A derivation of special and general relativity from algorithmic thermodynamics

Alexandre Harvey-Tremblay

August 2017

In this paper, I investigate a prefix-free universal Turing machine (UTM) running multiple programs in parallel according to a scheduler. I found that if, over the course of the computation, the scheduler adjusts the work done on programs so as to maximize the entropy in the calculation of the halting probability Ω, the system will follow many laws analogous to the laws of physics. As the scheduler maximizes entropy, the result relies on algorithmic thermodynamics which connects the halting probability of a prefix-free UTM to the Gibb's ensemble of statistical physics (which also maximizes entropy). My goal with this paper is to show specifically, that special relativity and general relativity can be derived from algorithmic thermodynamics under a certain choice of thermodynamic observables applied to the halting probability.

Contents

0.1. *Notation*

I will use the following notation: The symbol $\frown$, for example as in $X \frown Y$, means the concatenation between the strings X and Y. The double vertical lines $|X|$ means the length of the string X. The suffix b, for example in 110_b refers to the binary notation.

1. Introduction

A universal Turing machine (UTM) is a Turing machine (TM) that has the property that it can correctly simulate every TM for every program p. Said in first order logic, for any UTM this must hold;

$$\forall p \forall TM[UTM(\langle TM\rangle \frown \langle p\rangle) = \langle TM(p)\rangle] \tag{1}$$

The bracket notation $\langle TM\rangle$, $\langle p\rangle$ and $\langle TM(p)\rangle$ simply indicates that the input is encoded in the language of the UTM. As each TM outputs a list of symbols, the equal sign means that for any program p the output of the UTM must equal the encoded output of the TM.

1.1. *Algorithmic information theory*

In accordance with Gregory Chaitin's Ω construction (Chaitin(1975)), I can define a sum which encodes the solution to the halting problem as a probability;

$$\Omega = \sum_p 2^{-E(p)-|p|} \qquad \textit{where, } E(p) = \begin{cases} 0 & p \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{2}$$

I will now unpack this sum and use example values to help fix the idea. Consider the follow example values for $E(p)$ and $|p|$,

$$= 2^{-\infty}2^{-1} + 2^{-0}2^{-2} + 2^{-0}2^{-3} + 2^{-0}2^{-4} + 2^{-\infty}2^{-5} + ... \tag{3}$$

The presence of the negative infinity in the term of the exponential causes some terms to vanish to zero. Note that the suffix b indicates the binary notation.

$$= 0_b + 0.01_b + 0.001_b + 0.0001_b + 0_b + ... \tag{4}$$

$$= 0.01110..._b \tag{5}$$

which recovers Ω for the example values.

$$\tag{6}$$

Observable	Conjugate variable
Energy E	Temperature $\beta = 1/(k_b T)$
Volume V	Pressure $\gamma = p/(k_b T)$
Number of particles N	Chemical potential $\delta = -\mu/(k_b T)$

Table 1. *Typical observables of statistical mechanics.*

1.2. *Algorithmic thermodynamics*

Ω is similar to a Gibb's ensemble of statistical physics. In fact, this similarity has been noted by other authors before (Chaitin(1975); Li and Vitányi(1997); Tadaki(2008); Baez and Stay(2010)). Indeed, the Gibb's ensemble compares to the halting probability as follows;

Gibb's ensemble	Halting probability
$Z = \sum_x e^{-\beta(E+pV+Fx)}$	$\Omega = \sum_p 2^{-E(p)-\lvert p\rvert}$

(7)

To be upgraded to a full-fledge Gibb's ensemble, I only need to add a conjugate variable to the halting probability analogous to the temperature β. As suggested by Takadi, I multiply the terms of the exponential by a compression factor adjusting the packing density of the bits of Ω. I get

$$Z'_\Omega = \sum_p 2^{-\beta[E(p)+D|p|]} \tag{8}$$

$E(p)$, as it is either 0 or ∞ will absorb β, so its contribution to Ω remains the same. For $\beta F|p|$, the effect is to "compress" or "decompress" the bits of Ω. If $\beta F > 1$, no bit erasure take places. To fix the idea, I unpack the sum taking $\beta F = 2$ as an example.

$$= 2^{-2\times1} + 2^{-2\times2} + 2^{-2\times3} + 2^{-2\times4} + 2^{-2\times5} + ... \tag{9}$$

$$= 2^{-2} + 2^{-4} + 2^{-6} + 2^{-8} + 2^{-10} + ... \tag{10}$$

$$= 0.01_b + 0.0001_b + 0.000001_b + 0.000000001_b + ... \tag{11}$$

$$= 0.0101010101..._b \tag{12}$$

The result is that some zero-valued bits have been injected between the bits of Ω. To recover Ω, it suffices to eliminate the extra bits. No halting information is lost.

1.3. *Statistical physics*

Before continuing to the next section, I will do a quick recall of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy

$$S = -k_b \sum_{x\in X} p(x) \ln p(x) \tag{13}$$

subject to the fixed macroscopic observables. The solution is the Gibbs ensemble. Taking the observables listed in Table 1 as examples, the partition function becomes

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{14}$$

The probability of occupation of a micro-state is;

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{15}$$

The average values and their variance for the observables are;

$$\overline{E} = \sum_{x \in X} p(x) E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{16}$$

$$\overline{V} = \sum_{x \in X} p(x) V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{17}$$

$$\overline{N} = \sum_{x \in X} p(x) N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{18}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial E}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial V}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial N}\right|_{E,V} = -\frac{\mu}{T} \tag{19}$$

which can be summarized as

$$dE = TdS - pdV + \mu dN \tag{20}$$

This is known as the state equation of the thermodynamic system. We are now ready to continue.

1.4. *An "entropic UTM"*

A UTM can attempt to calculate Ω by starting each program in dovetail, and as they halt, add their contribution to Ω. After an infinite amount of time, Ω will indeed be recovered. However, the calculation does not converges towards Ω as it progresses and discontinuously yields Ω only at infinity. To see why, consider the case where the first zero-valued bit of Ω is at position i. Since the general non-halting problem is unsolvable, at most the calculation of Ω differs from the real value of Ω by 2^{-i}. The error rate does not decreases during the calculation and only vanishes at infinity when all halting programs are known.

To make the laws of physics come out, I must adjust the calculation so that it converges towards Ω even during the calculation. In other words, the error rate must be made to monotonically decrease during the calculation. This can be done with entropic dovetailing.

