

QUANTUM FUSION IN NUCLEAR ENGINEERING

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Abstract

This article provides new insights in the interaction of light and matter and describes physical processes and technologies that enable the creation of quantum fusion in materialization of the **Quantum Fusion (QF) Hypothesis** which is applicable in the fields of molecular, quantum and computational mechanics,, material science, quantum physics and astrophysics, and electronics.

Theory

We present here our quantum propagation and entanglement model for **quantum fusion generator**. This concept is based on **1935 Einstein-Podolsky-Rosen Argument in Quantum Theory**. Einstein maintains there (Bacciagaluppi and Valentini 2009, p. 488): “the interpretation, according to which $|\psi|^2$ expresses the probability that *this* particle is found at a given point, assumes an entirely peculiar mechanism of action at a distance, which prevents the wave continuously distributed in space from producing an action in two places on the screen. Einstein continues: “in my opinion, one can remove this objection only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time **one localizes the particle during propagation.**”

In this article we offer to enhance the virtual model of a bosonic superconducting cosmic string (**fig. 1, p. 2**), and model it as our **quantum harmonic system (fig. 2, p. 2)** that enables the creation of quantum fusion generator for quantum energy, quantum nanocomputers and TVs, engineless quantum transmission and propelling devices for cars and aircrafts, superfluid propulsion for rockets and spacecrafts, levitation and teleportation based on three fundamental laws of physical-chemical kinetics: (1) **the law of entire equilibrium**, (2) **the law of the duality of elementary processes** (or the equality of direct and reverse transition probabilities), and (3) **the law of equal *a priori* probabilities**. It is shown that said three laws follow from the law of the symmetry of time, and furthermore, that the first and third of these laws are both derivable from the second.

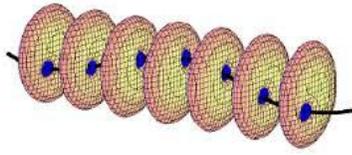
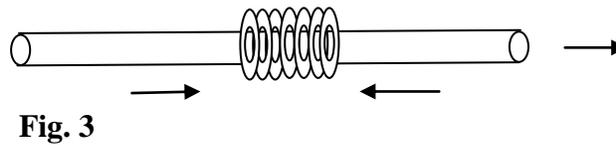
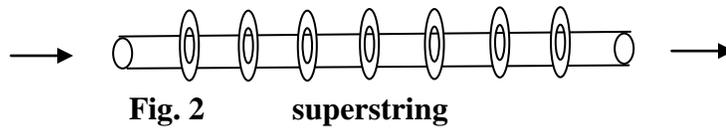


Fig 1. **Cosmic bosonic string**

Elaboration (confirmed by references)

Accordingly, and contrary to the common bosonic string model in **fig 1**, we model the ultracold hollow cylindrical superstring (**fig 2**) as a space-time piercing quantum tube with Casimir effect in interacting Bose-Einstein condensate inside a cylindrical tube (ref. 2 on p. 6) in overlapping counter-rotating magnetic fields (**fig. 3 here, and fig. 4 on p. 9**) and in **quantum fusion** in quantum entanglement and tunneling (**ill. 1**).. (Compare with the space-time piercing characteristics of neutrinos and their left-right counter-spinning ability).

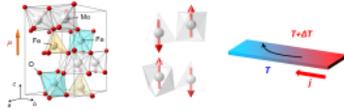


Ill. 1

Our tunneling superstring system in **fig. 2** consists of open left entry to trap fermionic atoms in the vacuum vortex core. There, quantum Hall* effect (QHE) is realized in a 2d electron gas subjected to a strong perpendicular magnetic field (**fig. 3**) under the influence of the nuclear spin fields that are then harmonized in vertex by shifting counter-rotating magnetic fields in **dynamical Casimir effect** (ref. 2) to unify them in a superimposed magnetic field in quantum squeezejunction (fig 3 above). The unified matter is then superconducted via superstring's open right exit in mass propagation due to induced Casimir and Zeeman effects and Feshbach resonance, making helium and hydrogen to interact in nuclear fusion: **hydrogen** nuclei into **helium, whereby the** matter of the fusing nuclei is converted to **heavy or dark photon** (high energy).

