Is special relativity in algorithmic information theory?

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My purpose with this paper is to show that the theory of special relativity is derivable from algorithmic information theory (AIT). Specifically, I construct a partition function of Ω such that the program-runtime and the program-size are observables of a prefix free universal Turing machine. This partition function implies a maximum speed in bits per iterations of the UTM, which has a similar interpretation as the speed of light does in special relativity.

Contents

1 *Algorithmic information theory*

We start with Gregory Chaitin's Ω construction.

Definition 1.1. *Ω is the halting probability of a prefix free universal Turing machine (UTM)*[1]*. Ω is a normal, non-computable, algorithmically random and transcendental real number.*

For unitary encoded programs, Ω is obtained via the following sum,

$$\Omega = \sum_{x=1}^{\infty} 2^{-E(x)-x} \tag{1.2}$$

where x is a program and where $E(x)$ is the halting-event function and is defined as

$$E(x) = \begin{cases} 0 & x \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{1.3}$$

[1] Gregory J. Chaitin. An algebraic equation for the halting probability. https://www.cs.auckland.ac.nz/~chaitin/berlin.pdf, 1988; Gregory J. Chaitin. How real are real numbers? http://www.worldscientific.com/doi/abs/10.1142/S0218127406015726 and https://www.cs.auckland.ac.nz/~chaitin/olympia.pdf, 2006; Gregory Chaitin. Mathematics, complexity and philosophy. https://www.academia.edu/31320410/Mathematics_Complexity_and_Philosophy_full_bilingual_text_, 2010; and Ming Li and Paul Vitányi. An introduction to kolmogorov complexity and its applications. Springer, 1997

Note that as $E(x)$ is the carrier of non-halting information, it is connected to the halting problem of computer science. As a result, it is a non-computable function.

We note the similarities between 1.2 and the Gibb's ensemble of thermodynamics. In fact, these similarities have been noted by other authors before [2]. Simple replacements (changing the name of the variables) are enough to switch back and forth between the two representations. The Gibb's ensemble is,

[2] K. Tadaki. A statistical mechanical interpretation of algorithmic information theory. https://arxiv.org/pdf/0801.4194.pdf, 2008; John C. Baez and Mike Stay. Algorithmic thermodynamics. arXiv:1010.2067 [math-ph], 2010; and Ming Li and Paul Vitányi. An introduction to kolmogorov complexity and its applications. Springer, 1997

$$\begin{array}{cc} \text{Gibb's ensemble} & \text{Halting probability} \\ Z = \sum_x e^{-\beta(E+pV+Fx)} & \Omega = \sum_{x=1}^{\infty} 2^{-E(x)-x} \end{array} \tag{1.4}$$

In the Gibb's ensemble, p is the pressure, V the volume, F a force, x a length and E is the energy. Fx would describe an entropic force, such as polymer tension.

As a result of this similarity, we hypothesize that the halting probability can be given a thermodynamic interpretation. To investigate this further, we import the notions of thermodynamics into AIT to augment Ω with additional program observables, analogous to thermodynamic observables. This augmentation is required as the Ω function does not yet have sufficient observables so as to recover the complexity of physics. As it stands the only law we could recover from it would be $dE = TdS$.

First, we augment the rightmost term of the exponential, x, with a conjugate variable. Lets call it F. Second we multiply the terms of the exponential by β. As $E(x)$ is either 0 or ∞, it absorbs β. With these additions, we obtain Tadaki's D-random number [3] .

[3] K. Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. http://arxiv.org/abs/nlin/0212001, 2002; and K. Tadaki. A statistical mechanical interpretation of algorithmic information theory. https://arxiv.org/pdf/0801.4194.pdf, 2008

$$\Omega^D = \sum_{x=1}^{\infty} 2^{-\beta[E(x)-Fx]} \tag{1.5}$$

Third, we augment the exponential with the observable of the program-runtime observable t. Its conjugate variable is the compute-power, P. We obtain a partition function of algorithmic information theory (AIT).

$$\Omega^Z = \sum_{x=1}^{\infty} 2^{-\beta[E(x)-Pt-Fx]} \tag{1.6}$$

This partition function of AIT relates halting event to program size and to program runtime. The program-observables are conjugated to the compute-power P and the compressibility F of Ω, respectively. It has the following state equation,

$$dE = TdS - Pdt - Fdx \qquad (1.7)$$

2 Statistical Physics interpretation

We can give an thermodynamics interpretation of 1.6 by using the following replacements and justifications.

