The Geometry of Accelerated Expansion

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Abstract

The internal Schwarzschild solution is examined in the context of a cosmological model where the intergalactic vacuum is described by the internal metric. It is shown that the model predicts an accelerated expansion that agrees with current observations of the expansion history of our Universe, namely that the initial expansion is infinitely fast, and then the expansion slows for some time followed by an accelerated expansion. An examination of the Hubble parameter and redshift is made, and it is shown that the model agrees with cosmological data in predicting the transition redshift when the expansion of the Universe changes from deceleration to acceleration. Distance modulus is plotted against redshift and compared to cosmological data. The angular portion of the metric is interpreted, and it is discussed in terms of the celestial spheres of the CMB and Big Bang.

Introduction

The currently accepted geometry used to describe the Universe as a whole is the Friedman-Robertson-Walker (FRW) metric. The basic form of the metric was assumed based on the observation that the Universe is expanding and then the details about the scale factor are calculated using Einstein's filed equations. According to this metric, at the Big Bang, the Universe has no spatial size and expands forever after that.

There are a couple of problems with this metric. Firstly, since the form of the metric is assumed and not based on any deeper principle, it does not give any insight into why the Universe began expanding in the first place, or what geometry, if any, existed before that moment. Secondly, we now know the expansion of the Universe is accelerating, a fact that the FRW metric cannot account for in and of itself. The introduction of Dark Energy in the form of the Cosmological constant is required to account for this accelerated expansion. Finally, there is the problem of the Big Bang having no spatial dimension. When we look out into the Universe, we can think of the very distant galaxies and other objects that we see has being on different celestial spheres. Each celestial sphere corresponds to a specific time in the past. The larger the celestial sphere, the father in the past we are looking. The largest sphere we can see is the Cosmic Microwave Background. It is a sphere of finite size in our reference frame and the size gets bigger as we move through time. In the future, the radius of the CMB will be larger in our reference frame and the light that we see from it will be coming from a greater coordinate distance than the light we see now. It is tempting to say then that the radius of the sphere is a function of coordinate distance. But suppose we were magically able to see beyond the CMB all the way back to the Big Bang (and imagine the light from that time was not infinitely redshifted). It would appear to us as a celestial sphere of finite size, slightly larger than the CMB, that would, in our frame, appear to grow over time. But the radius of that celestial sphere as described by the FRW metric is unclear at best. The expansion factor is zero there and given that all of space is contracted, what coordinate distance would be assigned to the sphere and how could it change over time? Nonetheless, the Big Bang must be a sphere that surrounds us behind the CMB because we are surrounded by the past and the Big Bang is the most distant past.

In this paper, these issues are resolved by modelling the expanding Universe using the internal Schwarzschild metric. In this metric, the radius of the angular term is a time, rather than a distance. This is what solves the Big Bang celestial sphere problem. In the internal metric, the Big Bang sphere is a sphere of finite surface area where all points on the surface are identical (a perfect sphere). The metric also predicts the accelerated expansion and the moment where the expansion transitions to accelerating is calculated in this paper and it is found that the calculated transition is in line with observation. The metric is also based on two assumptions. The first is that the Universe is spherically symmetric. The second is that the energy of the Universe actually flows through time, meaning that the future spacetime geometry of the Universe exists, but the energy of the Universe only exists in the present and flows into the future (the present is analogous to the surface of an expanding star). In Schwarzschild coordinates with this model, the Universe is effectively falling through time.

Freefall Through Time

The current Big Bang model of the Universe says that the Universe expanded from an infinitely dense gravitational singularity at some time in the past. Current cosmological data suggests that this expansion was slowing down for some time, but is now continuing to expand at an accelerated rate. The Cosmological Principle suggests that from any reference frame in the Universe, the mass distribution is spherically symmetric and isotropic. It is proposed here that the observed expansion of the Universe is the result of a freefall in the time dimension. To analyze the spherically symmetric Universe freefalling through the time dimension, we need the Schwarzschild solution where the radial coordinate is the timelike coordinate. The interior ($r < 1$) solution of the Schwarzschild field gives us precisely that. For $r < 1$, the signature of the Schwarzschild metric flips and the radial coordinate becomes a dimension measuring time while the t coordinate becomes a dimension measuring space.