"Giant thermal Hall effect in multiferroics."Nature Materials.

. T. Ideue, T. Kurumaji, S. Ishiwata, and Y. Tokura . University of Tokyo.



The system in fig 3 functions similar to musical squeezebox harmonika or accordion (**ill. 2**) which expands and contracts its bellows by using trapped air to create pressure and vacuum and produce musical sounds.



Ill. 2 Accordion

Similar to accordion functions, our **quantum harmonic system** in fig. 3 shifts external magnetic fields back and forth over ultracold Majorana fermions trapped and compressed in the rotating tube of the superconducting superstring. In the lab such system can be modeled as a carbon tube with Bose-Einstein condensate with graphene membrane integrated within the counter-rotating **ferror- or nanomagnets*** sliding back and forth over the tube and its trapped ultracold particles similar to Casimir plates. Note that graphene membrane is impermeable to standard gases, including helium.

To make this system work as a modular cold fusion reactor, we would direct the particles beam from our **quantum harmonic generator** into the chamber with liquid helium and neon to interact there with solar neutrinos.

*Greek scientists from the University of Crete and the Foundation for Research and Technology-Hellas (FORTH), in collaboration with U.S. scientists from Ames Research Center of NASA, have discovered a new way to create small magnets by using short laser light pulses.

The discovery of the phenomenon called **Quantum Femto-Magnetism** was made by the University of Crete physics professor Ilias Perakis and his group in Greece, in co-operation with Ames Laboratory and Iowa State University physicist Jigang Wang and his team in the USA. 2016 Nobel Prize winner Duncan *Haldane* discovered how topological concepts can be used to understand the properties of **chains of small magnets**.

Our quantum model in **fig. 3** above represents the classical and quantum motion of photons, etc. in a rotating string. The spin motion per Bargmann-Michel-Telegdi equation is considered in the rotation tube and rotating system in acceleration of charged particles. In fact, neutral particles photons, neutrons, etc. can be accelerated by rotating tube. The specific characteristics of the mechanical systems in the rotating framework follow from the differential equations describing the massive body in the noninertial systems. (Landau, 1965). Let the Lagrange function of a point particle in the inertial system be as follows:

$$L_0 = \frac{mv^2}{2} - U \tag{1}$$

with the following equation of motion

$$m \frac{Dv_0}{dt} = - \frac{\partial U}{\partial r'} \tag{2}$$

where the quantities with index 0 correspond to the inertial system. The Lagrange equations in the noninertial system is of the same form as that in the inertial one, or,

$$\frac{d}{dt} \frac{\partial L}{\partial v} = \frac{\partial L}{\partial r} \tag{3}$$

However, the Lagrange function in the noninertial system is not the same as in eq. (1) because it is transformed. Specific extraordinary properties of our quantum vacuum tube in fig. 3 is that it simultaneously revolves, and rotates around its axis **due to the forces acting on the electron in the Hydrogen atom and the centrifugal force (which appears to be the result of conservation of angular momentum)**, creating thereby atomic vortex and superfluidity of trapped supercold gaseous helium, which in quantum Hall effect becomes superfluid in percolation of its housing tube and acts thereby as a lubricant and coolant for external magnets sliding over our quantum tube in fig. 3. Note that bosonic quasi-particles, known as exciton-polaritons, can be created in Bose-Einstein condensate (BEC) through strong coupling between bound electron-hole pairs and the photon field. Recently, a non-equilibrium BEC and superfluidity have been demonstrated in such structures.

