Bell's dilemma resolved, nonlocality negated, QM demystified, etc.

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Abstract: Eschewing naive realism, we define true (classical/quantum) realism:= some existents (ie, some Bell-beables) may change interactively. We then show that Bell's mathematical ideas re local causality—from his 1964:(1)-(2) to his 1990a:(6.9.3)—are valid under true realism. But we refute Bell's analyses (and his *local realism*), as we resolve his consequent 'action-at-a-distance' dilemma in favor of true locality:= no influence propagates superluminally. In short: defining beables by properties and values—and allowing that locally-causal interactions may yield new beables—we predict the probabilities of such interaction outcomes via equivalence-classes that are weaker (hence more general) than the corresponding classes in EPR/Bell. In this way delivering the same results as quantum theory and experiment—using EPRB, CHSH, GHZ and 3-space—we also advance QM's reconstruction in spacetime with a new vector-product for geometric algebra. True local realism thus supports local causality, resolves Bell's dilemma, negates nonlocality, demystifies QM, rejects naive realism, eliminates the quantum/classical divide (since observables are clearly beables; being or not being, prior to an interaction, but certainly existing thereafter), etc: all at the level of undergraduate math and logic, and all contra the analyses and impossibility-claims of Bell and many others. We also show that Bayes' Law and Malus' Law hold, undiminished, under true local realism and the quantum.

Keywords: Bell's theorem, causality, completeness, equivalence, GA, GHZ, true locality, true realism

Preamble: 0.1. (i) This draft replaces most of my earlier essays. (ii) Paragraphs, equations, figures, etc, are numbered to aid discussion, improvement, correction. (iii) Key texts are freely available online (see References). (iv) The term *particle* is used in accord with quantum conventions: a *pristine* particle here is in its initial ex-source/pretest state; *spin* is intrinsic angular momentum. (v) It is often difficult to understand what is meant by the generic term *realism*; eg, see Norsen (2006). Here, *true realism* is well-defined and experimentally supported. (vi) Taking math to be the best logic, it may flow for several lines before we comment. (vii) The resultant probabilities predict the outcomes of experiments in full accord with quantum theory and experiment. (viii) In this way advancing the ideas of EPR and Bell (but not Bell's conclusions), we demystify QM and refute claims like these:

> (ix) Bell (1964:199), "In a theory in which parameters are added [to QM] to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence [via an instantaneous signal] the reading of another instrument, however remote." Bell (1972: 880), "The nonlocal nature of quantum mechanics ... ." Aspect (2004:9), 'Bell discovered that the search for [local-realistic] models is hopeless.' Wiseman (2005:1), 'Bell (1964) strengthened Einstein's theorem (but showed the futility of [Einstein's] quest) by demonstrating that either reality or locality is a falsehood.' Goldstein *et al* (2011:1), "In light of Bell's theorem, [many] experiments ... establish that our world is non-local." Maudlin (2014:25), "Non-locality is here to stay ... the world we live in is non-local." Gisin (2014:4), "For a realistic theory to predict the violation of some Bell inequalities, the theory must incorporate some form of nonlocality." Brunner *et al* (2014:1), "Bell's 1964 theorem [a profound development] ... states that the predictions of quantum theory cannot be accounted for by any local theory." Norsen (2015:1), "In 1964 Bell demonstrated the need for non-locality in any theory able to reproduce the standard quantum predictions." Bricmont (2016:112), 'There are nonlocal physical effects in Nature.' *Annals of Physics* Editors (2016:67; unanimously), in the context of Bell's theorem 'it's a proven scientific fact that a violation of local realism has been demonstrated theoretically and experimentally.'

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0.2. For us (using Bell's handy term for existents), true realism in physics: (i) affirms the existence of objective (ie, mind-and-theory-independent) properties and values for well-defined beables; (ii) allows that such beables may change interactively (such changes long clear to us from Malus' optics c1810 and Bohr's 'disturbance' insight); (iii) rejects as naive any brand of realism that negates/neglects (ii).