Definition 21 (Dovetailing). Dovetailing is a program execution strategy for a Turing machine to guarantee that progress will be made on arbitrarily-many programs even in the presence of non-halting programs.

Definition 22 (Standard dovetailing). Consider the case of standard dovetailing. First, we start the shortest program and perform one iteration. Then, we start the second program and perform one iteration on the first and second program. Then, we start the third program and perform one iteration on the first, second and third program. And so on. Using dovetailing, progress will eventually be made on every program and no program will cause the TM to hang.

To convert (2) into a dovetailing calculation of Ω, it suffices that I add the program-action conjugate to program-frequency observable $\mathcal{S}f$ to the sum. Also, note that f, the frequency is related to the time t via $f^{-1} = t$. It yields,

$$Z_\Omega = \sum_p 2^{-\beta[E(p)+D|p|+\mathcal{S}f]} \qquad \text{Partition function} \qquad (23)$$

$$dE = TdS - \mathcal{S}df - Dd|p| \qquad \text{State equation} \qquad (24)$$

What are these program observables and why am I allowed to add them? Recall that $Z_\Omega^{'}$ and Z_Ω are Gibb's ensembles. As a result observables of program properties can be added. I will now look at $\mathcal{S}f$ into more detail to understand the impact it as on the calculation of Z_Ω. Starting with an example, suppose the following values of $\mathcal{S}$ for the first three programs,

$$S_1 = 5 \qquad S_2 = \infty \qquad S_3 = 5 \qquad (25)$$

Note that I do not, nor am I trying to, escape the non-computability of Ω. Indeed, $\mathcal{S}$ is non-computable because $\mathcal{S}$ bears the solution to the general non-halting problem. Z_Ω simply shifts the non-computability from $E(p)$ to $\mathcal{S}$. In my example, the sum Z_Ω becomes

$$Z_\Omega = 2^{-1-\frac{5}{t}} + 2^{-2-\frac{\infty}{t}} + 2^{-3-\frac{5}{t}} + ... \qquad (26)$$

As an example, consider these values of $Z_\Omega(t)$ for specific values of t along with the error rate $\xi(t) = \Omega - Z_\Omega(t)$

$$\Omega = 0.101..._b \qquad \xi = 0 \tag{27}$$

$$\lim_{t\to 0^+} Z_\Omega(t) = 0 \qquad \xi = \Omega \tag{28}$$

$$Z_\Omega(1) = 0.00000101..._b \qquad \xi = 0.10011011..._b \qquad \xi \leq 2^{-0} \tag{29}$$

$$Z_\Omega(5) = 0.0101..._b \qquad \xi = 0.0101..._b \qquad \xi \leq 2^{-1} \tag{30}$$

$$Z_\Omega(10) = 0.01110001001000..._b \qquad \xi = 0.00101110110111..._b \qquad \xi \leq 2^{-2} \tag{31}$$

$$Z_\Omega(1000) = 0.10011010100011..._b \qquad \xi = 0.00000101011100..._b \qquad \xi \leq 2^{-5} \tag{32}$$

$$\lim_{t\to\infty} Z_\Omega(t) = 0.101... \qquad \xi = 0 \qquad \xi \leq 2^{-\infty} \tag{33}$$

As I grow t from 0 to ∞, the error rate monotonically diminish until it eventually vanishes. I will now prove two theorems; 1) Z_Ω can recover Ω at $t \to \infty$ and 2) Z_Ω calculates Ω through time with a monotonically decreasing error rate.

Theorem 34. To prove that Z_Ω recovers Ω at $t \to \infty$, I will show that Ω is computable from $Z_\Omega^{'}$ and that

$$\lim_{t\to\infty} Z_\Omega \to Z_\Omega^{'}$$

Proof. A program p can have any value of $\mathcal{S}_p$ within $[0, \infty]$. If the program halts immediately, $\mathcal{S}_p = 0$. If it never halts, $\mathcal{S}_p = \infty$. If it halts after a certain time, $\mathcal{S}_p \in \mathbb{N}$. A program that never halts will not contribute to the halting partition. This will be the case if $\mathcal{S}_p = \infty$. This yields,

$$\lim_{\tau\to 0^+} \tau S_p = \lim_{t\to\infty} \frac{\mathcal{S}_p}{t} = \begin{cases} 0 & \text{p halts} \\ \infty & \text{otherwise} \end{cases} \tag{35}$$

As this is the definition of $E(p)$, see 2, we obtain

$$\lim_{t\to\infty} \frac{\mathcal{S}_p}{t} = E(p) \tag{36}$$

Lemma 1.1. $E(p) + E(p) = E(p)$

Proof. $E(p)$ is either 0 or ∞. Since $0 + 0 = 0$ and $\infty + \infty = \infty$, the lemma holds. ☐

Therefore,

$$\lim_{t\to\infty} Z_\Omega = \lim_{t\to\infty}\left(\sum_p 2^{-\beta[E(p)+\mathcal{S}f+D|p|]}\right) \tag{37}$$

$$= \sum_p 2^{-\beta[E(p)+E(p)+D|p|]} \tag{38}$$

$$= \sum_p^{\infty} 2^{-\beta[E(p)+D|p|]} \tag*{(lemma 1.1)}$$

$$= Z_\Omega^{'} \tag{39}$$

Is knowing $Z_\Omega^{'}$ enough to compute Ω? The answer is yes as I just need to remove the zero-valued bits inserted in between the bits of Ω. □

Theorem 40. To show that equation (23) dovetails programs, it suffices to show the following. For $0 < t < \infty$, the partition function Z_Ω is

$$Z_\Omega(t) = \Omega - 2^{-k(t)}$$

where $2^{-k(t)}$ is an error rate that is monotonically decreasing to 0 as $t \to \infty$. As a result of increasing the time, the calculation of Z_Ω produces an ever more precise estimation of Ω.

Proof. Using a similar argument as the one provided by John C. Baez and Mike Stay, I argue that as $\mathcal{S}$ exponentially suppresses programs with long halting time, there will always be a time t such that the contribution of programs that have not yet halted will be less than $2^{-k(t)}$. □

As a result, the partition function $Z_\Omega^{'}$ produces a monotonically improving estimation of Ω over time. The fact that the error rate is able to decrease monotonically implies that the calculation does not hang. Hence it is a type of dovetailing. Furthermore, since I have defined the calculation with a Gibb's ensemble, I am guaranteed that the calculation maximizes the entropy during the calculation.