Our quantum tube in fig. 3 is encapsulated by hydrogen solution in Feshbach resonance, creating thereby a dual quantum model in coherent entanglement (ref. 1) i.e., subquanta within quanta. Such a quantum-subquanta introvert-extrovert duo is the building block of universal quantum web predicted by Einstein, so our quantum model on macroscale explains the phenomenon of wave particle duality in perpetuum mobile of the in-and-out flows of matter and energy of a black hole with curved horizon due to rotation and inflated gases in corona of matter where energy circulates in a **Möbius band** (superconductive and polarized under magnetic field) in a partially visible and mostly invisible spectrum (**dark matter**). Compare with charged particles that have been caught in the magnetic field of earth and that can move on a Möbius band.

Hence, our model and physical system in fig. 3 materializes the quantum vacuum and quantum space theories where superfluid vacuum is constructed from quanta. The assumption that the vacuum is a superfluid (or a BEC), enables us to derive Schrödinger's non-linear wave equation, also known as the Gross-Pitaevskii equation, from first principles. Furthermore, by treating the vacuum as an acoustic metric, it becomes the analogue for general relativity's curved spacetime within regimes of low momenta.

This kaleidoscopic matter explains the mystery of mass generation, the question of how the Higgs boson gets its mass, because it manifests the mass generation similar to gap generation mechanism in superconductors or superfluids. In other words, mass becomes a consequence of symmetry, vectoring quantum **vortices** formed in vacuum condensate.

Because our ultracold superstring in fig. 2. p.2 is nonrelativistic, it is not constrained to the multidimensional space-time in which superstrings are usually studied in high-energy physics. So, our string is the actual **harmonic condensed matter system**, where superconductivity in **macroscopic quantum phenomena** can be studied experimentally, and quantum energy teleported.

It means that in our above shown quantum model, physical/molecular data of the object can be photonically compressed, tunneled via our quantum tube and then amplified/reassembled at a given destination. See **ref. 2**.

Ref. 1:

Streltsov, A. et al. Measuring quantum coherence with entanglement. *Phys. Rev. Lett.* 115, 020403 (2015):

A team of researchers from India, Spain and the UK has mathematically proved that it is possible to convert an amount of ‘quantum coherence’ in a system into an equal amount of ‘quantum entanglement’. The team, which included Alexander Streltsov from ICFO-The Institute of Photonic Sciences, Barcelona, Spain, and Gerardo Adesso from the University of Nottingham, provided a mathematically rigorous approach to resolve this question using a common frame to quantify quantumness in terms of coherence and entanglement. They show that any non-zero amount of coherence in a system could be converted to entanglement via incoherent operations.

Ref. 2

Shyamal Biswas, Saugata Bhattacharyya.

Amit Agarwal

03/2015

ABSTRACT

“We explore **Casimir effect** on an interacting Bose-Einstein condensate (BEC) inside a cylindrical tube. The Casimir force for a confined BEC comprises of a mean field part arising from the inhomogeneity of the condensate order parameter, and a quantum fluctuation part which results from the phononic Bogoliubov excitations of the BEC. Considering Dirichlet boundary conditions for the condensate wave function as well as for the Bogoliubov excitations we explicitly calculate the Casimir force and scaling function. For low densities of the condensate, the mean field part dominates over the quantum fluctuation part, while for high densities, as the BEC order parameter becomes homogenous, the quantum fluctuations start playing a more dominant role.”

Ref. 3

Observation of the dynamical Casimir effect in a superconducting circuit:

C. M. Wilson,

G. Johansson,

A. Pourkabirian,

M. Simoen,

J. R. Johansson,

T. Duty, F. Nori & P. Delsing

Nature **479**, 376–379, (17 November 2011) doi:10.1038/nature10561

We accordingly apply Einstein's concept in quantum plasma (see image below) in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space of all possible values of position and momentum variables where every degree of freedom or parameter of the system is represented as an axis of a multidimensional space. The concept of phase space was developed in the late 19th century by Ludwig Boltzmann, Henri Poincaré, and Willard Gibbs.