But the Schwarzschild metric is a vacuum solution to the field equations. Given that observation tells us that the Universe is not a total vacuum, the only way that this solution can be valid is if the future is truly void of energy. In this way, the present Universe would be like the surface of a star and the future would be the external vacuum. This means that the energy of the quantum and local gravitational fields would truly be propagating into the future as the Universe evolves; the future only contains the empty spacetime described by the internal Schwarzschild solution and the present is a three dimensional shell that falls into that vacuum.

So let us take the center of our galaxy as the origin of an inertial reference frame. We can draw a line through the center of the reference frame that extends infinitely in both directions radially outward. This line will correspond to fixed angular coordinates (θ, ϕ).

There are infinitely many such lines, but since we have an isotropic, spherically symmetric Universe, we only need to analyze this model along one of these lines, and the result will be the same for any line.

The radial distance in this frame is kind of a compound dimension. It is a distance in space as well as a distance in time. The farther away a galaxy is from us, the farther back in time the light we currently receive from it was emitted. Fortunately the $r < 1$ spacetime of the Schwarzschild solution plotted in Kruskal-Szekeres coordinates provides us with a method to understand this radial direction. Figure 1 shows the $r < 1$ solution on a Kruskal-Szekeres coordinate chart where, in this model, the hyperbolas of constant r represent spacelike slices of constant cosmological time and the rays of t represent radial distances (each point on this plot is a 2-sphere and each hyperbola is a 3-sphere). We will not be considering differences in angles until a later section in the paper, so we only need to consider the two halves of Figure 1. We will focus on the upper half where the half represents an observer pointed in a particular direction and the positive t's represent the coordinate distance from the observer in that particular direction while the negative t's represent coordinate distance in the opposite direction.

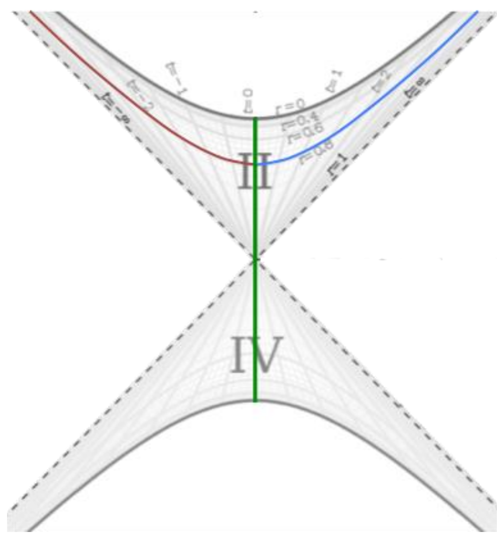

Figure 1 – Freefall Through Cosmological Time[1]

We must first determine the paths of inertial observers in the spacetime. For this we need the internal Schwarzschild metric and the geodesic equations for the internal Schwarzschild metric [1]. In these equations u represents a time constant that in the external metric would be the Schwarzschild radius. In Figure 1, the value of u is 1.

$$d\tau^2 = \frac{r}{u-r}dr^2 - \frac{u-r}{r}dt^2 - r^2 d\Omega^2 \tag{1}$$

[1] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

$$\frac{d^2t}{d\tau^2} = \frac{u}{r(u-r)}\frac{dr}{d\tau}\frac{dt}{d\tau} \tag{2}$$

$$\frac{d^2r}{d\tau^2} = \frac{u}{2r^2}\left[\frac{u-r}{r}\left(\frac{dt}{d\tau}\right)^2 - \frac{r}{u-r}\left(\frac{dr}{d\tau}\right)^2\right] \tag{3}$$

In Equations 1, 2, and 3, we use units where $c = 1$ and equations 2 and 3 assume no angular motion. Looking at points $0 < r < u$, then by inspection of Equation 2 it is clear that an inertial observer at rest at *t* will remain at rest at *t* ($\frac{d^2t}{d\tau^2} = 0$ if $\frac{dt}{d\tau} = 0$). Also, we see that if an observer is moving inertially with some initial $\frac{dt}{d\tau}$, then if $\frac{dr}{d\tau} < 0$, the coordinate speed of the observer will be reduced over time (the coordinates are expanding beneath her) and if $\frac{dr}{d\tau} > 0$, the coordinate speed will be increased over time (the coordinates are collapsing beneath her).