In the end, what I have described is a dovetailing algorithm which maximizes the entropy in the calculation of Ω.

1.5. *Prior and related work*

How then are the laws of physics revored from (23)? To recover the laws of physics, I will make use of the properties of the Gibb's ensemble notably by studying its state equation. Although not necessary, it helps to give (23) a physical interpretation of its observables. Let me start by considering the prior work.

In their paper (Baez and Stay(2010)), John C. Baez and Mike Stay suggest an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be observables. Starting from Gregory Chaitin's Ω number, the halting probability

$$\Omega = \sum_{p \text{ halts}} 2^{-|p|} \tag{41}$$

is extended with algorithmic observables to obtain

$$\Omega' = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{42}$$

Noting the similarity between the Gibb's ensemble of statistical physics (14) and (42), they suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1 $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2 $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the tradeoff between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.
3 $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (42), they derive analogues of Maxwell's relations and they consider thermodynamic cycles such as the Carnot cycle or Stoddard cycle. For this they introduce the concepts of *algorithmic heat* and *algorithmic work*.

Other authors have suggested other somewhat arbitrary correspondance (Li and Vitányi(1997); Tadaki(2008)).

1.6. *Physical interpretation*

I suggest to map the program-observables to physical-observables as follows. As I will show, this interpretation correctly maps the algorithmic thermodynamics interpretation of special relativity to its physical interpretation. Hence, it would appear to the be the preferred mapping.

— The program-runtime is the number of *Iterations* a UTM needs to perform until a program halts. It is therefore natural to associate it with the physical *Time* in *seconds*. Indeed, a program requiring more iterations to halt will also require more time to terminate. If a system performs iterations at a faster or slower rate, the conjugate variable to time, the *Power* in *Watts*, can be adjusted to account for this variation.

Table 2. *The preferred correspondence between algorithmic thermodynamics and statistical physics.*

Observable	Variable	Units	Conjugate	Variable	Units
Halting event	E	J	Temperature	T	K
Program-size (length)	x	m	Force	F	N
Program-size (area)	A	m^2	Stiffness	k	N/m
Program-size (volume)	V	m^3	Pressure	p	N/m^2
Program-frequency	f	$1/s$	Action	$\mathcal{S}$	Js
Program-runtime	t	s	Power	P	W

— Its inverse, the program-frequency, is associated with the reverse of the second, s^{-1}, and its conjugate variable is the *Action* in *Joules-seconds*.

— The program-size is expressed in number of *bits*. Writing the bits one after the other on any medium (paper, disk drive, etc.) will require a certain physical size for each bit. As the line is the lowest dimensional geometry to spread bits, the program-size is naturally associated with the physical *length* as its simplest case. Furthermore, if an encoding medium would allow greater or lesser "packing-tightness" of the bits, it can be modelled with its conjugate variable the *Force* in *Newtons* pushing the bits together or pulling them apart. If one wishes instead to investigate geometries of higher dimensions, one can use different units. For the $2D$ case, it can be mapped to an *Area* in m^2 and its conjugate variable will be the *Surface tension* in N/m. For the $3D$ case, the program-size can be mapped to a *Volume* in m^3 and its conjugate variable will be the *Pressure* in N/m^2. Even higher dimensions could be used, but their physical interpretation, if any, would be less clear.

— Only the halting event remains. As it is the only quantity with *no units*, it is natural to map it to the *Energy* in *Joules*. Indeed, in the Gibb's ensemble, the energy is the only observable not multiplied by a conjugate variable. Adding extra units to the halting event only to have them cancelled out by a conjugate variable would be futile.

Summarizing the points above, I obtain Table 2 as the mapping of choice between *algorithmic thermodynamics* and *physical thermodynamics*.

1.7. *State equation*

The state equation for the partition function (23) is,

Definition 43 (Algorithmic state equation).

$$dE = TdS - \mathcal{S}df - Dd|p|$$

This is analogous to the law of conservation of energy, interpreted as a law of conservation of halting information. I will now take the Taylor series of $Dd|p|$. To do so I first pose $L(p) := |p|$ then I obtain,

$$DL(p) = DL(0) + DL'(0)p + D\frac{1}{2}L''(0)p^2 + D\frac{1}{6}L''(0)p^3 + ... \tag{44}$$

$$DdL(p) = DL'(0)dp + DL''(0)pdp + D\frac{1}{2}L'''(0)p^2dp + ... \tag{45}$$

switching the notation from p to x, I get

$$DdL(x) = DL'(0)dx + DL''(0)xdx + D\frac{1}{2}L'''(0)x^2dx + ... \tag{46}$$

then further posing $F := DL'(0)$, $k := DL''(0)$, $p := DL''(0)$ (here p means the pressure, not a program), I get

$$DdL(x) = Fdx + kxdx + px^2dx + ... \tag{47}$$

To recover the physical interpretation, it suffices that I replace $Dd|p|$ with its Taylor expansion. The state equation, in the physical interpretation is,

Definition 48 (Physical state equation).

$$dE = TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx$$

and if a three-dimensional simplification cutoff is desired, we get

$$dE = TdS - \mathcal{S}df - (F + kx + px^2)dx \tag{49}$$

Solutions to this state equation yields entropic and computed "universes" generated by the calculation of Ω with a UTM.

2. Thermodynamics

2.1. *Time*

Theorem 50. The state equation (48) implies an halting entropy decreasing with time.

Proof.

$$\begin{aligned} dE &= TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx && \text{state equation} \quad (51)\\ 0 &= TdS - \mathcal{S}df && \text{posing } dE = dx = 0 \quad (52)\\ 0 &= TdS + t^{-2}\mathcal{S}dt && df = -t^{-2}dt \quad (53)\\ 0 &= TdS + Pdt && P = t^{-2}\mathcal{S} \quad (54)\\ -\frac{P}{T} &= \frac{dS}{dt} && \text{(decreasing entropy)} \end{aligned}$$

□

Definition 55 (Halting entropy). The halting entropy is the entropy exclusively associated with the calculation of Ω over time. It is the entropy obtained when $dE = dx = 0$.