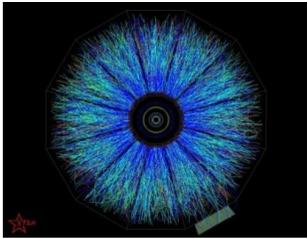
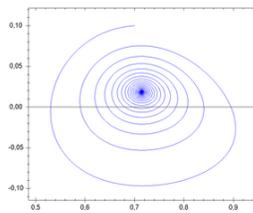


Image of quantum plasma

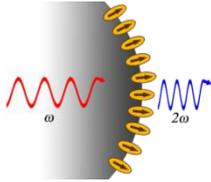


Compare with phase space of a dynamic system with focal instability, showing one phase space trajectory

This allows us to get generic boundary conditions for the quantum oscillator on N dimensional complex projective space (CPS) and on its non-compact version i.e., Lobachevsky space (LN) in presence of constant magnetic field. As a result, we get a family of energy spectrums for the oscillator. Motion of a classical particle in 3-dimensional Lobachevsky and Riemann spaces is studied in the presence of an external magnetic field which is analogous to a constant uniform magnetic field in Euclidean space. In both cases three integrals of motions are constructed and equations of motion are solved exactly in the special cylindrical coordinates on the base of the method of separation of variables. In Lobachevsky space there exist trajectories of two types, **finite and infinite** in radial variable, while in Riemann space all motions are finite and periodical. The invariance of the uniform magnetic field in tensor description and gauge invariance of corresponding 4-potential description is demonstrated explicitly. The role of the symmetry is clarified in classification of all possible solutions, based on the geometric symmetry group, $SO(3,1)$ and $SO(4)$ respectively.

Elaboration

Our **quantum fusion system** will be based on a **quantum carrier (quantum ball or sphere)** and **jump-resonance phenomena** of nonlinear feedback control system. Second harmonic generation (see image below) with resonant enhancement is applicable.



This cartoon depicts ordered molecules at a small spherical surface. An ultrafast pump laser pumps light with frequency ω which generates light at 2ω from the locally non-centrosymmetric media.

The nonlinearities are those whose outputs are single-valued odd functions of the inputs and are independent of frequencies of the photonic inputs. The general conditions under which **jump-resonance** occurs will be given and the system with saturation nonlinearity will be analyzed. The essential objective is to define **the contours on the complex plane** for the constant values of system variables, e.g., input amplitude, amplitude ratio, and phase shift.

Common **Frequency Hopping Spread Spectrum (FHSS)** will be upgraded by us in our quantum plasma system to randomly propagate atomic particles by photonically switching from one signal carrier (quantum tube) to other quantum channels in thereby achieved dynamic equilibrium beyond chaotic interference. Fermi-Dirac distribution function and electrovacuum solutions of the Einstein-Maxwell field equations are applicable. Same concept can be applied in our remote **quantum loops fusion system** (see the diagram in **fig. 1** and **fig. 2**).

We accordingly introduce here the notions of a **spinning quantum spring** (fig. 2) to constitute a **spinning quantum ball or sphere** created by a multimodal quantum structure, fig. 1:

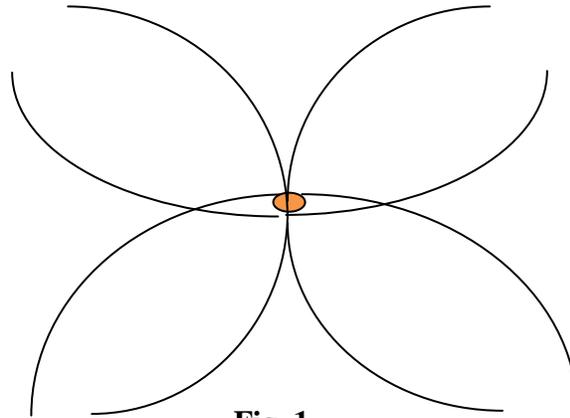


Fig. 1.