Let us therefore examine Equation 3 for an observer with no angular motion. Combining Equations 1 and 3 with $d\Omega = 0$, equation 3 becomes:

$$\frac{d^2r}{d\tau^2} = -\frac{u}{2r^2} \tag{4}$$

Notice that the observer's acceleration through cosmological time is similar to the form of Newton's law of gravity, where *r* (a time coordinate) varies from *u* to 0 (If the Schwarzschild constant was 2*GM*, as it would be in the external solution, Equation 4 would be Newton's gravity).

So we will first use Figure 1 to describe the freefall of the galaxies through the cosmological time dimension where galaxies (or galaxy clusters) follow lines of constant *t* (and any such observer can choose $t = 0$ as their coordinate). The 'Big Bang' will have occurred at the center of Figure 1 at $r = 1$. We know this because the above analysis showed that space expands if $\frac{dr}{d\tau}$ is negative, so for our current cosmological time, our worldlines must be moving toward $r = 0$.

How we see the Universe

Looking at Figure 1, we should note that light signals travel on 45-degree angles. So when we look out at the Universe, we can imagine that we are seeing light emitted from concentric 2-spheres from when the energy of the Universe was at the particular coordinate time corresponding to a particular 2-sphere. They are 2-spheres because each sphere represents a specific coordinate time in the past and distance from us, they are not independent. We can choose to observe the Universe at any arbitrary past time, but we cannot choose to observe the Universe at an arbitrary distance and time, the distance from us we observe depends on the present age of the Universe and the age of the 2-sphere we observe. Nonetheless, each 2-sphere will appear to us to be spatially homogeneous and isotropic and this is reflected in Equation 2 (if we fix the r of a 2-sphere, the space will be homogenous and isotropic).

The Scale Factor

Expressions for the proper time interval along lines of constant t and Ω and the proper distance interval along hyperbolas of constant r and Ω from Equation 1 are:

$$\frac{dr}{d\tau} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{5}$$

$$\frac{ds}{dt} = \pm\sqrt{\frac{u-r}{r}} = \pm a \tag{6}$$

Where a is the scale factor. First we should notice that neither Equation 5 nor 6 depend on the t coordinate. This is good because the t coordinate marks the position of other galaxies relative to ours. Since all galaxies are freefalling in time inertially, the particular position of any one galaxy should not matter. The proper velocity and proper distance only depends on the cosmological time r.

What is notable here is that in Schwarzschild coordinates, the scale factor is equal to the velocity through the time dimension for an observer at rest $\left(\frac{dt}{d\tau} = \frac{d\Omega}{d\tau} = 0\right)$. When $r = u$, Equations 5 and 6 are both 0. At this point (the Big Bang), it is our proper velocity in time that is zero. So at that instant, we are no longer moving through time and therefore all points in space are coincident (the observer can reach every point in space without moving through time, all paths are light-like). So this why the scale factor goes to zero there and why the lines of t in Figure 1 converge at that point; it is an instant where our velocity through cosmological time goes to zero as our speed through cosmological time changes from positive to negative (we can see that if we draw a worldline through the center point, $\frac{dr}{d\tau}$ will change signs as it passes the $r = 1$ point). In fact, for any choice of time coordinate, that point will be a stationary point in those coordinates.

At $r = 0$, both equations 5 and 6 are infinite. So when the worldlines enter or exit one of the $r = 0$ hyperbolas, they do so at infinite proper speed *through the time dimension*. If something is travelling through space at the speed to light, the proper distance between points in space is zero. In this case, since we have infinite proper velocity in the time dimension, the proper distance between points in space will be infinite, because you would traverse an infinite amount of time in order to move through an infinitesimal amount of space. What we see then is that at $r = 0$ space will be infinitely expanded and thus the scale factor is infinite. A plot of the scale factor vs. r (with $u = 1$) is given in Figure 2 below:

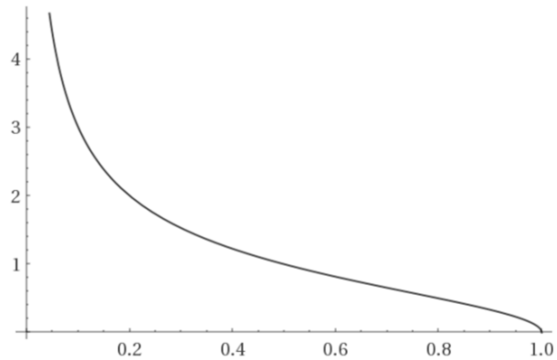

Figure 2 – Scale Factor vs. r

In Figure 2, there is an inflection point at $r = 0.75$. This is the point at which the expansion changes from decelerating to accelerating.

