

# Spatial coherence restriction in quantum non-mereological compounds

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July 25, 2017

## Abstract

Employing the symmetrization postulate for two quantum particles we prove a non-linear differential relation that imposes certain constraints on the admissible values of wavefunction solutions and rules out spatial superposition states. Combining this with the two-particle Schrödinger equation in the position representation, a non-linear partial differential equation is deduced which is time-asymmetric and remains invariant under the permutation of particle indices. Extension of the analysis to several particle composites is also sketched and the mereological implications following from this formalism are explored.

## 1 Introduction

One of the most distinct features of quantum mechanics, relies on the fact that the wavefunction inhabits the abstract configuration space in a manifold of  $3N$  dimensions, where  $N$  being the number of particles involved, and it is most clearly seen in the case of two or more interacting particles [1]. In one of the early papers on wave mechanics, Darwin [2] examined an interesting set up involving a coherent pair of particles as he named it that illustrates quite convincingly that the wave function propagates in the configuration and not the physical space. To the contrary, if we associate each particle with a wave in the three dimensional space we are led to inconsistencies and fail to adequately explain collision effects (for a rather different interpretation see the discussion [3]). Interparticle correlations between two bodies became notable with the work of Einstein, Podolsky and Rosen[4, 5] and since then have been at the heart of quantum mechanics. The issue of non-separability between interacting particles had been particularly emphasized by Einstein in his own version of the EPR thought experiment [6] and Schrödinger claimed quite succinctly that according to quantum theory maximum knowledge of a composite system does not necessarily mean fullest knowledge of the individual particles [7, 8]. Almost three decades later, especially through Bell's groundbreaking work [9, 10]

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joint probability functions were started being intensively exploited in the field of quantum mechanical foundations and the celebrated Bell inequalities led to similar investigations by various researchers and even experimental verification [11]. The relational character of quantum theory and correlations between subsystems have also been discussed thoroughly [12, 13, 14]. Mermin in particular emphasized the physical reality of these subsystem correlations in contradistinction to the *correlata* underlying them. On the experimental side, two-particle interference effects have been observed in a number of occasions [15]. Lastly, the coincidence probabilities have become an indispensable tool in the study of coherence and tests of quantum optical physics, especially two-photon interferometry have considerably enriched our understanding of both interference and correlations [16, 17].

It should be nevertheless noted that the mereological [18, 19] aspects of the theory, in other words the investigation of the relation between parts and whole or how parts constitute the whole in the quantal domain, have not attracted particular attention. What has prevailed in the literature is the examination of measurability consistency conditions, as it was indicated in an early paper by Bohr [20] and most clearly established in the profound analysis given by Bohr and Rosenfeld for the problem of field operators measurability [21]. Mereological considerations then have been rarely discussed (see though [22] and references therein).

In this Letter we will follow a different route and try to establish a connection between mereology and the quantal formalism. By constructing dynamical equations that take into account both the several body time dependent Schrödinger equation and its complex conjugate along with a permutation differential equation to be proved in the next section that relies on the symmetrization principle, we will argue that a basic principle of minimal mereology does not hold and non-mereological composition must be considered as an option in the quantum domain. It is also to be noted that the equation of motion that will be synthesized is time-irreversible. Following this we extend our results for the multiparticle case and finally we recapitulate our findings in the final section.

## 2 Dynamical equations for the biparticle quantum compound

We consider initially two neutral, spinless interacting particles with the same mass  $m$ , assuming that the time varying coupling binary potential  $V(\mathbf{r}_1, \mathbf{r}_2, t)$  depends on their relative distance  $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ . The fundamental quantum mechanical equation of motion for the two-particle probability amplitude reads as

$$\left(-\frac{\hbar^2}{2m}(\nabla_1^2 + \nabla_2^2)\right)\Psi_{12} = \left(i\hbar\frac{\partial}{\partial t} - V(\mathbf{r}_1, \mathbf{r}_2, t)\right)\Psi_{12}. \quad (1)$$

Our task will be to derive an equation for the two-body probability amplitude

$\Psi_{12} = \Psi_{12}(\mathbf{r}_1, \mathbf{r}_2, t)$  and the coincidence probability density  $|\Psi_{12}|^2$ , that furthermore incorporates the permutation relation

$$\nabla_1^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} = \nabla_2^2 \Psi_{12}^* \nabla_1^2 \Psi_{12}. \quad (2)$$

This may be proved defining  $J_{12}$  as

$$J_{12} = \nabla_1^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} - \nabla_2^2 \Psi_{12}^* \nabla_1^2 \Psi_{12}. \quad (3)$$