As time increases, the entropy from the calculation of Ω decreases according to the term $-P/T$. Why does it decreases over time? Consider that, at the beginning of the calculation, none of the bits of $\Omega(t)$ are known hence the error rate is at its maximum. Each bit whose value is unknown contribute $k_B T \ln 2$ to the entropy. As the calculation progresses and the error rate is diminished, then each additional and correct bit that has been calculated becomes fixed and their entropy contributions are reduced to 0.

As a result, an arrow of time is perceived. This arrow of time is quite stronger than than anything physics would suggest via the statistical second law of thermodynamics. Indeed, an increase in time is associated with an increase in halting information. Since each bit of Ω is algorithmically random, then the future which can only be described with more bits is guaranteed to be non-computable. While the past, which contains less valid bits than the present is guaranteed to be computable. This corresponds more closely to our human experience, as we can remember and even deduce the past based on present evidence, but cannot precisely know the future until it happens.

Furthermore, as the entropy of the valid bits of Ω is exactly 0, then it means that the past of the system is fixed and cannot be changed. Again, this more closely matches our human experience as we cannot change our past, so why would its halting entropy be anything other than 0?

Please note that the laws of physics do not currently explain why the past is fixed, nor why the future is non-computable. Both of which are a direct consequence of (50).

2.2. *Exfoliation*

As an entropy decreasing with time would violate the second law of thermodynamics, I suggest that an entropic exfoliation to space occurs so as to make the second law hold. In this scenario, the entropy reduction from the calculation of Ω is compensated by an increase in entropy associated with the exfoliation observables. Consider the following theorem.

Theorem 56. The state equation (48), the second law of thermodynamics and theorem (50) implies an entropic exfoliation to space.

Proof.

$$dE = TdS - \mathcal{S}d\tau - (F + kx + px^2 + ...)dx \qquad \text{state equation} \qquad (57)$$

$$0 = TdS - \mathcal{S}d\tau - (F + kx + px^2 + ...)dx \qquad \text{posing } dE = 0 \qquad (58)$$

$$0 = TdS + t^2\mathcal{S}dt - (F + kx + px^2 + ...)dx \qquad d\tau = -t^2 dt \qquad (59)$$

$$0 = TdS + Pdt - (F + kx + px^2 + ...)dx \qquad P = t^2\mathcal{S} \qquad (60)$$

$$\frac{dS}{dt} = \frac{1}{T}\frac{dx}{dt}\left(F + kx + px^2 + ...\right) - \frac{P}{T} \qquad \text{(exfoliation)}$$

□

Definition 61 (Exfoliation entropy). The exfoliation entropy is the entropy contribution by the term $\frac{1}{T}\frac{dx}{dt}\left(F + kx + px^2 + ...\right)$ to the entropy.

To investigate this result, let us look at three cases;

$$\frac{1}{T}\frac{dx}{dt}\left(F+kx+px^2+...\right)<\frac{P}{T} \implies \frac{dS}{dt}<0 \quad \text{decreasing entropy} \tag{62}$$

$$\frac{1}{T}\frac{dx}{dt}\left(F+kx+px^2+...\right)=\frac{P}{T} \implies \frac{dS}{dt}=0 \quad \text{constant entropy} \tag{63}$$

$$\frac{1}{T}\frac{dx}{dt}\left(F+kx+px^2+...\right)>\frac{P}{T} \implies \frac{dS}{dt}>0 \quad \text{increasing entropy} \tag{64}$$

At (63), a shift occurs in the direction of the production of entropy over time. It is the point at which the exfoliation entropy overtakes and exceeds the reduction in halting entropy. The second law of thermodynamics which states that $dS/dt \geq 0$ will hold for (63) and (64) but will be violated for (62). In any case, if $\frac{1}{T}\frac{dx}{dt}\left(F+kx+px^2+...\right)>0$ then the second law of thermodynamics applicable to the exfoliation observables will be observed.

This derivation more closely matches human experience. Indeed,

1 at the beginning of time, the future of the system is un-actualized, hence the possibilities are endless. To reflect this, the halting entropy is at its maximum at $t=0$, and the exfoliation entropy is equal to 0. This matches our current belief that the exfoliation entropy at the Big Bang is very low.

2 during the evolution, the future becomes past which is "set in stone". As the past is "set in stone" the halting entropy of the bits defining it are equal to 0. This is because we "remember" or "observe" only one past. This reduction in halting entropy is offset by a growth in exfoliation entropy which is related to the size and complexity of the space encoded by the exfoliation observables. This growth in space entropy obeys the second law of thermodynamic.

3 at the end of time, there is no future. The value of Ω as been calculated, and the full history of the system is now "set in stone". The halting entropy is 0 and the exfoliation entropy is at its maximum. This matches the hypothesis of the heat death.

Note that contrary to the halting entropy, the exfoliation entropy of an observer's past need not be equal to 0 as multiple exfoliated micro-states could be compatible with an observer's present. Indeed, as per the second law of thermodynamics, the observer sees a monotonically increasing exfoliation entropy.

How then do we understand this result from the perspective of algorithmic information theory? The exfoliation variables represent the entropy in the choice of available prefix-free encodings for the programs that are input of the UTM. When no bits of Ω are known, it doesn't make sense to speak of the ways to encode this information as there is nothing to encode. Hence the entropy should be 0. As more bits of Ω are known then more ways to encode this information exists and the entropy associated with the possible encodings increases.

2.3. *Holographic principles*

Theorem 65. The state equation (48) implies an holographic principle in the area-dominant regime, where xdx is the dominant contributor to the exfoliation entropy.

Proof.

$$dE = TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx \qquad \text{state equation} \quad (66)$$

$$TdS = (F + kx + px^2 + ...)dx \qquad \text{posing } dE = dt = 0 \quad (67)$$

$$TdS = kxdx \qquad \text{approx. } kx \gg F + px^2 + ... \quad (68)$$

$$\int TdS = \int kxdx \quad (69)$$

$$TS = k\frac{1}{2}x^2 + C \quad (70)$$

$$\implies S \propto A \qquad \text{(holographic principle)}$$

□

The laws of physics which will be derived from the area-dominant approximation $kx \gg (F + px^2 + ..)$ will certainly contain an holographic principle linking the entropy to the area enclosing the volume. However, the holographic principle need not necessarily hold at other entropic growth scales, for example, where the volumetric entropy x^2dx is dominant. Indeed, state equation (48) would appear to suggests three different scales each having a "holographic principle" of a different dimensional size.