Redshift and the Hubble Parameter

We can use the fact that $\sqrt{\frac{u-r}{r}}$ is the scale factor and get the expression for cosmological redshift caused by the expansion [1]:

$$z = \sqrt{\frac{r_{emit}}{(u-r_{emit})}}\sqrt{\frac{u-r}{r}} - 1 \tag{7}$$

We can use Equation 7 to predict the redshift of the Universe at the time the expansion changed from decelerating to accelerating. First, we must find the value of u. For the external metric, this constant has the value of the Schwarzschild radius of a mass given by $2GM$. For the interior metric, this constant will need to be a time; specifically, it will be the coordinate time in years from the 'Big Bang' to $r = 0$. We can use the known Hubble parameter and current age of the Universe to find this constant. The Hubble parameter is given by:

$$H = \frac{\dot{a}}{a} = \frac{d}{dr}\left(\sqrt{\frac{u-r}{r}}\right)\sqrt{\frac{r}{u-r}} = \frac{u}{2r(u-r)} \tag{8}$$

We know that the Universe is around 13.8 billion years old, so in Equation 8 we can make the substitution $r = u - 13.8$ (because the Big Bang occurs at $r = u$). The Hubble parameter at this time has been measured to be around 67.8 (km/s)/Mpc. Converting that value to units of 1/(billion years), setting Equation 8 equal to that value and solving for u we get an approximate value of:

$$u \approx 28.8\ billion\ years \tag{9}$$

We can now express r in units of billions of years from $r = 0$ (the Big Bang occurs at $r = 28.8$). A plot of Equation 8 with the value $u = 28.8$ and the ΛCDM model [2] with $\Lambda = 0.013$ is given in Figure 3 below (our current time is shown as the dashed vertical line):

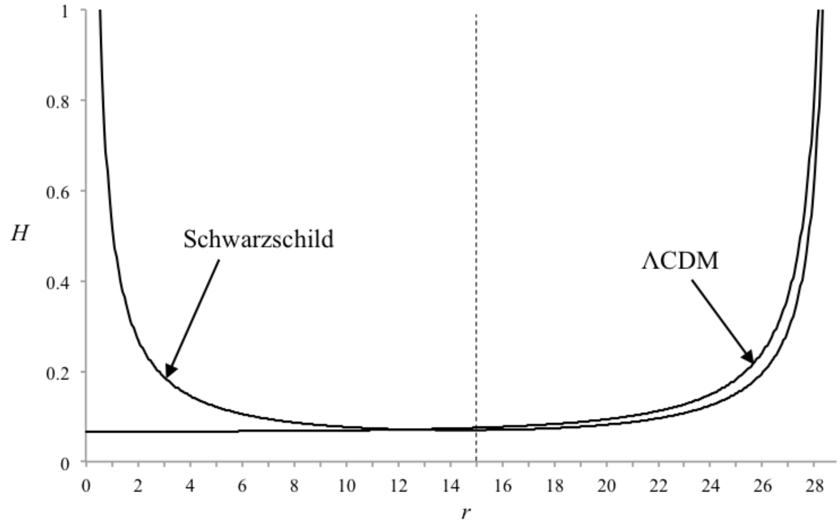

Figure 3 – Hubble Parameter vs. r ($u = 28.8, \Lambda = 0.013$)

Equation 7 can be used to find the transition redshift, which is the redshift we observe at the point when the Universe transitioned from a decelerating expansion to an accelerating expansion. In Equation 7, this transition occurs at $r_{emit} = 21.6$ and our current time is $r = 14.98$. Plugging those values into Equation 8 we get an estimated transition redshift of:

$$z_t = 0.66 \tag{10}$$

This value is within the 2σ bound for the parameter [3,4], and therefore it does appear to be in agreement with cosmological measurements. A plot of redshifts measured at our current time vs. time is given in Figure 4 below:

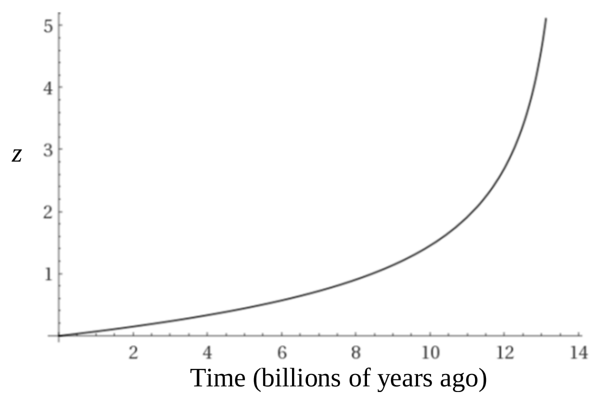

Figure 4 – Redshift vs. Time

Finally, the deceleration parameter is given by:

$$q = \frac{\ddot{a}a}{\dot{a}^2} = \frac{4r}{u} - 3 = \frac{r}{7.2} - 3 \tag{11}$$

A plot of the deceleration parameter is given in Figure 5 below:

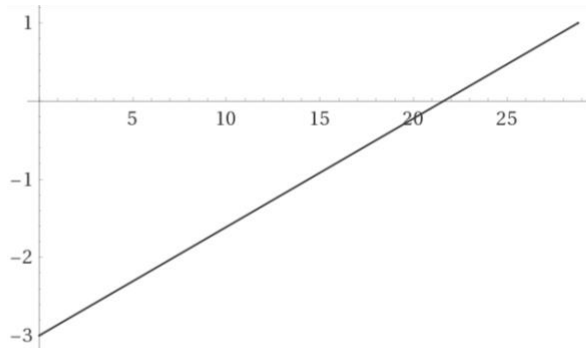

Figure 5 – Deceleration Parameter vs. *r*

Coordinate Distance & Distance Modulus

Figure 1 is a plot of the metric on a Kruskal-Szekeres coordinate chart where the *T*-axis is the vertical axis and the *X*-axis is the horizontal axis. The definition of *T* and *X* are given below for $u = 28.8$:

$$X = \sinh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \quad (12)$$

$$T = \cosh\left(\frac{t}{57.6}\right)\sqrt{(28.8 - r)e^{\frac{r}{28.8}}} \quad (13)$$

Light travels on 45-degree lines in these coordinates so if we consider our current reference frame at $t = 0$ and $r = 15$, we can find the coordinate distance *t* of some galaxy we observe along the 45-degree line at some *r* by setting $\Delta X = -\Delta T$ and solving for *t*. When we do this, we get:

$$t = 28.8 \ln\left(\frac{23.23}{28.8 - r}\right) - r \quad (14)$$

Where *t* is in billions of light years and $15 \leq r \leq 28.8$. Note that Equation 14 is only valid for the current cosmological time. The 23.23 constant is specific to this time so for some other time, a different constant would be required and is given by the value $C = (28.8 - r_0)e^{\frac{r_0}{28.8}}$. We can also use Equation 7 to find r_{emit} as a function of *z* and substitute that into Equation 14 to get the coordinate distance as a function of redshift. If we set $r = 15$ for $u = 28.8$ in Equation 7 and solve for r_{emit} we get:

$$r_{emit} = 28.8\frac{z^2+2z+1}{z^2+2z+1.92} \quad (15)$$

Substituting Equation 15 into 14 will give the coordinate distance as a function of measured redshift. A commonly used parameter in cosmology is the distance modulus, μ, which is defined as:

$$\mu = 5\log_{10}\left(\frac{d}{10}\right) \quad (16)$$

Where d is the distance measured in parsecs. A plot of distance modulus vs. redshift obtained by combining Equations 14, 15, and 16 (where we use t measured in parsecs for d in Equation 16) is shown in Figure 6 below plotted over data obtained from the Supernova Cosmology Project [6]:

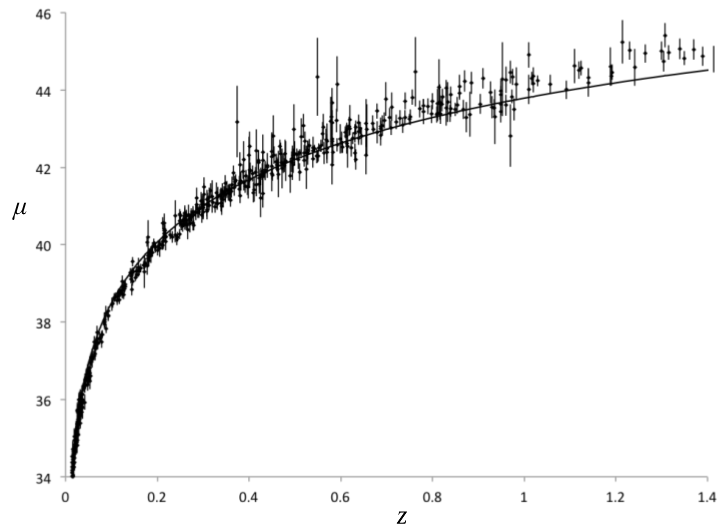

Figure 6 – Distance Modulus vs. Redshift

Note that all these predictions only required the spherical symmetry assumptions of the Schwarzschild metric and calculation of a single parameter, u, from cosmological data; it requires no information regarding the detailed energy distribution within the Universe. In fact, the value of u only determines the units we are working in; it does not affect the form of the model. This reflects the fact that the details of the expansion are the result of the vacuum solution alone.

Proper Time of the Rest Observer

Figure 7 shows the past light cone of an inertial observer at a given time during the expansion:

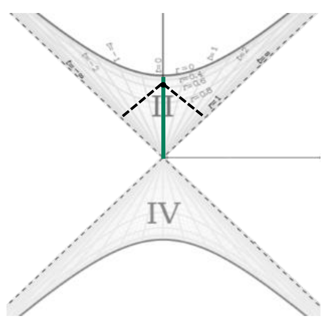

Figure 7 – Past Light Cone of Inertial Observer During the Expansion[2]

[2] Diagram modified from: "Kruskal diagram of Schwarzschild chart" by Dr Greg. Licensed under CC BY-SA 3.0 via Wikimedia Commons - http://commons.wikimedia.org/wiki/File:Kruskal_diagram_of_Schwarzschild_chart.svg#/media/File:Kruskal_diagram_of_Schwarzschild_chart.svg

We can calculate the duration of the expansion of the Universe in the frame of an inertial observer at rest by integrating Equation 5 from 0 to u. The total time of expansion is therefore:

$$\tau = \frac{\pi}{2} u \tag{17}$$

Where τ is measured in billions of years. Equation 17 tells us that in the frame of an observer at rest at *t*, the time elapsed from the Big Bang to $r = 0$ measured by her clock would be around 45.2 billion years and there is only about 8.8 billion years of proper time between now and $r = 0$ for her.

Thinking of τ in Equation 17 as a 'Universal Period' allows us to define a Universal constant $U = \frac{\pi}{2} u$ for time and space. Equation 17 is the maximum amount of time that can be measured between the Big Bang and $r = 0$. So if we set $U = \frac{\pi}{2} u = c = 1$ then we are working in units where space and time have the same units and all measurable times will be between 0 and 1. When working in these units, the constant in the interior Schwarzschild metric will be $u = \frac{2}{\pi}$.

Metric and Geodesics in Terms of the Hubble Parameter and Scale Factor

We can re-express equations 1-4 in terms of the scale factor *a* and the Hubble parameter:

$$d\tau^2 = a^{-2} dr^2 - a^2 dt^2 - r^2 d\Omega^2 \tag{18}$$

$$\frac{d^2 t}{d\tau^2} = 2H \frac{dr}{d\tau} \frac{dt}{d\tau} \tag{19}$$

$$\frac{d^2 r}{d\tau^2} = a^2 H \left[a^2 \left(\frac{dt}{d\tau}\right)^2 - a^{-2} \left(\frac{dr}{d\tau}\right)^2 \right] \tag{20}$$

$$\frac{d^2 r}{d\tau^2} = -a^2 H \tag{21}$$

Equation 21 gives us a quantity analagous to the surface gravity used in the external solution. The non-zero Christoffel symbols of the model (for $d\Omega = 0$) in terms of *H* are:

$$\Gamma^r_{rr} = -H \tag{22}$$

$$\Gamma^t_{tr} = \Gamma^t_{rt} = H \tag{23}$$

$$\Gamma^r_{tt} = a^4 H \tag{24}$$

The Angular Term

We have to this point ignored the angular portion of the Schwarzschild metric. For the internal metric, the angular term seems initially curious because the radius associated with it is a time rather than a distance. According to Figure 1, if we look out at the Universe to a sphere of fixed r, we are also seeing a slice of the Universe that is a fixed t from our position at that r. Thus, in our frame, *dt* between objects on that shell is zero. But we know that some distance separates them, and that distance must come from the angular part of the metric. But the radius of the angular part of the metric is independent of the distance of a shell from us. This means that as the celestial spheres, which are spheres of constant *r* appear to expand over time, it is the angle associated with these spheres that is changing, not the radius.