This diagram shows the central **quantum nuclei** with quantum loops of **spinning quantum spring** that defines the horizon of the rotating plain constituting external quantum carrier – **quantum ball** -- a **3D plasma** (fig. 2 below, fig 3 on p. 11 and fig. 4 on p. 12) in self-generated and contained e.m. field, as in a ball lighting (ref. 3 and 4), to be activated by a tunable pulse laser via spectral prism and acoustic membrane. (See **Rutherford's model** on p. 28 below and **G. Gamow: Quantum** (1928))



Fig. 2. Our **3D quantum space display concept for a quantum network in quantum fusion** in total angular momentum **J**, which combines both the spin and orbital angular momentum of a particle or system: **$J = L + S$** .

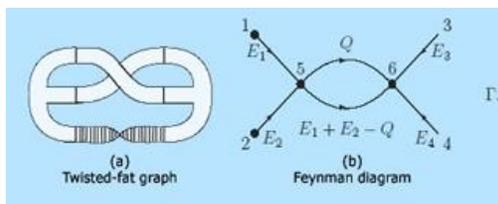
In re: Rudolf Peierls noted that electrons in a magnetic field can be regarded as moving in a quantum spacetime. See also *Snyder, H. (1947), "Quantized space-time", Phys. Rev. D, 67: 38–41, doi:10.1103/PhysRev.71.38*

"Entangled particles behave as one, independent of distance. Any observation of such entangled electrons result in correlated information," Professor Ronald Hanson in Delft explains. Measuring one particle therefore instantaneously influences the other, even when they are light-years apart. Physicists at Australian National University have engineered a spiral laser beam and used it to create a whirlpool of hybrid light-matter particles called **polaritons**.

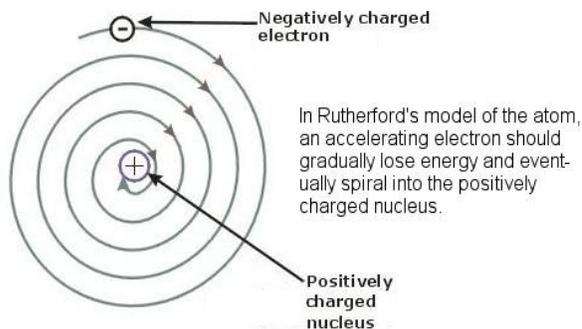
"Creating circulating currents of polaritons – vortices – and controlling them has been a long-standing challenge," said leader of the team, theoretician Dr Elena Ostrovskaya, from the Research School of Physics and Engineering, Australian National University "We can now create a circulating flow of these hybrid particles and sustain it for hours.

It appears that our **quantum ball particles' carrier** is a continuum of possible energies in quantum fusion. When the carrier is confined to a 3D space, the quantum energy levels begin to spread out and the quantum nature becomes detectable, i.e., electrons will settle in the quantum ball and not in the adjacent layers. This carrier will then exhibit quantum effects imposed on it, where the number of particles trapped in the carrier can be controlled by an external voltage.

Compare our multimodal quantum loops 3D diagram in **fig. 1 above, p. 27** with Feynman diagram of quantum field geometry and Rutherford's model of the atom below:



Feynman diagram of quantum field geometry



To visualize the spinning nature of our multimodal quantum system in fig 1 above, we show in **fig. 3** below a **central nuclei loop** of quantum spring with spinning “blades” that create the whirlpool of charged particles in **Coulomb force** as quantum ball’s overall structure.

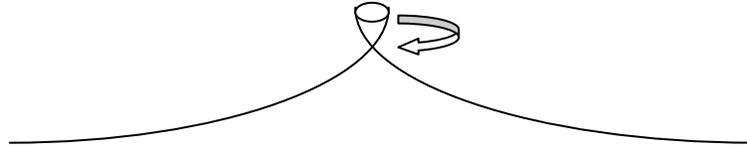
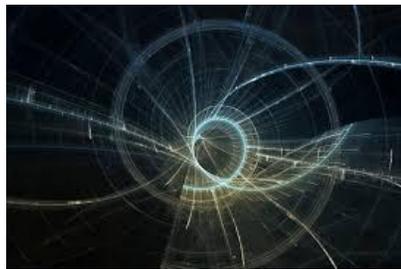


Fig. 3

Our **model of a spinning quantum spring** in fig 3 is the **core of a quantum ball** (or **plasmoid** as below) in **collective quantum plasma effects** when the quantum nature of its particles significantly affects its macroscopic properties in quantum-controlled fusion in **femtochemistry**, in which femtosecond-long laser flashes trigger chemical reactions. There, nuclei can be pushed close enough to overcome the **Coulomb barrier** that forces atoms of similar charge to repel each other, fuse and release heat through neutron scattering. When more energy is created than it takes to sustain the reaction, **sustained fusion** becomes viable.

