the Universe is freefalling through time toward the temporal center of the metric.

Time being the radial dimension of the metric combined with the fact that the solution is a vacuum solution gives a mathematical justification for our intuitive notions of past, present, and future. The anisotropy along the radial direction gives us an arrow of time that distinguishes the 'past' and 'future' analogous to the way the external solution gives us an absolute distinction between 'up' and 'down'. And the vacuum as described above gives us a boundary between them, that boundary being the 'present' time.

Metric Anisotropy

According to the Cosmological Principle, which is supported by observation, the Universe is homogenous and isotropic. But the Schwarzschild metric is anisotropic, so how can this metric be a true description of our Universe? It is important to note that the anisotropy of the internal metric lies exclusively along the time dimension. On the top half of Figure 1, light travels on 45 degree lines upwards to an observer (for our example, we'll put the observer at $t = 0$) at some time r. The diagram is showing the spacetime for a fixed θ and ϕ. The observer at *r* will see 2 light signals intercepting it: one coming from the positive *t* direction and one coming from the negative *t* direction. Thus, if the observer is oriented in a particular direction, the light coming to her face is the signal from the positive *t* direction, while the light coming to the back of her head is coming from the negative *t* direction. But both of these directions face outward from the center of the sphere (they face the past). Therefore, the Universe in front of her will look just like the Universe behind her. The anisotropy of the metric does not manifest itself as an anisotropy in space, but rather of time. Since we did not specify θ and ϕ, we can conclude that this is true for every direction. The metric is anisotropic in past, present, and future, but one cannot see this anisotropy in space; we cannot choose to look toward or away from the center of the Universe because the center is a point in time, not space. We are always looking away from the center no matter what direction we face – there is no preferred direction, no observable anisotropy. Contrast the above argument with the external metric where on a diagram similar to Figure 1, it shows that one light signal always comes from the direction of the center of the metric while the other signal comes from the opposite direction. So in that metric one can look toward or away from the center at will and the external spacetime is therefore observably anisotropic.

But, the angular portion of the metric does collapse while the spatial dimension expands, so how do we account for the observation that the Universe looks more dense in the past? The brightness of a given object that we observe will be proportional to its position in time, r, relative to us since the radius of the sphere is time. According to Equation 1 along a null geodesic, $\frac{dt}{dr} = \frac{r}{u-r}$, implying that the variation in space for a given variation in observed brightness will increase exponentially as we look at galaxies closer in time to the Big Bang.

Consider two equal-area patches of sky. In the first patch we observe galaxies from the near past while in the other patch we observe galaxies from the distant past. After measuring the brightness of the galaxies in each patch, we determine that the variation in brightness (Δr) is approximately equal in both patches. In this circumstance, we might see a more dense collection of galaxies in the second patch even though the galaxies in the first patch should have less angular separation due to the collapsing angular term in the metric. This is due to the fact that for the same variation in brightness, we see a larger variation in space in the second patch compared to the first. In other words, the amount of space being 'projected' onto the second patch is much greater that the amount of space being projected onto the first because the second patch is farther in the past where the spatial dimension is contracted. Therefore, we would still expect to observe a more dense collection of galaxies in the second patch than we would on the first. The increase in density is a kind of relativistic optical illusion.

Freefall Through Time

Let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ). There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $\frac{u}{r} \geq 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 showed the $\frac{u}{r} \geq 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant *r* represent spacelike slices of constant cosmological time and the rays of *t* represent spatial distances. We will not be considering differences in angles until a later section in the paper, so we only need to consider the two halves of Figure 1. We will focus on the upper half where the half represents an observer pointed in a particular direction and the positive *t*'s represent the coordinate distance from the observer in that particular direction while the negative *t*'s represent coordinate distance in the opposite direction.

We must first determine the paths of inertial observers in the spacetime. For this we need the geodesic equations for the internal Schwarzschild metric [1] given in Equation 1. In these equations u represents a time constant that in the external metric would be the Schwarzschild radius (in Figure 1, the value of u is 1). The following equations are the geodesic equations for *t* and *r* ($r \leq u$):

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2}\left[\frac{u-r}{r}\left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r}\left(\frac{dr}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2}\left[1 + \left(\frac{d\Omega}{d\tau}\right)^2\right] - (u-r)\left(\frac{d\Omega}{d\tau}\right)^2 \tag{4}$$

For $\frac{d\Omega}{d\tau} = 0$, notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the Schwarzschild constant was 2*GM*, as it would be in the external solution, Equation 4 would be Newton's gravity). Also, anyone moving inertially starting with non-zero $\frac{dt}{d\tau}$ will experience the same acceleration through time as someone with zero $\frac{dt}{d\tau}$ since dt does not appear in Equation 4.