Dimension	Dominant force	Approximation	Entropy	
$1D$	Fdx	$F \gg kx + px^2 + ...$	$S \propto L$	(71)
$2D$	$kxdx$	$kx \gg F + px^2 + ...$	$S \propto A$	(72)
$3D$	px^2dx	$px^2 \gg F + kx + ...$	$S \propto V$	(73)
...				

In this scenario, the universe would be dominated by the linear scale at short distances, then would be overtaken by the area scale and finally by the volume scale. In the coming section, I will show that special relativity and the law of inertia are derivable in the $S \propto L$ scale, that general relativity is derivable at the $S \propto A$ scale and will provide citations suggesting that dark energy might be derivable from the $S \propto V$ scale.

2.4. *Spacetime*

Theorem 74. The state equation (48) implies a relation between space and time.

Proof.

$$
\begin{aligned}
dE &= TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx && \text{state equation} && (75)\\
0 &= -\mathcal{S}df - (F + kx + px^2 + ...)dx && \text{posing } dE = dS = 0 && (76)\\
0 &= -\mathcal{S}df - Fdx && \text{approx. } F \gg kx + px^2 + ... && (77)\\
0 &= t^{-2}\mathcal{S}dt - Fdx && dt = -t^{-2}dt && (78)\\
0 &= Pdt - Fdx && P = t^{-2}\mathcal{S} && (79)\\
Fdx &= Pdt && \text{add } Fdx && (80)\\
dx &= \frac{P}{F}dt && && \text{(fundamental relation of spacetime)}
\end{aligned}
$$

□

The units of P/F are meters per second. This implies that any system described by (48) will have a characteristic power and force that relates time to space. Indeed, if the system is the universe, then taking the characteristic Planck power and force, we do recover the speed of light,

$$P\frac{1}{F} = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{81}$$

In lieu of an appeal to the Planck constant, we are permitted to pose $c := P/F$ as a definition and rewrite the result as

$$dx = cdt \tag{82}$$

which is the fundamental relation of special relativity and c is a constant connecting space to time.

2.5. *Limiting relations*

Theorem 83. The state equation (48) implies a maximum speed.

Proof.

$$
\begin{aligned}
dE &= TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx && \text{state equation} && (84)\\
0 &= -\mathcal{S}df - (F + kx + px^2 + ...)dx && \text{posing } dE = dS = 0 && (85)\\
0 &= -\mathcal{S}df - Fdx && \text{approx. } F \gg kx + px^2 + ... && (86)\\
0 &= TdS + t^{-2}\mathcal{S}dt - Fdx && d\tau = -t^{-2}dt && (87)\\
0 &= TdS + Pdt - Fdx && P = t^{-2}\mathcal{S} && (88)\\
Fdx - Pdt &= TdS && \text{add } Fdx - Pdt && (89)\\
\frac{dx}{dt} - \frac{P}{F} &= \frac{T}{F}\frac{dS}{dt} && && \text{(maximum speed)}
\end{aligned}
$$

□

To see why this implies a maximum speed, first consider that the units of this equation are meters per second. Second, consider the following three cases;

$$\frac{dx}{dt} = \frac{P}{F} \implies T\frac{dS}{dt} = 0 \tag{90}$$
$$\frac{dx}{dt} < \frac{P}{F} \implies T\frac{dS}{dt} < 0 \tag{91}$$
$$\frac{dx}{dt} > \frac{P}{F} \implies T\frac{dS}{dt} > 0 \tag{92}$$

To prove that the speed P/F is a maximum it suffices to note the presence of a reversal of the second law of thermodynamics at the P/F barrier. Furthermore, as the irreversibility of the second law of thermodynamics is well established, it follows that the barrier cannot be overcome. A system evolving faster than c will experience a reversal of the second law compared to a system slower than c (and vice-versa), but neither will be able to cross c and flip its direction.

Please note that in standard physics the speed of light is accepted as an axiom and is not derived from more fundamental principles. Here the speed of light is a direct consequence of (48).

Theorem 93. The following relations each characterize a maximum quantity.

Approximation			
none	$T\frac{dS}{dt} = \frac{dE}{dt} - P$	maximum power (J/s)	(94)
$S \propto L$	$\frac{T}{F}\frac{dS}{dt} = \frac{dx}{dt} - \frac{P}{F}$	maximum speed (m/s)	(95)
$S \propto A$	$\frac{T}{k}\frac{dS}{dt} = \frac{xdx}{dt} - \frac{P}{k}$	maximum viscocity (m^2/s)	(96)
$S \propto V$	$\frac{T}{p}\frac{dS}{dt} = \frac{x^2dx}{dt} - \frac{P}{p}$	max. vol. flow rate (m^3/s)	(97)

Proof. Each relation can easily be obtained from (48) by posing the other observables to 0. To prove that the quantities are a maximum, it suffices to notice that they each correspond to the point at which the second law of thermodynamics is reversed. □

Theorem 98. The partition function (23) implies a discrete halting entropy with a minimum step.

This theorem has a stronger requirement than the previous two theorems on maximum quantities. It is not enough to just prove an extremum value, but a minimum value that is also discrete.

Proof. To prove it, recall how Z_Ω calculates an estimation of Ω valid within a monotonically deceasing error rate ξ. Knowing the precise value of ξ is equivalent to knowing Ω, as Ω can simply be recovered by adding ξ to Z_Ω. The implication is that the bits of ξ must also be non-computable. As each bit of ξ is enough to recover one bit of Ω, it follows that, as ξ is the error rate, no bits of ξ can be known beyond the position of its first one-valued bit.

Second, let us define a non-divergent entropy for the system $\Omega = Z_\Omega + \xi$. As the system is infinitely complex, its entropy will be convergent only for the first $N \in \mathbb{N}$ bits. The bits of Z_Ω have an entropy of 0, and the bits of ξ have an entropy of $N_\xi k_B \ln 2$.

$$S = N_{Z_\Omega} k_B \ln 1 + N_\xi k_B \ln 2 \tag{99}$$
$$= N_\xi k_B \ln 2 \tag{100}$$

As a result, the smallest entropy of the system S_0 is $k_B \ln 2$. Furthermore, the entropy increases by steps of $k_B \ln 2$, as N_ξ is a natural number. This proves the theorem. □

Theorem 101. Exfoliation observables and exfoliation conjugates are discrete as per the discrete halting entropy.