By interchanging the particle indices and due to the symmetrization postulate [1] it readily follows that

$$J_{21} = \nabla_2^2 \Psi_{21}^* \nabla_1^2 \Psi_{21} - \nabla_1^2 \Psi_{21}^* \nabla_2^2 \Psi_{21} = J_{12} \quad (4)$$

and

$$J_{21} = \nabla_2^2 \Psi_{12}^* \nabla_1^2 \Psi_{12} - \nabla_1^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} = -J_{12}. \quad (5)$$

From the last two equations we obtain (2) and this concludes our proof for two particles with equal masses. We notice that this relation, which is independent of Planck's constant, remains invariant under the permutation of 1 and 2 indices, it is identical to its complex conjugate and contains non-linear terms so it violates the superposition principle and may also be seen as a constraint for the admitted solutions that obey Schrödinger's equation (1). Effectively then, (2) acts as a superselection rule that suppresses spatial superposition of states but it should be emphasized that in the present case the restriction of coherence is not a result imposed by a measuring device or the environment [23] but it is dictated by the symmetrization postulate. Obviously, (2) holds when  $\Psi$  is real, but they may be less trivial solutions of (2). Quadratic terms of the form  $\nabla \Psi^* \nabla \Psi$  are well known from the Lagrangian description of the Schrödinger field, in Eq. (2) though, the Laplacian operators are not identical and one of them acts on the quantum state and the other on its complex conjugate. It is possible using the same methodology to prove similar relations for biharmonic operators or other higher derivative terms instead of the Laplacians, but in this work we will only consider second order spatial derivatives as they are naturally related to the kinetic energy operators so they have a more transparent physical interpretation.

Since we have assumed that the two particles considered have spin zero this means that their wavefunctions is symmetric. It is evident though that an equation like (2) holds also for antisymmetric wavefunctions which describe fermions. Nevertheless, a coherent superposition of symmetric and antisymmetric wavefunctions would violate (2) since the cross terms that appear change sign when we interchange indices. We may also consider states distinguishable by their different joint probability densities  $|\Psi_{12}|^2 \neq |\Phi_{12}|^2$  which satisfy though

$$\nabla_1^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} = \nabla_1^2 \Phi_{12}^* \nabla_2^2 \Phi_{12}. \quad (6)$$

Eq. (6) contains two equal real terms, potentially verifiable by experiment. It is a different question how exactly to measure this value and we will not address it here.

The standard procedure when we treat a quantum mechanical problem is to solve the the Schrödinger equation, calculate the wave wavefunction and take its absolute square since connection with experiment is made through Born's rule. The terms appearing in (2) though, suggest the multiplication of the time dependent Schrödinger equation with its complex conjugate, which incidentally it is not identical with the former, so in a way we will incorporate (1) and the permutation differential relation (2) in a broader mathematical framework and then explore some of its consequences. We recall that the multidimensional configuration space represents the arrangement of all the system particles in the three dimensional physical space, it defines the position of all the particles in the system. We employ the position representation since mereology which is our main point of interest is traditionally related to location and spatiality [24]. By multiplying (1) with its complex conjugate and taking into account (2) it follows an equation that includes the equal cross terms  $\nabla_1^2 \Psi_{12} \nabla_2^2 \Psi_{12}^*$ ,

$$\begin{aligned} \frac{\hbar^4}{4m^2} (\nabla_1^2 \Psi_{12}^* \nabla_1^2 \Psi_{12} + \nabla_2^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} + 2\nabla_{1,2}^2 \Psi_{12}^* \nabla_{2,1}^2 \Psi_{12}) = \\ \hbar^2 \frac{\partial \Psi_{12}^*}{\partial t} \frac{\partial \Psi_{12}}{\partial t} - i\hbar V_{12} \left( \Psi_{12}^* \frac{\partial \Psi_{12}}{\partial t} - \Psi_{12} \frac{\partial \Psi_{12}^*}{\partial t} \right) + V_{12}^2 |\Psi_{12}|^2, \quad (7) \end{aligned}$$

or, taking into account Born's rule

$$\begin{aligned} \frac{\hbar^4}{4m^2} (\nabla_1^2 \Psi_{12}^* \nabla_1^2 \Psi_{12} + \nabla_2^2 \Psi_{12}^* \nabla_2^2 \Psi_{12} + 2\nabla_{1,2}^2 \Psi_{12}^* \nabla_{2,1}^2 \Psi_{12}) = \\ \frac{\hbar^2}{2} \left( \ddot{\rho}_{12} - \Psi_{12} \frac{\partial^2 \Psi_{12}^*}{\partial t^2} - \Psi_{12}^* \frac{\partial^2 \Psi_{12}}{\partial t^2} \right) - i\hbar V_{12} \left( \dot{\rho}_{12} - 2\Psi_{12} \frac{\partial \Psi_{12}^*}{\partial t} \right) + V_{12}^2 \rho_{12}. \end{aligned}$$