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant *t* (and any such observer can choose $t = 0$ as their coordinate). The 'Big Bang' will have occurred in Figure 1 along the line $r = 1$. We know this because the above analysis showed that space expands if $\frac{dr}{d\tau}$ is negative, so for our current cosmological time, our worldlines must be moving toward $r = 0$.

The Scale Factor

Expressions for the proper time interval along lines of constant *t* and Ω and the proper distance interval along hyperbolas of constant *r* and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \quad (5)$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \quad (6)$$

Where *a* is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the *t* coordinate. This is good because the *t* coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time *r*.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$. When $r = u$, Equations 5 and 6 are both 0. At this point (the Big Bang), it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident (the observer can reach every point in space without moving through time, all paths are light-like). So this why the scale factor goes to zero there and why the lines of *t* in Figure 1 converge at that point; it is an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from positive to negative (we can see that if we draw a worldline through the center point, $\frac{dr}{d\tau}$ will change signs as it passes the $r = 1$ point). In fact, for any choice of time coordinate, that point will be a stationary point in those coordinates.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed of light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite. A plot of the scale factor vs. r (with $u = 1$) is given in Figure 2 below:

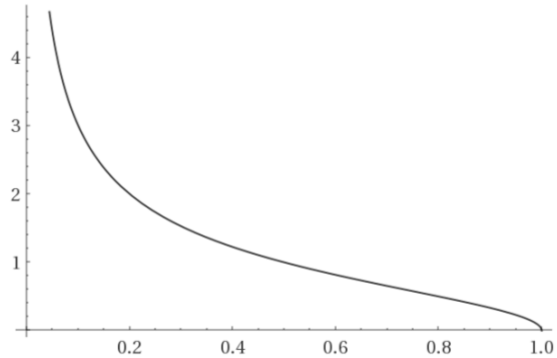

Figure 2 – Scale Factor vs. r

In Figure 2, there is an inflection point at $r = 0.75$. This is the point at which the expansion changes from decelerating to accelerating.

Redshift and the Hubble Parameter

We can use the fact that $\sqrt{\frac{u-r}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1] (note that this Equation was derived from the FRW metric in the reference, but the internal metric, when setting $d\Omega = 0$, can be put in the same form as the FRW metric with a coordinate change, so the equation below is still valid for the internal metric):

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{u-r}{r}} - 1 \qquad (7)$$

We can use Equation 7 to predict the redshift of the Universe at the time the expansion changed from decelerating to accelerating. First, we must find the value of u. For the external metric, this constant has the value of the Schwarzschild radius of a mass given by $2GM$. For the interior metric, this constant will need to be a time; specifically, it will be the coordinate time in years from the 'Big Bang' to $r = 0$. We can use the known Hubble parameter and current age of the Universe to find this constant. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \qquad (8)$$

We know that the Universe is around 13.8 billion years old, so in Equation 8 we can make the substitution $r = u - 13.8$ (because the Big Bang occurs at $r = u$). The Hubble parameter at this time has been measured to be around 67.8 (km/s)/Mpc. Converting that value to units of 1/(billion years), setting Equation 8 equal to that value and solving for u we get an approximate value of:

$$u \approx 28.8\ billion\ years \tag{9}$$

We can now express r in units of billions of years from $r = 0$ (the Big Bang occurs at $r = 28.8$). A plot of Equation 8 with the value $u = 28.8$ and the ΛCDM model [2] with $\Lambda = 0.013$ is given in Figure 3 below (our current time is shown as the dashed vertical line):