Proof. Consider the following relations connecting the halting entropy to the exfoliation variables, $dS = dE$, $dS = (F + kx + px^2 + ...)dx$ and the halting variables $dS = \mathcal{S}df$ and $dS = -Pdt$. If dS is discrete, then it implies that these variables dE, $\mathcal{S}$, F, k, p, df, P, dx and dt are also discrete.

Without loss of generality, consider the pair $\mathcal{S}df$. Since dS is discrete (98), then both $\mathcal{S}$ and df must be discrete. This reasoning can be applied to all exfoliation and halting variables. Why must both $\mathcal{S}$ and df be discrete? Suppose that either $\mathcal{S}$ or df are real. Then a real multiplied by a real is a real, and a real multiple by a whole number is also a real. Hence for dS to be a whole number, both $\mathcal{S}$ and df must both be whole number.

Another way to see it is that the entropy of a real number can be infinite, but we are only allowed an entropy $N k_B \ln 2$, hence none of these variables can be arbitrary real numbers. □

The discretization of observables, notably the action observable, has been used since the early days of quantum physics to justify it. Indeed, the explanation of the photoelectric effect and the black body radiation, two early successes of quantum physics were both explained via the discretization of what I refer in here as the exfoliation variables. Could the discretization of exfoliation variables imply a certain quantum character to the partition function? Further connections to quantum physics will be investigated by the author in a future paper.

3. Relativity

3.1. *Light cones as thermodynamic cycles*

In this section, I look at the thermodynamic cycle of the system transiting through time and space starting at O to A to B and back to O as illustrated on Figure 1. During the transitions and to keep the energy constant, tradeoffs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles such as those involving pressure and volume but also of relativistic light cones.

We pose that $dE = 0$ and the $S \propto L$ approximation throughout the cycle.

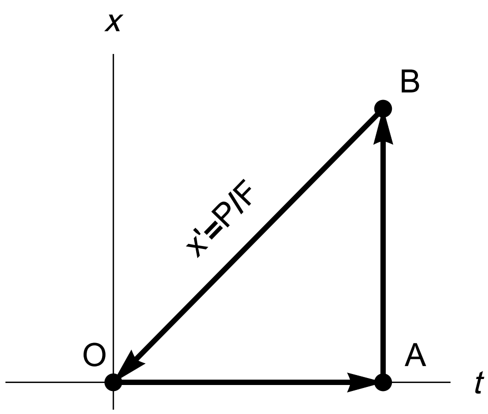

Fig. 1. A thermodynamic cycle through space, time and entropy as observables.

$$TdS = Fdx - Pdt \tag{102}$$

O to A: As we translate O in time to A while keeping the distance constant ($dx = 0$), the halting entropy must decrease over time to compensate.

$$(TdS = Fdx - Pdt|_{dx=0} \tag{103}$$

$$\implies \frac{dS}{dt} = -\frac{P}{T} \tag{104}$$

To travel forward in time, the system acquires knowledge of the values of previously unknown bits of Ω, reducing the halting entropy. Conversely, to travel backward in time, the system must erase halting bits from its pool of information so as to increase its halting entropy. Traveling backward in time is equivalent to erasing halting information about the system's present.

A to B: As we translate A in space to B while keeping the time constant ($dt = 0$), the exfoliation entropy must increase over space to compensate.

$$(TdS = Fdx - Pdt|_{dt=0} \tag{105}$$

$$\implies \frac{dS}{dx} = \frac{F}{T} \tag{106}$$

We conclude that the further away from A a region is, the higher its exfoliation entropy will be. Since $dt = 0$, no change in time is experienced.

O to B: As we translate O forward both in time and in space to B while keeping the entropy constant ($dS = 0$), we have movement at the speed c.

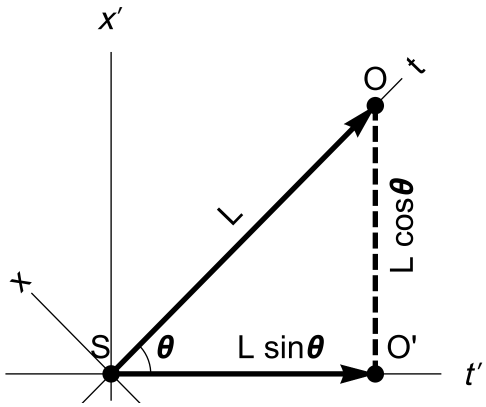

Fig. 2. The spacetime intervals between two observers. Here O' travels at speed $|v|$ in O's reference frame.

$$(TdS = Fdx - Pdt|_{dS=0} \tag{107}$$

$$\implies \frac{dx}{dt} = \frac{P}{F} = c \tag{108}$$

We conclude that an object travelling at speed c is neither encouraged nor discouraged by entropy. However, the type of entropy changes. The rate P/F is the rate of conversion of halting entropy to exfoliation entropy. At O the system is comprised exclusively of halting entropy as its future is not yet determined. As the system evolves towards B, its halting entropy is decreased over time as the system accrues a definite past, and its exfoliation entropy is increased over space to offset the reduction.

We conclude that travelling backward in time is restricted for the same reasons that the speed of light is an upper bound. In both cases, halting information is erased. Furthermore, as a system accrues a past, its exfoliation entropy is increased to compensate the reduction of halting entropy. P/F is the rate at which the production of exfoliation entropy equals the reduction of halting entropy.

3.2. *Lorentz's transformation*

To recover the Lorentz's factor γ, let us consider figure 2. Two observers start at the origin S and travel in spacetime respectively to O and O'. We regard O' as traveling at speed $|v|$ in O's reference frame. From standard trigonometry, we derive the following values for the lengths;

Segment	Length	
$\lvert\overline{SO'}\rvert$	L	(109)
$\lvert\overline{OO'}\rvert$	$L\sin\theta$	(110)
$\lvert\overline{SO}\rvert$	$L\cos\theta$	(111)

Let us start with the Pythagorean theorem and solve for $\cos\theta$.

$$\left|\overline{SO}\right|^2 = \left|\overline{OO'}\right|^2 + \left|\overline{SO'}\right|^2 \tag{112}$$
$$L^2 = (L\sin\theta)^2 + (L\cos\theta)^2 \tag{113}$$
$$1 = (\sin\theta)^2 + (\cos\theta)^2 \tag{114}$$
$$\sqrt{1-(\sin\theta)^2} = \cos\theta \tag{115}$$