If we imagine that we could observe the celestial spheres over billions of years, they would appear to us to be expanding. For simplicity, let us only consider the CMB sphere. Over those billions of years, we will always see the CMB, but it is not that the CMB is expanding, it is fixed in the past, so what we see is the CMB at greater and greater coordinate distance to us. What is happening is that we are falling away from the CMB through time and that is why we would see the more distant light from it over time. We can never get closer or father to the CMB in space because the CMB itself only exists in the past and the only way our distance from it can increase is through the time dimension. Since the radius of the CMB as described by the metric is fixed and it appears larger and larger to us over time, it must be the angle of the metric that is changing. Since the entire CMB is in our past and we are falling away from it through time, the angle from one point on the CMB to the diametrically opposed point is not 180 degrees. Rather, a great circle on the CMB (or any celestial sphere) in our reference frame corresponds to the lip of a cone who's tip is at $r = t = 0$. Thus, in terms of the metric, the great circle is not the equator of a sphere but rather a circular secant on a sphere defined by the intersection of a sphere of radius *r* and a cone whose opening angle depends on the value of *t* corresponding to the circle and whose tip is at the center of the sphere. Therefore, when we see the CMB get bigger over time, it is the opening angle of the cone that is getting larger, not the radius. In fact, the maximum observable opening angle would be 90 degrees beause any angle larger than that would require faster than light travel.

The radius in the metric being a time rather than a space is also important when considering the Big Bang itself. Even though we cannot see beyond the CMB, if we could the Big Bang itself would need to be a slightly bigger sphere surrounding everything. The FRW cannot handle this problem. How can the Big Bang, a time when the Universe was fully contracted, have a spatial dimension when viewed at later times? This is only possible if the true metric radius of the celestial spheres (where the Big Bang would be the largest/oldest one) is a time rather than a space. Also, if we could see the celestial sphere of the Big Bang, it would appear completely black to us because any signals from that time would be infinitely redshifted when viewed at any later time and every point on the sphere would be necessarily identical. Thus, when observed from the future, the Big Biang looks very much like the surface of a black hole, and these similarities will be more formally investigated in the final section of the paper.

The ‘Big Bounce’?

A plot of τ vs. r from the uppermost to lowermost hyperbola in Figure 1 is given in Figure 9 below. It illustrates well the relationship to typical spatial projectile motion (for $u = 1$).

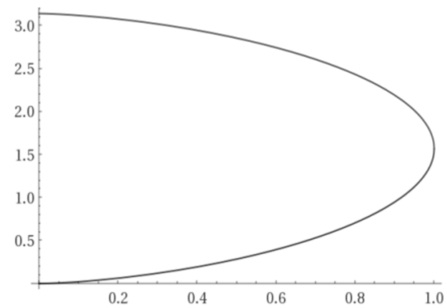

Figure 9 - τ vs. r

Consider a perfectly rigid and elastic ball in simple Newtonian mechanics. If we throw it straight up in the air with initial velocity $\frac{dx}{d\tau}$, the velocity will continuously decrease until at some height $\frac{dx}{d\tau} = 0$, at which point the ball will reverse direction and fall with increasingly negative $\frac{dx}{d\tau}$ until it returns to the ground. When it hits the ground (which we will assume has infinite inertia), since the ball is perfectly rigid and elastic, it will experience an infinite acceleration that will bounce it back toward its maximum height and this cycle will continue ad infinitum. So, there are two turnaround points for the ball. One point is maximum height, where the ball does not experience any special acceleration; it just stops moving through space as it turns around. The second point is a hard acceleration that the ball can really feel a (infinite) force changing its direction.

Likewise, we can see that the Schwarzschild cosmology is a similar situation except that the Universe is the ball and the acceleration is through time rather than space. The Big Bang corresponds to maximum height, where the Universe’s velocity through time changes sign. The $r = 0$ hyperbolas are, perhaps, the ‘bounce’. When the ball bounced, it experienced an infinite acceleration. In the cosmological case, when $r = 0$ the curvature of the spacetime is infinite [1]. This infinite curvature may be a point in time where the worldlines of the Universe turn back on themselves as if the spacetime is folded there and the worldlines go up one side and down the other (the infinite curvature is at the fold).