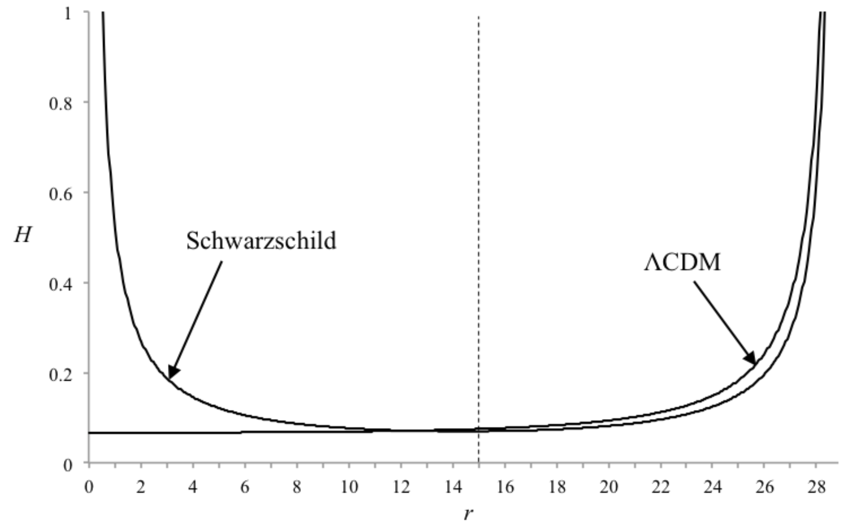

Figure 3 – Hubble Parameter vs. r ($u = 28.8, \Lambda = 0.013$)

Equation 7 can be used to find the transition redshift, which is the redshift we observe at the point when the Universe transitioned from a decelerating expansion to an accelerating expansion. In Equation 7, this transition occurs at $r_{emit} = 21.6$ and our current time is $r = 14.98$. Plugging those values into Equation 8 we get an estimated transition redshift of:

$$z_t = 0.66 \tag{10}$$

This value is within the 2σ bound for the parameter [3,4], and therefore it does appear to be in agreement with cosmological measurements. A plot of redshifts measured at our current time vs. time is given in Figure 4 below:

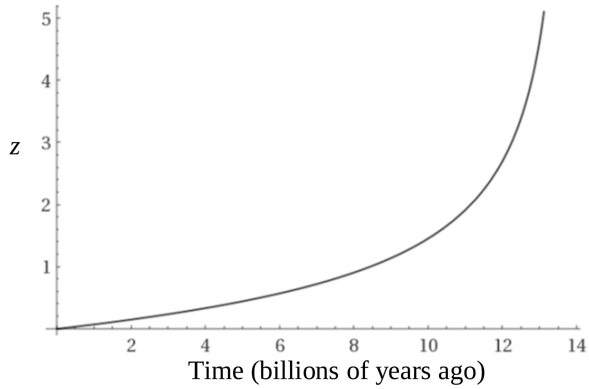

Figure 4 – Redshift vs. Time

Finally, the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 = \frac{r}{7.2} - 3 \tag{11}$$

A plot of the deceleration parameter is given in Figure 5 below:

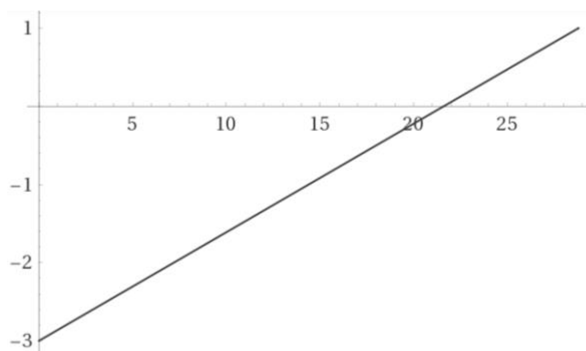

Figure 5 – Deceleration Parameter vs. *r*

Coordinate Distance & Distance Modulus

Figure 1 is a plot of the metric on a Kruskal-Szekeres coordinate chart where the *T*-axis is the vertical axis and the *X*-axis is the horizontal axis. The definition of *T* and *X* are given below for $u = 28.8$:

$$X = \sinh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \tag{12}$$

$$T = \cosh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \tag{13}$$

Light travels on 45-degree lines in these coordinates so if we consider our current reference frame at $t = 0$ and $r = 15$, we can find the coordinate distance t of some galaxy we observe along the 45-degree line at some *r* by setting $\Delta X = -\Delta T$ and solving for *t*. When we do this, we get:

$$t = 28.8 \ln\left(\frac{23.23}{28.8-r}\right) - r \tag{14}$$

Where *t* is in billions of light years and $15 \leq r \leq 28.8$. Note that Equation 14 is only valid for the current cosmological time. The 23.23 constant is specific to this time so for some other time, a different constant would be required and is given by the value $C = (28.8 - r_0)e^{\frac{r_0}{28.8}}$. We can also use Equation 7 to find r_{emit} as a function of *z* and substitute that into Equation 14 to get the coordinate distance as a function of redshift. If we set $r = 15$ for $u = 28.8$ in Equation 7 and solve for r_{emit} we get:

$$r_{emit} = 28.8\frac{z^2+2z+1}{z^2+2z+1.92} \tag{15}$$

Substituting Equation 15 into 14 will give the coordinate distance as a function of measured redshift. A commonly used parameter in cosmology is the distance modulus, μ, which is defined as:

$$\mu = 5\log_{10}\left(\frac{d}{10}\right) \tag{16}$$

Where *d* is the distance measured in parsecs. A plot of distance modulus vs. redshift obtained by combining Equations 14, 15, and 16 (where we use *t* measured in parsecs for *d* in Equation 16) is shown in Figure 6 below plotted over data obtained from the Supernova Cosmology Project [6] (The R^2 value of the data is 0.99):

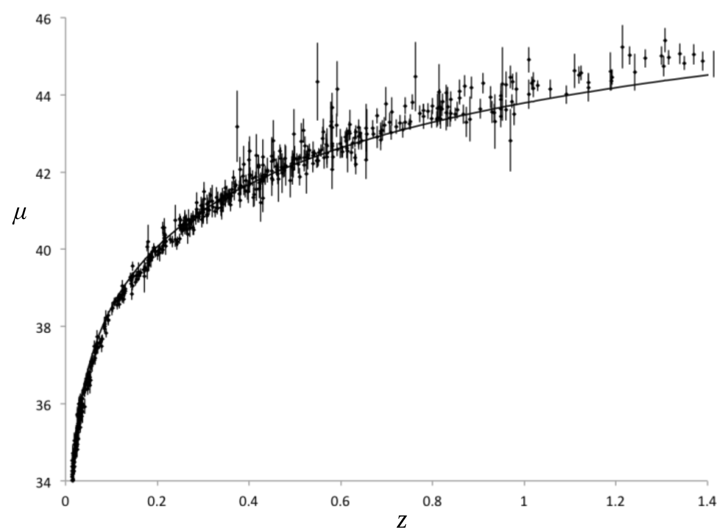

Figure 6 – Distance Modulus vs. Redshift

Note that all these predictions only required the spherical symmetry assumptions of the Schwarzschild metric and calculation of a single parameter, *u*, from cosmological data; it requires no information regarding the detailed energy distribution within the Universe. In fact, the value of *u* only determines the units we are working in; it does not affect the form of the model. This reflects the fact that the details of the expansion are the result of the vacuum solution alone.

Proper Time of the Rest Observer

Figure 7 shows the past light cone of an inertial observer at a given time during the expansion:

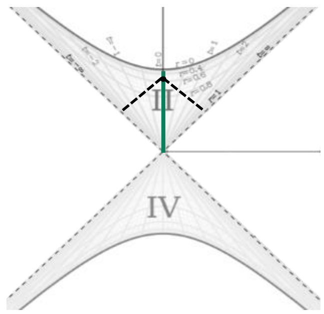

Figure 7 – Past Light Cone of Inertial Observer During the Expansion[2]

We can calculate the duration of the expansion of the Universe in the frame of an inertial observer at rest by integrating Equation 5 from 0 to u. The total time of expansion is therefore:

$$\tau = \frac{\pi}{2} u \tag{17}$$

Where τ is measured in billions of years. Equation 17 tells us that in the frame of an observer at rest at t, the time elapsed from the Big Bang to $r = 0$ measured by her clock would be around 45.2 billion years and there is only about 8.8 billion years of proper time between now and $r = 0$ for her.