We consider that the distance between two observers moving at constant speed is simply vt. Hence, $\left|\overline{OO'}\right| = vt$. Solving for $\sin\theta$, we obtain

$$\left|\overline{O_1O_2}\right| = vt = L\sin\theta \tag{116}$$
$$\implies \sin\theta = \frac{vt}{L} \tag{117}$$

From equation (115) and (117), we get the reciprocal of the Lorentz factor,

$$\sqrt{1-\frac{v^2t^2}{L^2}} = \cos\theta = \gamma^{-1} \tag{118}$$
$$\implies \gamma = \frac{1}{\sqrt{1-\frac{v^2t^2}{L^2}}} \tag{119}$$

Finally, we consider that L is the distance travelled in time by O in its own reference frame. This is given via the relation $dx = cdt$. Hence $L = ct$. We obtain,

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{120}$$

which is the well known Lorentz factor and is the multiplication constant connecting $\left|\overline{SO}\right|$ to $\left|\overline{SO'}\right|$.

3.3. *General relativity (GM)*

I will show how the state equation (48) suggests that general relativity is an emergent entropic phenomenon attributable to the second term, $kxdx$, of the Taylor decomposition of $d|p|$. But first, let is look at the implications of the first term, Fdx.

Theorem 121. The $S \propto L$ scale implies the law of inertia, $F = ma$.

Proof. First, consider that as the entropy, S, is related to the bits, then $dS = k_BdN$ where N is the number of bits. Second, I will derive the equation for an entropic force.

$$\begin{aligned}
dE &= TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx && \text{state equation} && (122)\\
0 &= TdS - (F + kx + px^2 + ...)dx && \text{posing } dE = df = 0 && (123)\\
0 &= TdS - Fdx && \text{approx. } F \gg kx + px^2 + ... && (124)\\
Fdx &= TdS && \text{add } Fdx && (125)\\
F &= T\frac{dS}{dx} && \text{divide } dx && (126)\\
F &= Tk_B\frac{dN}{dx} && && \text{(entropic force)}
\end{aligned}$$

As my goal is to recover $F = ma$, I must link T to the acceleration. To do so I will use the Unruh temperature experienced by a body undergoing constant acceleration as suggested by Erik Verldinde in (Verlinde(2010)). The existence of a well defined temperature allows me to conclude that the system is described at thermodynamic equilibrium, hence the state equation holds.

$$\begin{aligned}
F &= \left(\frac{ha}{ck_B}\right)k_B\frac{dN}{dx} && \text{Unruh temperature} && (127)\\
F &= \left(\frac{h}{c}\frac{dN}{dx}\right)a && \text{clean up} && (128)
\end{aligned}$$

Finally, the equation $F = ma$ can be recovered provided that the ratio dx/dN is the Compton wavelength.

$$\implies \frac{dx}{dN} = \frac{h}{mc} \tag{129}$$

What does this means from the algorithmic perspective and why is dx/dN the Compton wavelength? dx/dN is the ratio between the position of an object and the number of bits required to express such position. It implies that each increment of an object's position by its Compton wavelength must use one additional bit of entropy. The algorithm to encode position used by the UTM is of the form $x = n\lambda$ where n indicates the number of times its Compton wavelength is repeated to reach its position. The entropy usage is optimized as the position of an object does not need to be specified more accurately than its Compton wavelength. □

Theorem 130. The area-dominant regime implies general relativity.

Proof. My goal in this proof is to derive the Einstein field equation of general relativity starting from the holographic principle,

$$TdS = kxdx \implies S \propto A \tag{131}$$

$$\implies dS = \gamma dA \tag{132}$$

This has indeed been done before in the literature, notably by Ted Jacobson in (Jacobson(1995)), then later (and differently) by Erik Verlinde in (Verlinde(2010)). Furthermore, Christoph Schiller in (Schiller(2008)) argues that a maximum power implies the Field equation. In here, I will provide a sketch of the proof by Ted Jacobson. As Schiller explains;

Jacobson, starting from $dE = TdS$, first connects dE to an arbitrary coordinate system and energy flow rates,

$$dE = \int T_{ab} k^a d\Sigma^b \tag{133}$$

Here T_{ab} is a energy-momentum tensor, k is a killing vector field and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that, assuming that the holographic principle holds (and in here it does according to 65), the right part of (132) can be rewritten to,

$$dA = \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{134}$$

Where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-huri equation giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab} k^a d\Sigma^b = \gamma \frac{c^2}{a} \int R_{ab} k^a d\Sigma^b \tag{135}$$

which is only be satisfied if

$$T_{ab} = \gamma \frac{c^2}{a} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{136}$$

Here, the full field equations of general relativity are recovered including the cosmological constant.

□

Finally, I suggest an interpretation of the third term of the Taylor series expansion of the program-length encoding, $TdS = px^2 dx$, as related to dark energy.

$$\begin{aligned} dE &= TdS - \mathcal{S}df - (F + kx + px^2 + ...)dx && \text{state equation} \quad (137) \\ 0 &= TdS - (F + kx + px^2 + ...)dx && \text{posing } dE = df = 0 \quad (138) \\ 0 &= TdS - px^2 dx && \text{approx. } px^2 \gg F + kx + ... \quad (139) \\ TdS &= px^2 dx && (140) \end{aligned}$$

A derivation of dark energy is outside the scope of this paper. Therefore we will refer to this paper (Verlinde(2016)) by Erik Verlinde. In it, he makes a compelling argument that a volumetric entropy can account for the observed dark energy. He links a volumetric entropy overtaking other forms of entropy over large distances to the theory of elasticity.

4. Characteristic units

My goal in this section is to show how the definition of the Planck units naturally flows from the state equation (48). To do so, I must first obtain definitions for G, c and $\hbar$ by deriving from (48) known laws of physics which contains them. I by obtaining the gravitational constant G from Newton's law of gravitation.

Theorem 141. The gravitational constant G is defined as $c^3 L^2/\hbar$.