Eq. (7) is our main result. This hybrid wave-mechanical equation includes both two-particle amplitudes and the coincidence probability density, it is identical to its complex conjugate and the interchange of indices 1 and 2 leave it invariant, exactly like (2). Its solutions by construction are also solutions of the Schrödinger equation but violate the superposition principle so the converse conclusion is not true, not every solution of the Schrödinger equation is acceptable. It should be noted though that (7) may have solutions distinct from the the Schrödinger one.

It is interesting to note the time asymmetry of (7), even when we substitute  $\Psi(t) \rightarrow \Psi^*(-t)$  the resulting expression remains time-asymmetrical which implies non-unitary evolution. This means in other words that if we start with an initial wave function  $\Psi_{12}(t)$  it will become  $\Psi_{12}(t')$  as time evolves but changing the direction of time will not return it necessarily to its initial value. In comparison, had we started with the stationary Schrödinger equation and followed the same analysis we would instead obtain at the right hand of (7)  $(E - V_{12})^2 |\Psi_{12}|^2$  (E the energy eigenvalue of the system), which is of course time-reversible.

We notice that the left hand side of equation (7) is composed of a sum of terms that refer to the constituent parts of the system, but also a cross-term that acts on both the wavefunction and its complex conjugate. Loosely speaking it is a kind of an entangled form which suggests non-separability and illustrates the irreducible nature of the biparticle compound. The interconnectedness nevertheless of the system constituents is expressed not through the form of the quantum mechanical state but through the particular form of the operators acting on  $\Psi_{12}$  and  $\Psi_{12}^*$ . It is reasonable to deduce then that we cannot examine each particle *singulatim* neither we should consider the biparticle as simple mereological arrangement of distinct co-present entities, but rather as a substantial unified whole [27, 28, 29]. This view seems to be in accordance with the expressed opinion of many physicists (including Schrödinger) about the loss of physical reality for the individual quantum particles [30].

In order to make this statement more precise we may introduce at this point of the discussion the Weak Supplementation Principle (WSP) [18, 19, 31] according to which, if a formal object has a proper part then it has at least another proper part entirely distinct from the first, its mereological remainder. In other words, nothing can have a proper part that is not supplemented by another that is disjoint from it. This principle will be violated if an object has many disjoint proper parts but also has a proper part that shares parts with, or is not disjoint from all the others. In the case of the two-body Schrödinger equation (1) we may view the kinetic operator for one of the particles acting on the wavefunction as a proper part of the total kinetic operator and the kinetic operator of the second as its supplement or mereological remainder. Due to the property of additivity of kinetic energies in the Hamiltonian we conclude that the WSP holds. It is not possible though to reach the same conclusion in the case of Eq. (7). If on the right hand side of (7) we supposedly had only the sum of the terms  $\nabla_1^2 \Psi_{12}^* \nabla_1^2 \Psi_{12}$ ,  $\nabla_2^2 \Psi_{12}^* \nabla_2^2 \Psi_{12}$  we could still argue that WSP is satisfied since each term is the mereological remainder of the other. Including nevertheless the term  $2\nabla_{1,2}^2 \Psi_{12}^* \nabla_{2,1}^2 \Psi_{12}$  make impossible to argue this anymore, since there is no mereological remainder or supplement in this case and we cannot disentangle the action of the operators corresponding to each particles as they act on  $\Psi_{12}$  and  $\Psi_{12}^*$ . We conclude then that this mereological principle is violated.

Seeking an interpretation of these statements, we may embrace the conceptual framework of Thomism and one of its central positions that defends the unicity of the compound substantial form denying simultaneously the actual presence of individual substantial forms for its elements [32, 33, 34, 35, 36, 37], which means that the particles are present virtually, e.g. by their powers, in the composite. Employing the scholastic idiom then we may say that the quantum biparticle is a substantial form and not an artefact or an accidental form [38], or to put it in simpler terms the biparticle compound constitutes the production of a new physical reality, it is not a mere collection of particles accidentally unified. The component parts even though are retrievable in principle, they are not intrinsically unaffected by their union. Similar conclusions have been reached through Aristotelian argumentation in a different context (see [27, 28])

as previously cited).