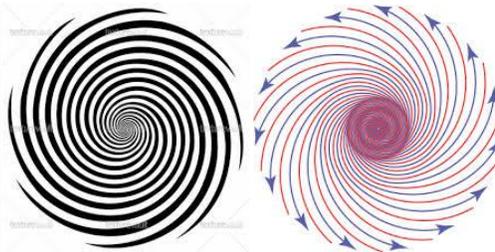
Our quantum device as a **quantum fusion generator** is a spherically symmetric quantum plasmoid based on radial plasma oscillations. Such plasmoid can exist in dense plasma of electrons, ions, and neutral particles in virtual acoustic wave, which is excited in the neutral component of plasma. This interaction can be attractive and form ion pairs.



(Compare our ball with the ball lighting images in ref. 1, 2 below, and fig.4, p. 12)



Ref. 1. Ball lightning



Ref. 2. Spiral propagation of light

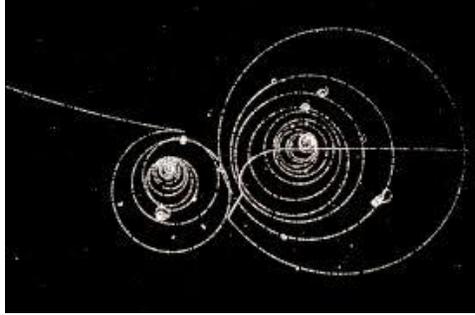


Fig. 4. It shows a remote spinning quantum spiral in 3D space display per classical field theory with space-time manifold M (fig. 5 below) and field space F , where $\phi : F \rightarrow M$, and action critical points are $S[\phi], dS = 0$.



Fig. 5. Space-time manifold M as toroidal ring model, known originally as **plasmoid ring** with electrons and protons and was first proposed by Alfred Lauck Parson in 1915.

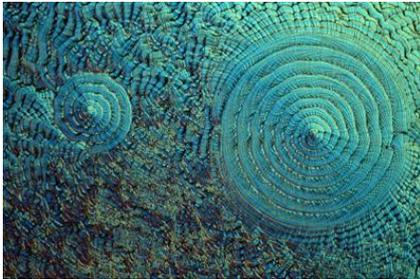
Theory

Instead of a single orbiting charge, the **toroidal ring** was conceived as a collection of infinitesimal charge elements, which orbited or circulated along a common continuous path or "loop" as in our model in **fig 1** and **3.** In general, this path of charge could assume any shape, but tended toward a circular form due to internal repulsive electromagnetic forces. In this configuration the charge elements circulated, but the ring as a whole did not radiate due to changes in electric or magnetic fields since it remained stationary.

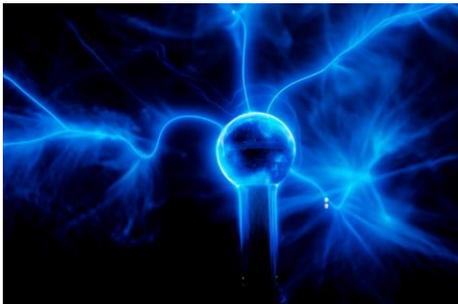
The ring produced an overall magnetic field ("spin") due to the current of the moving charge elements. These elements circulated around the ring at the speed of light c , but at frequency $\nu = c/2\pi R$, which depended inversely on the radius R . The ring's inertial energy increased when compressed, like a spring, and was also inversely proportional to its radius, and therefore proportional to its frequency ν . The theory claimed that the proportionality constant was Planck's constant h , the conserved angular momentum of the ring.