Relationship to the External Solution

Let us consider a meter stick at rest at the center of a collapsing spherically symmetric collapsing shell. The meter stick inside the shell stretches from the center of the shell out to a distance *2GM* (the shell is at a radius greater than *2GM* so the entire stick is in flat space). An observer in freefall on the collapsing shell does so with speed (in natural units measured by her clock) [5]:

$$\frac{dr}{d\tau} = -\sqrt{\frac{2GM}{r}} \tag{29}$$

Therefore, the freefall observer will see observers at rest at *r* moving past her at the speed given in Equation 29. Since the meter stick is also at rest relative to observers at rest at any *r*, Equation 29 will also give the relative velocity between the freefall observer and the meter stick when the shell is at *r*. Since the spacetime between the freefall observer and central observer is flat, they will each see the other's clock dilated by the Special Relativity Relationship:

$$d\tau = dt\sqrt{1-V^2} = dt\sqrt{1-\frac{2GM}{r}} \tag{30}$$

Because the meter stick will appear to be moving in the frame of the freefalling observer, its length in her frame would be:

$$L = 2GM\sqrt{1-\frac{2GM}{r}} \tag{31}$$

We see from Equation 31 that as the freefalling observer approaches $r = 2GM$ the length of the meter stick in her frame will contract to zero length. So observers in freefall will see the space beyond $r = 2GM$ fully contracted as they approach $r = 2GM$.

Thus, just as the internal and external solutions of the Schwarzschild metric must match up mathematically, so do the black hole event horizon and Big Bang celestial sphere. Both have a finite radius. In the external solution, the radius appears smaller as you fall toward it in space, in the internal solution, the radius appears smaller as you move toward it in time (but the metric radius remains fixed regardless of observer). And as was just demonstrated, the apparent radius of both shrinks to zero for observers approaching $r = u$. Finally, all signals from both are infinitely redshifted such that they appear perfectly black and featureless to distant observers.

But the freefalling observer of the external solution will never fall into a 'black hole'. It would take an infinite amount of time in the frame of an observer at infinity for the freefalling observer to reach the event horizon. But the Universe will expand infinitely (and possibly recollapse) in a finite amount of time in the frame of the infinite observer and therefore the freefalling observer will only reach the $r = 2GM$ location when the Universe itself has recollapsed (if it does indeed recollapse). We know this because the proper time of an observer at rest in the internal solution is the coordinate time of the external solution:

$$dt_{external} = adr_{internal} \tag{34}$$

Since it takes a freefalling observer an infinite amount of coordinate time to reach the horizon in the external solution, but there is only a finite amount of proper time to $r = 0$ and then back to $r = u$ in the internal solution, the freefaller can never reach the horizon during the expansion or collapse of the Universe. When she reaches $r = 2GM$, the entire Universe will be fully contracted (it will have reached the $a = 0$ state described in the previous sections) as though everything in the Universe has collapsed to the same $r =$

$2GM$, and the observer as well as the entire Universe will have reached the next 'Big Bang' state at which point it will presumably begin its expansion once more. This is how the internal and external Schwarzschild solutions relate to one another, they both correspond to the 'Big Bang' state of the Universe.

It is also notable that the external and internal solutions seem to turn smoothly into one another as one crosses the horizon, but consider the external metric measured in some arbitrary units of space and time. In that case, one must include the speed of light in the metric:

$$d\tau^2 = c^2 \frac{r-u}{r} dt^2 - \frac{r}{r-u} dr^2 - r^2 d\Omega^2 \tag{35}$$

In equation 35, we put c^2 in the dt term because *r* and *t* are measured in common units of space and time. If we now allow *r* to be less than *u* such that we get the internal solution, Equation 35 becomes:

$$d\tau^2 = \frac{r}{u-r} dr^2 - c^2 \frac{u-r}{r} dt^2 - r^2 d\Omega^2 \tag{36}$$

For the internal solution, *t* is supposed to be the spatial term and *r* is the time term. But we see from Equation 36 that if one just allows *r* to become less than *u* as though an observer crosses the horizon, the units of the metric no longer make sense as a result of the c^2 (the second term ends up with units like ${m^4}/{s^2}$). This may be evidence that the internal and external solutions are in fact unique, separate solutions to the field equations meaning that black holes are not actually a facet of General Relativity.

Conclusion

It has been shown that the internal Schwarzschild metric will give observations that very closely resemble cosmological observations in our Universe. So either the internal solution is in fact a cosmological solution, or observers inside a Black Hole will see a spacetime that evolves in a strikingly similar way to the evolution of large-scale Universe we ourselves observe.

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