Thinking of τ in Equation 17 as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2} u$ for time and space. Equation 17 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2} u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

Metric and Geodesics in Terms of the Hubble Parameter and Scale Factor

We can re-express equations 1-4 in terms of the scale factor a and the Hubble parameter (for $\frac{d\Omega}{d\tau} = 0$):

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[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

$$d\tau^2 = a^{-2}dr^2 - a^2dt^2 - r^2d\Omega^2 \tag{18}$$

$$\frac{d^2t}{d\tau^2} = 2H\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{19}$$

$$\frac{d^2r}{d\tau^2} = a^2H\left[a^2\left(\frac{dt}{d\tau}\right)^2 - a^{-2}\left(\frac{dr}{d\tau}\right)^2\right] \tag{20}$$

$$\frac{d^2r}{d\tau^2} = -a^2H \tag{21}$$

Equation 21 gives us a quantity analagous to the surface gravity used in the external solution. The non-zero Christoffel symbols of the model (for $d\Omega = 0$) in terms of H are:

$$\Gamma^r_{rr} = -H \tag{22}$$

$$\Gamma^t_{tr} = \Gamma^t_{rt} = H \tag{23}$$

$$\Gamma^r_{tt} = a^4H \tag{24}$$

The Angular Term

We now return to the angular term of the internal Schwarzschild metric. According to the metric, the radius of an infinitesimal angular distance depends only on the point in time at which the arc exists, rather than its spatial distance from any point. This makes calculating angular distances a more challenging exercise than it would be with the FRW metric. For instance, consider an observer, we'll call her Scout, who is surrounded by a ring not much bigger than her such that every point on the ring is equidistant from her. In her frame this ring encompasses 360 degrees of angular distance around her. But according to the metric, the ring has a radius of approximately 15 billion years (our current coordinate time). But she could also be surrounded by an even larger ring at the same time that, according to the metric, will also have a radius of 15 billion years. So if their radii are the same, they must have differing angles associated with them (and those angles must be very small to offset the enormously large radius). The resolution to this is depicted in Figure 9 below.

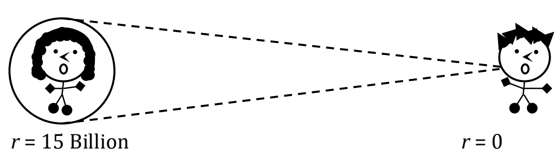

Figure 9 – Local Angles vs. Metric Angles

In Figure 9, Scout is at our current cosmological time and she measures the circumference of the ring surrounding her using a single angle of 360 degrees. But the true angles to be used in the metric to measure that circumference are those seen by Jem, who sits at $r = 0$ and a Δt from Scout such that the light emitted from each point on the ring at $r = 15\ billion$ is seen by Jem at $r = 0$. Therefore, Jem needs two angles to measure the circumference, θ and ϕ, because the ring does not surround him, but rather occupies a small region on a distant surface that surrounds him. Furthermore, given the enormous distance between Scout and Jem in both space and time, we can see that these angles would be very small in this case such that the circumference of the ring would be a reasonable size when calculated with the metric.

Note that the ring in Figure 9 could also be a collection of galaxies that Scout sees surrounding her. We can conclude from this that the angular distances between galaxies we currently observe on the celestial spheres may be smaller than what we currently measure since those distances are currently based on their angles relative to us, rather than an observer at $r = 0$.

Another interesting artifact of the metric is that while the light from distant galaxies is being increasingly redshifted as time goes on and our ability to travel through space becomes increasingly difficult (Equation 2), angular distances are converging as the celestial spheres collapse toward $r = 0$. Therefore, we should expect that there is a kind of surface tension in the celestial spheres, squeezing the objects on them closer together over time. This surface tension may even account for some of the observed Dark Matter effects, but that is beyond the scope of this paper.

The 'Big Bounce'?

A plot of τ vs. r from the uppermost to lowermost hyperbola in Figure 1 is given in Figure 10 below. It illustrates well the relationship to typical spatial projectile motion (for $u = 1$).

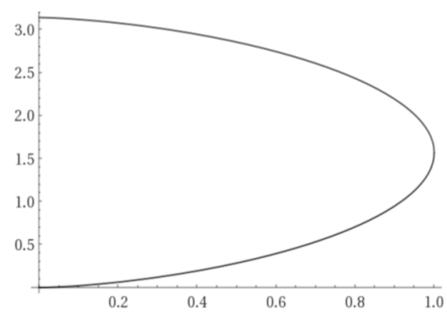

Figure 10 - τ vs. r

Consider a perfectly rigid and elastic ball in simple Newtonian mechanics. If we throw it straight up in the air with initial velocity $\frac{dx}{d\tau}$, the velocity will continuously decrease until at some height $\frac{dx}{d\tau} = 0$, at which point the ball will reverse direction and fall with increasingly negative $\frac{dx}{d\tau}$ until it returns to the ground. When it hits the ground (which we will assume has infinite inertia), since the ball is perfectly rigid and elastic, it will experience an infinite acceleration that will bounce it back toward its maximum height and this cycle will continue ad infinitum. So, there are two turnaround points for the ball. One point is maximum height, where the ball does not experience any special acceleration; it just stops moving through space as it turns around. The second point is a hard acceleration that the ball can really feel a (infinite) force changing its direction.