According to the model, electrons or protons could be viewed as bundles of "fibers" or "**plasmoids**" with total charge $\pm e$. The electrostatic repulsion force between charge elements of the same sign was balanced by the magnetic attraction force between the parallel currents in the fibers of a bundle, per Ampère's law. These fibers twisted around the torus of the ring as they progressed around its radius, forming a Slinky-like helix. Circuit completion demanded that each helical plasmoid fiber twisted around the ring an integer number of times as it proceeded around the ring. This requirement was thought to account for "quantum" values of angular momentum and radiation.

The toroidal or "helicon" model did not demand a constant radius or inertial energy for a particle. In general its shape, size, and motion adjusted according to the external electromagnetic fields from its environment. These adjustments or reactions to external field changes constituted the emission or absorption of radiation for the particle. The model, then, claimed to explain how particles linked together to form atoms.



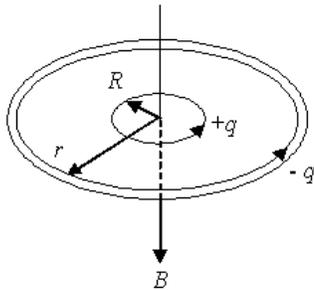
Ref. 3. Compare this ball lightning structure with fig 2, p. 9, fig 4, p. 12, and ref. 4 below.



Ref. 4 Electron-ionic model of ball lightning (from Wikiversity)

The **electron-ionic model of ball lightning** was presented by Sergey G. Fedosi , a physicist and the philosopher from Perm, Russia, and Sergey A. Kim, from Perm state university, in a number of works. Fedosin S.G., Kim A.S. The Physical Theory of Ball Lightning. Applied Physics (Russian Journal) , No. 1, 2001, P. 69 – 87.

In their model, ball lightning is a cluster of the very hot ionized air with the positive charge in general, whose shell consists of the rapidly revolving electrons with the total current up to $1,4 \cdot 10^5$ A. Ball lightning as whole is supported by the balance of the electromagnetic forces, which act between the charges. Positive ions inside the lightning are distributed freely as a result of the spherical symmetry, and attract to themselves the electrons of shell, retaining them from the dispersion. According to the model the ball lightning is formed from two close branches of a linear lightning at the time of termination of current in the main channel with the subsequent closure of branches in a current ring.



Equatorial cross-section model of ball lightning as a distinct ring on the current sheet spheroidal shape. R - radius of rotation of ions in the equilibrium shell around the magnetic field with induction B , r - radius of the outer electron shell.

Electronic currents in the shell create strong magnetic field inside the lightning. These currents are perpendicular to rotational axis, the diameter of rotation decreases to the poles, where magnetic field grows. This retains positive ions from the dispersion along the rotational axis due to the effect of magnetic bottle. Basic magnetic field inside the lightning is directed along the rotational axis. I.e., ions can move along the axis along the lines of magnetic field. From other side, the ions revolve in the circle perpendicularly to axis under the action of Lorentz force with respect to their thermal velocity. As a result at a certain distance from the axis of lightning appears the intersection of two ion flows, which is observed as the luminous shells inside the lightning. Emission from the shells appears from friction and recombination of the being intersected ion flows.

Theory predicts from the first principles the maximum diameter of ball lightning 34 cm. With the larger size the summary charge of lightning, which has positive sign, grows to the value of 10^{-5} C and appears the electrical breakdown of air near the lightning. The energy of the lightning in this case reaches 10.6 kJ, the current in the shell $1,4 \cdot 10^5$ A, the internal magnetic field of 0.5 Tesla.

Because of its charge ball lightning does not simply float under the action of the force of Archimedes, but it is retained by electric force from clouds and the induced charge on the Earth. The formula for the maximum radius of ball lightning has the form:

$$r = \frac{mc^2}{qE_0},$$

Ref. 5:

Derivation of the Fermi-Dirac distribution function

We start from a series of possible energies, labeled E_i . At each energy we can have g_i possible states and the number of states that are occupied equals $g_i f_i$, where f_i is the probability of occupying a state at energy E_i .