If (7) satisfies the Schrödinger equation it follows that the probability density  $\rho_{12}(t)$  is governed by the continuity equation. This will ensure that  $\rho_{12}(t)$  will always remain equal to  $|\Psi_{12}(t)|^2$  assuming that the initial probability density is  $\rho_{12}(t_0) = |\Psi_{12}(t_0)|^2$  [39]. As it has been noted though, Eq. 7 may admit solutions distinct from the Schrödinger one, in which case the expression for the probability density needs to be modified. It may be added that from a technical point of view that due to the presence of higher order derivatives in (7), more initial and boundary value conditions need to be specified. It is also quite straightforward to generalize the above expressions for curvilinear coordinate systems.

### 3 Several identical particles

It is straightforward to extend our previous results for the case of a compound of several interacting particles of equal masses  $m$ , since the permutation of any two particle indices does not affect the wavefunction and we can still write down equations like (2) for every possible two-particle combination. The case of unequal masses  $m_1, m_2, \dots, m_n$  needs special handling though since it is not obvious that a permutation leaves the state invariant. What we need to do then is to consider every possible two-particle couple and then make a relative distance and a center of mass transformation for each of them as in the case of the well known two-body stationary problem [1]. Formally this means

$$\Psi_{12\dots n}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_n, t) \rightarrow \Psi_{12\dots n}\left(\mathbf{r}_{ij} = |\mathbf{r}_i - \mathbf{r}_j|, \mathbf{R}_{ij} = \frac{m_i \mathbf{r}_i + m_j \mathbf{r}_j}{m_i + m_j}, t\right). \quad (8)$$

We will denote in this subsection the multiparticle amplitude by  $\Psi$  for notational simplicity. A couple of useful relations which reduce to the familiar ones for  $i = 1, j = 2$  for the two-body problem are

$$\nabla_i^2 = \nabla_{\mathbf{r}_{ij}}^2 + \frac{2m_i}{M} \nabla_{\mathbf{r}_{ij}} \cdot \nabla_{\mathbf{R}_{ij}} + \left(\frac{m_i}{M}\right)^2 \nabla_{\mathbf{R}_{ij}}^2 \quad (9)$$

$$\nabla_j^2 = \nabla_{\mathbf{r}_{ij}}^2 - \frac{2m_j}{M} \nabla_{\mathbf{r}_{ij}} \cdot \nabla_{\mathbf{R}_{ij}} + \left(\frac{m_j}{M}\right)^2 \nabla_{\mathbf{R}_{ij}}^2, \quad (10)$$

with  $M = m_i + m_j$ . There are  $N(N-1)/2$  permutation equations of the form

$$\nabla_i^2 \Psi^* \nabla_j^2 \Psi = \nabla_j^2 \Psi^* \nabla_i^2 \Psi, \quad (11)$$

and the multiparticle equation of motion is written as

$$\left( \frac{\hbar^4}{4m_i^2} \sum_{i=1}^N \nabla_i^2 \Psi^* \nabla_i^2 \Psi + \frac{\hbar^4}{2m_i m_j} \sum_{i \neq j} \nabla_{i,j}^2 \Psi^* \nabla_{j,i}^2 \Psi \right) = \hbar^2 \frac{\partial \Psi^*}{\partial t} \frac{\partial \Psi}{\partial t} - i\hbar V \left( \Psi^* \frac{\partial \Psi}{\partial t} - \Psi \frac{\partial \Psi^*}{\partial t} \right) + V^2 |\Psi|^2, \quad (12)$$

where  $V = \sum V_{ij}$  and  $V_{ij}$  the binary potentials coupling each pair of particles. It is not necessary though that every particle should interact with all the rest, nearest neighbour interactions are sufficient. In the case of  $N$  particles there are  $N(N-1)/2$  possible binary potentials but only  $N-1$  of them suffice to bind the whole aggregate and even to induce correlations between those particles that do not interact. So the ratio of the minimum number to the total one is only  $2/N$ . We see then that as the number of particles is increasing the left hand side of Eq. (12) could become considerably smaller to the right one and the time-asymmetric term becomes less significant. Finally, we should mention that if we obtain a desired solution for the  $N$  body problem we can proceed to calculate the marginal probabilities by integrating out the proper variables.

## 4 Summary

In the present work we synthesized equations for the time evolution of coincidence probability functions and the several particle probability amplitudes, based on standard quantum mechanical formalism, namely the Schrödinger equation and an equation that follows directly from symmetrization postulate. A common feature of the various forms of the resulting wave-mechanical equations is that they are non-linear, suppressing then spatial coherence, identical to their complex conjugates, invariant under the exchange of particles but not invariant under time reversal. The present formulation is genuinely and irreducibly multipartite and ontological priority is given to the biparticle or more generally to the non-mereological compound, not to the monadic particle which appears to be a rather derivative entity.

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