Proof.
I work in the area-dominant regime where $kx \gg (F + px^2 + ...)$. This regime contains the holographic principle and as a result the entropy of the system grows via x^2, an area law. I further consider that the entropy of this area law is given by bits exclusively occupying a small area L^2 on the surface. In this case, the total number of bits on the surface is given by,

$$N = \frac{x^2}{L^2} \tag{142}$$

I then, via the state equation, relate the number of bits N to the energy of the system.

$$\begin{aligned}
dE &= TdS - \mathcal{S}df - (F + kx + px^2)dx && \text{state equation} && (143)\\
dE &= TdS && \text{posing } df = dx = 0 && (144)\\
dE &= Tk_B dN && \text{entropy from bits} && (145)\\
\int dE &= Tk_B \int dN && && (146)\\
E &= Tk_B N && && (147)\\
\implies T &= \frac{L^2}{k_B}\frac{E}{x^2} && && \text{(equilibrium temperature)}
\end{aligned}$$

I obtain a temperature constant throughout the system indicating that it is at thermodynamic equilibrium. As my goal is to recover the gravitational constant, I inject this temperature in the entropic force relation.

$$\begin{aligned}
F &= Tk_B \frac{dN}{dx} && \text{entropic force} && (148)\\
F &= \left(\frac{L^2}{k_B}\frac{E}{x^2}\right) k_B \frac{dN}{dx} && \text{derived temperature} && (149)
\end{aligned}$$

As the temperature was obtained from the area regime and therefore the force is produced as the result of a exfoliation of the area regime, I then replace the ratio dN/dx by reduced Compton wavelength.

$$F = \left(\frac{L^2}{k_B}\frac{E}{x^2}\right) k_B \left(\frac{mc}{\hbar}\right) \quad \text{clean up} \tag{150}$$

$$F = \left(\frac{L^2 c}{\hbar}\right)\frac{Em}{x^2} \tag{151}$$

I then convert E to its rest mass via $E = mc^2$.

$$F = \left(\frac{L^2 c^3}{\hbar}\right)\frac{Mm}{x^2} \tag{152}$$

I obtain the Newton's law of gravitation along with a definition for G.

$$F = G\frac{Mm}{x^2} \tag{153}$$

$$\implies G = \frac{L^2 c^3}{\hbar} \tag{154}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}} \tag{Planck's length}$$

□

Theorem 155. The speed of light c is defined by P/F.

Proof. I refer to the previous proof where P/F is a characteristic speed associated with an inversion in the direction of the second law of thermodynamic. Then, under the principle the second law is irreversible, the speed P/F is a boundary and defines c. □

Theorem 156. The action $\mathcal{S}$ is defined by $\hbar$.

Proof.

$$dE = \mathcal{S} df \tag{157}$$

$$\int dE \mathcal{S} \int df \tag{158}$$

$$E = \mathcal{S} f \implies \mathcal{S} = h \tag{159}$$

This is the photon frequency-to-energy relation $E = hf$. □

I have now obtained a definition for each of the fundamental constant used to define the fundamental Planck units of time, energy and length. Note that as I will combine the constants into the area regime dominated by spheres and circle, I will use the reduced Planck constant for $\mathcal{S}$ instead of h so as to absorb the 2π term. I obtain,

$$\hbar = \mathcal{S} \qquad c = \frac{P}{F} \qquad G = \frac{L^2 c^3}{\hbar} \tag{160}$$

I can now recover the definitions of the Planck units of length, energy and time for the UTM,

$$G = \frac{L^2c^3}{\hbar} \implies L = \sqrt{\frac{\hbar G}{c^3}} \quad \text{(Planck's length)}$$

$$t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}} \quad \text{(Planck's time)}$$

$$E = \mathcal{S}/t \implies E = \sqrt{\frac{\hbar c^5}{G}} \quad \text{(Planck's energy)}$$

$$P = t^{-2}\mathcal{S} = \frac{c^5}{G} \quad \text{(Planck's power)}$$

$$\frac{P}{F} = c \implies F = \frac{c^4}{G} \quad \text{(Planck's force)}$$

which agrees with the physical Planck units.

5. Conclusion

We note an affinity between an entropic UTM and the laws of physics. The affinity occurs when we consider a UTM calculating its Ω number in a manner so as to maximize the entropy throughout the calculation. When the entropy is maximized, the halting probability becomes a Gibb's ensemble. As a result, additional program observables can be added to the halting probability while preserving its connection to Ω. The laws of physics come out when a single observable is added, $\mathcal{S}df$, as it is enough to make the calculation converge towards Ω over time.

Understanding physics from the perspective of an entropic UTM holds several conceptual advantages. For one, we can now define a non-computable future with a computable past set in stone, whose halting entropy is 0. This provides us with an arrow of time closely matching human experience. The entropy of the complete system (which includes future possibilities as well as an encoding scheme for the past) is should stay constant over time as the change of entropy of one is offset by the other. The second law of thermodynamics understood as an increase in entropy over time is perceived in the exfoliation variables while the larger system made to include future possibilities has a constant entropy over time.

The decomposition of the program encoding scheme used by the UTM via a Taylor expansion produces terms which can be linked to a scale where a specific entropic force is dominant. For the first Taylor expansion term, we recover special relativity (speed of light (83), light-cones (figure 1) and the Lorentz's factor (figure 2)) and the law of inertia (121). For the second term, we recover general relativity (130) and the holographic principle (65). Finally, the third term might be related to an entropic explanation of dark energy (140). As a Taylor expansion tends to produce a series where the first term dominates locally, we obtain an interpretation where inertia dominates at local distances until it is overtaken by general relativity at stellar scales and eventually by dark energy at cosmological scales.

References

John C. Baez and Mike Stay. Algorithmic thermodynamics. arXiv:1010.2067 [math-ph], 2010.

Gregory J. Chaitin. A theory of program size formally identical to information theory. `https://www.cs.auckland.ac.nz/ chaitin/acm75.pdf`, 1975.

Ted Jacobson. Thermodynamics of spacetime:the einstein equation of state. `https://arxiv.org/pdf/gr-qc/9504004.pdf`, 1995.

Ming Li and Paul Vitányi. An introduction to kolmogorov complexity and its applications. Springer, 1997.

Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. `https://arxiv.org/pdf/physics/0607090.pdf`, 2008.

K. Tadaki. A statistical mechanical interpretation of algorithmic information theory. `https://arxiv.org/pdf/0801.4194.pdf`, 2008.

Erik Verlinde. On the origin of gravity and the laws of newton. arXiv:1001.0785v1 [hep-th], 2010.

Erik Verlinde. Emergent gravity and the dark universe. `https://arxiv.org/pdf/1611.02269.pdf`, 2016.