- The program-runtime is the number of *Iterations* a UTM needs to perform until a program halts. It is therefore natural to associate it with the physical *Time* in *seconds*. Indeed, a program requiring more iterations to halt will also require more time to terminate. If a system performs iterations at a faster or slower rate, the conjugate variable to time, the *Power* in *Watts*, can be adjusted to account for this variation.
- The program-size is expressed in number of *bits*. Writing the bits one after the other on any medium (paper, disk drive, etc.) will require a certain physical size for each bit. As the line is the lowest dimensional geometry to spread bits, the program-size is naturally associated with the physical *length* as its simplest case. Furthermore, if an encoding medium would allow greater or lesser "packing-tightness" of the bits, it can be modelled with its conjugate variable the *Force* in *Newtons* pushing the bits together or pulling them apart. If one wishes instead to investigate bit packing geometries of higher dimensions, one can use different units. For the $3D$ case, the program-size can be mapped to a *Volume* in m^3 and its conjugate variable will be the *Pressure* in N/m^2. For the $2D$ case, it can be mapped to an *Area* in m^2 and its conjugate variable will be the *Surface tension* in N/m.
- Only the halting event remains. As it is the only quantity with *no units*, it is natural to map it to the *Energy* in *Joules*. Indeed, in the Gibb's ensemble, the energy is the only observable not multiplied by a conjugate variable. Adding extra units to the halting event only to have them cancelled out by a conjugate variable would be futile.

Summarizing the points above, we obtain table 1 as our mapping of choice between *algorithmic thermodynamics* and *physical thermodynamics*.

3 Special Relativity

We will derive special relativity from 1.6 by taking its state equation 1.7 and investigating it.

Observable	Variable	Units	Conjugate	Variable	Units
Halting event	E	J	Temperature	T	K
Program-size (length)	x	m	Force	F	N
Program-size (area)	A	m^2	Stiffness	γ	N/m
Program-size (volume)	V	m^3	Pressure	p	N/m^2
Program-runtime	t	s	Power	P	W

Table 1: The preferred correspondence between *algorithmic thermodynamics* and *statistical physics*.

3.1 Maximum speed

Theorem 3.1. *An object travelling faster than the speed given by P/F will violate the second law of thermodynamics.*

Proof.

$$dE = TdS - Pdt - Fdx \qquad \text{(State equation)}$$

$$0 = TdS - Pdt - Fdx \qquad \text{(Posing } dE \text{ to } 0)$$

$$Fdx = TdS - Pdt \tag{3.2}$$

$$\frac{dx}{dt} = \frac{T}{F}\frac{dS}{dt} - \frac{P}{F} \tag{3.3}$$

Note that the units for each term of equation 3.3 describe a speed. In the thermodynamic interpretation the units are meters per seconds. Whereas in the AIT interpretation, the units are bits per iteration.

Let us look at three cases:

1. If $|dx/dt| > |-P/F|$, then $dS/dt < 0$ and the entropy decreases with time. This violates the second law of thermodynamics.
2. If $|dx/dt| < |-P/F|$, then $dS/dt > 0$ and the entropy increases with time. This is fine.
3. If $|dx/dt| = |-P/F|$, then $dS/dt = 0$ and the entropy remains constant. This is also fine.

So, if the second law of thermodynamics is to be believed, it follows that P/F is the fastest speed possible for a given system. Hence,

$$\frac{P}{F} = c \tag{3.4}$$

where c is a characteristic speed of the system dependant on P, the characteristic power and F the characteristic force. A speed faster than c would imply a reversal of the second law. This result agrees with special relativity where a speed higher than c describes a hypothetical tachyon travelling backward in time.

□

If the system is the universe, then taking P to be the characteristic Planck power, and F to be the characteristic Planck force, we do in fact recover the speed of light.

$$P\left(\frac{1}{F}\right) = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{3.5}$$

3.2 *Light-cone*

We look at the thermodynamic cycle of the system transiting through time and space starting at P_x to P_0 to P_t to P_x as illustrated on Figure 1. During the transitions and to keep the energy constant, trade-offs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles such as those involving pressure and volume, etc. The cycle presented here is reminiscent of relativistic light cones.

We work in the quasi static approximation

$$\Delta E = T\Delta S - P\Delta t - F\Delta x \tag{3.6}$$

and we pose that $\Delta E = 0$ throughout the cycle

$$T\Delta S = P\Delta t + F\Delta x \tag{3.7}$$

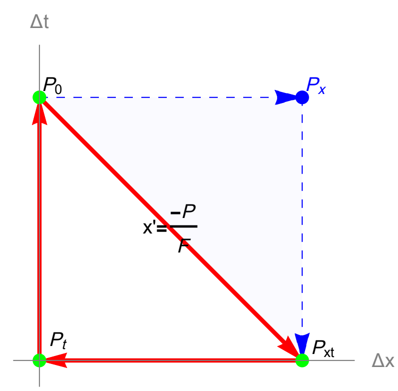

Figure 1: A thermodynamic cycle through space, time and entropy as observables.