The number of possible ways - called configurations - to fit $g_i f_i$ electrons in g_i states, given the restriction that only one electron can occupy each state, equals:

$$W_i = \frac{g_i!}{(g_i - g_i f_i)! g_i f_i!}$$

This equation is obtained by numbering the individual states and exchanging the states rather than the electrons. This yields a total number of $g_i!$ possible configurations. However since the empty states are all identical, we need to divide by the number of permutations between the empty states, as all permutations cannot be distinguished and can therefore only be counted once. In addition, all the filled states are indistinguishable from each other, so we need to divide also by all permutations between the filled states, namely $g_i f_i!$.

The number of possible ways to fit the electrons in the number of available states is called the multiplicity function.

The multiplicity function for the whole system is the product of the multiplicity functions for each energy E_i

$$W = \prod_i W_i = \prod_i \frac{g_i!}{(g_i - g_i f_i)! g_i f_i!}$$

Using Stirling's approximation, one can eliminate the factorial signs, yielding:

$$\ln W = \sum_i \ln W_i = \sum_i [g_i \ln g_i - g_i(1 - f_i) \ln(g_i - g_i f_i) - g_i f_i \ln g_i f_i]$$

The total number of electrons in the system equals N and the total energy of those N electrons equals E . These system parameters are related to the number of states at each energy, g_i , and the probability of occupancy of each state, f_i , by:

$$N = \sum_i g_i f_i$$

and

$$U = \sum_i E_i g_i f_i$$

According to the basic assumption of statistical thermodynamics, all possible configurations are equally probable. The multiplicity function provides the number of configurations for a specific set of occupancy probabilities, f_i . The multiplicity function sharply peaks at the thermal equilibrium distribution. The occupancy probability in thermal equilibrium is therefore obtained by finding the maximum of the multiplicity function, W , while keeping the total energy and the number of electrons constant.

For convenience, we maximize the logarithm of the multiplicity function instead of the multiplicity function itself. According to the Lagrange method of undetermined multipliers, we must maximize the following function:

$$\ln W - a \sum_j g_j f_j - b \sum_j E_j g_j f_j$$

where a and b need to be determined. The maximum multiplicity function is obtained from:

$$\frac{\partial}{\partial (g_i f_i)} \left[\ln W - a \sum_j g_j f_j - b \sum_j E_j g_j f_j \right] = 0$$

which can be solved, yielding:

$$\ln \frac{g_i - g_i f_i}{g_i f_i} - a - b E_i = 0$$

or

$$f_i = f_{FD}(E_i) = \frac{1}{1 + \exp(a + b E_i)}$$

which can be written in the following form

$$f_{FD}(E_i) = \frac{1}{1 + \exp\left(\frac{E_i - E_F}{\beta}\right)}$$

with $\beta = 1/b$ and $E_F = -a/b$. The symbol E_F was chosen since this constant has units of energy and will be the constant associated with this probability distribution.

Taking the derivative of the total energy, one obtains:

$$dU = \sum_i E_i d(g_i f_i) + \sum_i g_i f_i dE_i$$

Using the Lagrange equation, this can be rewritten as:

$$dU = \beta d(\ln W) + \sum_i g_i f_i dE_i + E_F dN$$

Any variation of the energies, E_i , can only be caused by a change in volume, so that the middle term can be linked to a volume variation dV .

$$dU = \beta d(\ln W) + \left[\sum_i g_i f_i \frac{dE_i}{dV} \right] dV + E_F dN$$

Comparing this to the thermodynamic identity:

$$dU = TdS - pdV + \mu dN$$

The energy, E_F , equals the energy associated with the particles, \square .

The comparison also identifies the entropy, S , as being the logarithm of the multiplicity function, W , multiplied with Boltzmann's constant. The Fermi-Dirac distribution function then becomes:

$$f_{FD}(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

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