Likewise, we can see that the Schwarzschild cosmology is a similar situation except that the Universe is the ball and the acceleration is through time rather than space. The Big Bang corresponds to maximum height, where the Universe's velocity through time changes sign. The $r = 0$ hyperbolas are, perhaps, the 'bounce'. When the ball bounced, it experienced an infinite acceleration. In the cosmological case, when $r = 0$ the curvature of the spacetime is infinite [1]. This infinite curvature may be a point in time where the worldlines of the Universe turn back on themselves as if the spacetime is folded there and the worldlines go up one side and down the other (the infinite curvature is at the fold).

Relationship to the External Solution

Let us consider a meter stick at rest at the center of a collapsing spherically symmetric collapsing shell in space. The meter stick inside the shell stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat space). An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [5]:

$$\frac{dr}{d\tau} = -\sqrt{\frac{2GM}{r}} \tag{29}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 29. Since the meter stick is also at rest relative to observers at rest at any *r*, Equation 29 will also give the relative velocity between the freefall observer and the meter stick when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1 - V^2} = dt\sqrt{1 - \frac{2GM}{r}} \tag{30}$$

Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1 - \frac{2GM}{r}} \tag{31}$$

We see from Equation 31 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. So observers in freefall will see the space beyond $r = 2GM$ fully contracted as they approach $r = 2GM$. Furthermore, the clock of an observer at the center of the shell will be slowed as the shell collapses (the clock of an observer at the center ticks at a rate equal to an observer at rest at the location of the shell) such that if she exchanges light signals with the shell as it collapses, the time she measures for the light to return will shrink to zero as the shell reaches the Schwarzschild radius. Thus, she also effectively sees the space within the shell shrink to zero as the shell approaches the Schwarzschild radius.

But the freefalling observer of the external solution will never fall into a 'black hole'. It would take an infinite amount of time in the frame of an observer at infinity for the freefalling observer to reach the event horizon. But the Universe itself will reach $r = 0$ in a finite amount of time in the frame of the infinite observer and therefore the freefalling observer will only reach the $r = 2GM$ location when the entire Universe has reached $r = 0$. Thus, she will never actually reach any event horizon, she will reach $r = 0$ when the entire Universe has reached $r = 0$.

It is also notable that the external and internal solutions seem to turn smoothly into one another as one crosses the horizon, but consider the external metric measured in some arbitrary units of space and time. In that case, one must include the speed of light in the metric:

$$d\tau^2 = c^2 \frac{r-u}{r} dt^2 - \frac{r}{r-u} dr^2 - r^2 d\Omega^2 \tag{35}$$

In Equation 35, we put c^2 in the dt term because *r* and *t* are measured in common units of space and time. If we now allow *r* to be less than *u* such that we get the internal solution, Equation 35 becomes:

$$d\tau^2 = \frac{r}{u-r} dr^2 - c^2 \frac{u-r}{r} dt^2 - r^2 d\Omega^2 \tag{36}$$

For the internal solution, *t* is supposed to be the spatial term and *r* is the time term. But we see from Equation 36 that if one just allows *r* to become less than *u* as though an observer crosses the horizon, the units of the metric no longer make sense as a result of the c^2 (the second term ends up with units like ${m^4}/{s^2}$). Furthermore, if the external solution was expressed in rectangular coordinates rather than radial, the transition becomes even less comprehensible. This may be evidence that the internal and external solutions are in fact unique, separate solutions to the field equations meaning that black holes are not actually a facet of General Relativity.

Conclusion

It has been shown that the internal Schwarzschild metric will give observations that very closely resemble cosmological observations in our Universe. So either the internal solution is in fact a cosmological solution, or observers inside a Black Hole will see a spacetime that evolves in a strikingly similar way to the evolution of large-scale Universe we ourselves observe.

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