P_x to P_0: As we translate P_x closer in space to P_0 while keeping the time fixed, the entropy must decrease to compensate. This situation occurs when $\Delta x < 0$ and when $\Delta t = 0$.

$$(T\Delta S = P\Delta t + F\Delta x|_{\Delta t=0} \tag{3.8}$$

$$\implies \Delta S = \frac{F}{T}\Delta x \tag{3.9}$$

From the equation above, we note that ΔS is negative when $\Delta x < 0$. Since entropy tends to increase, we conclude that objects have a tendency to resist being returned to the origin and are instead encouraged to expand away from each other.

P_0 to P_t: As we translate P_0 backward in time to P_t while keeping the distance fixed, the entropy must decrease to compensate. This situation occurs when $\Delta t < 0$ and when $\Delta x = 0$.

$$(T\Delta S = P\Delta t + F\Delta x|_{\Delta x=0} \tag{3.10}$$

$$\implies \Delta S = \frac{P}{T}\Delta t \tag{3.11}$$

From the equation above, we note that ΔS is negative when $\Delta t < 0$. We conclude that an object is encouraged by entropic considerations to evolve forward in time and is discouraged from evolving backward in time.

P_x to P_t: As we translate P_{xt} forward in time and backward in space to P_0 keeping the entropic constant ($\Delta S = 0$), we have movement at the speed c.

$$(T\Delta S = P\Delta t + F\Delta x|_{\Delta S=0} \tag{3.12}$$

$$\implies \left|\frac{\Delta x}{\Delta t}\right| = \frac{P}{F} = c \tag{3.13}$$

From the equation above, an object travelling at speed c is neither encouraged nor discouraged by entropic considerations.

From the results of this section, we conclude that objects traveling forward in time, and objects travelling further apart are two sides of the same entropic coin. The natural direction for the evolution of time and space are justified by the same entropic argument.

Hence, this derivation predicts an expanding universe where time moves in the forward direction. Reversing this natural direction would require reversing the second law of thermodynamics in both cases.

4 Conclusion

From the AIT perspective, these results suggest that a UTM would run programs in dovetail. In this scenario, shorter programs have benefited from longer execution time such that their lead running time is related to longer program-size via P/F. This dynamic is governed as per special relativity. Programs exceeding P/F would violate dovetailing standards.

These results would seem to indicate a possible link between universal Turing machines and physics. Do universal Turing machines necessarily imply the laws of physics? This link is further investigate in my previous paper[4]. In it and using similar arguments as in this paper, I show that the following laws can be derived from AIT; a) Schrödinger's equation (QM), b) general relativity (GR), c) the quantum mechanical spin (QM), d) law of inertia, e) the holographic principle, f) the Bekenstein-Hawking entropy, g) Hawking radiation and the h) Schwarzschild radius. As QM and QR are both derived from the equation, it is suggestive that they are unified. I investigate this unification and show how QM and QR combine to resolve the singularities in black holes.

[4] Alexandre Harvey-Tremblay. An axiomless derivation of the theory of everything. https://www.academia.edu/33079029/An_axiomless_derivation_of_the_Theory_of_Everything, 2017

References

John C. Baez and Mike Stay. Algorithmic thermodynamics. arXiv:1010.2067 [math-ph], 2010.

Gregory Chaitin. Mathematics, complexity and philosophy. https://www.academia.edu/31320410/Mathematics_Complexity_and_Philosophy_full_bilingual_text_, 2010.

Gregory J. Chaitin. An algebraic equation for the halting probability. https://www.cs.auckland.ac.nz/~chaitin/berlin.pdf, 1988.

Gregory J. Chaitin. How real are real numbers? http://www.worldscientific.com/doi/abs/10.1142/S0218127406015726 and https://www.cs.auckland.ac.nz/~chaitin/olympia.pdf, 2006.

Alexandre Harvey-Tremblay. An axiomless derivation of the theory of everything. https://www.academia.edu/33079029/An_axiomless_derivation_of_the_Theory_of_Everything, 2017.

Ming Li and Paul Vitányi. An introduction to kolmogorov complexity and its applications. Springer, 1997.

K. Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. http://arxiv.org/abs/nlin/0212001, 2002.

K. Tadaki. A statistical mechanical interpretation of algorithmic information theory. https://arxiv.org/pdf/0801.4194.pdf, 2008.