0.3. Thus, given Malus/Bohr, we show that naive realism (as in EPR) is no longer tenable: for 'the results of observation are not always given prior to and independent of observation,' after Zeilinger (2011:56). So we reject the following *realisms*: Bell (2004:89) calls Einstein's 'deep commitment to realism and locality' the 'EPR axioms'. Clauser & Shimony (1978) unclearly, 'Realism is a philosophical view, according to which external reality is assumed to exist and have definite properties, whether or not they are observed by someone.' Miller (1996:35) helpfully, 'The usual criterion for realism' is EPR (1935). Janotta & Hinrichsen (2014:31) obscurely, 'A theory obeys *realism* if measurement outcomes can be interpreted as revealing a property of the system that exists independent of the measurement.'

0.4. Thus, under true local realism: True realism allows that beables may change during interactions. True locality allows that the 'direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light,' Bell (1990a:105).

1 Introduction

> 1.0. (i) 'This action-at-a-distance business will pass. If we're lucky it will be to some big new development like the theory of relativity. Maybe someone will just point out that we were being rather silly. But anyway, I believe the questions will be resolved,' after Bell (1990:9). (ii) 'Nobody knows where the boundary between the classical and quantum domain is situated. More plausible is that we'll find that there is no boundary,' after Bell (2004:29-30). [Under the theory here, Wholistic Mechanics (WM)—the name for our theory (with its basis in true local realism) since 1989—we agree and deliver.]

1.1. Studying EPR (1935) in the context of EPRB—the EPR-based experiment in Bohm & Aharonov (1957)—Bell (1964:199) claims that EPR's program requires a grossly non-local mechanism. However, instead of correcting EPR's error—as we do at ¶1.5—Bell creates a personal dilemma [see ¶1.6(i)]: not seeing that a theory of the type that he (and we and EPR) favored could succeed. Thus, after

> Bertlmann (2017:40): "Bell wondered, *'Where does the quantum world stop and the classical world begin?'* He wanted to get rid of that division. [Agreeing, that's what we do.] For him it was true that hidden variable theories [HVTs], where quantum particles do *have* definite properties governed by hidden variables, would be appropriate to reformulate quantum theory: *'Everything has definite properties*,' Bell said." Thus (see ¶2.8) Bell (1980:7) endorses d'Espagnat's inferences to preexisting properties. So—contrary to QM orthodoxy, Bohr's insight at ¶2.9, and our theory—Bell's HVT seems bound by Bertlmann's (*ibid*) generalization that "HVTs [not orthodox QM] postulate that the properties of individual systems—[like the orientation of a particle's spin]—do *have* preexisting values revealed by the act of measurement". Care is needed here, however: '*Predetermined* is Bell's original phrasing. If there is for Bell an identity between *predetermined* and *preexisting* I cannot say ad hoc, but for a realist—as Bell was—there is clearly a close connection between both phrasings,' after R. Bertlmann (pers. comm., 14 June 2017). [See *determinism* at ¶2.13.]

1.1a. nb: for us, a *revealed property* (eg, charge) may preexist, a revealed spin-orientation may not. So—under Bertlmann's generalization (*ibid*)—ours in not an HVT. Instead, we allow an observable to be made from beables whose (*preexisting*) pretest values *may be forever hidden* under interactions/transitions/transformations. Then, to advance our understanding, we encode the consequent incomplete—but adequate—information in conservation laws and probability relations. Thus:

1.2. Cautiously seeking consensus, we begin by accepting d'Espagnat's (1979:158) Bell-endorsed principles of local realism: (i) *realism* (regularities in observed phenomena are caused by some physical reality whose existence is independent of human observers); (ii) *locality* (no influence of any kind can propagate superluminally); (iii) *induction* (legitimate conclusions can be drawn from consistent observations). So this is not a dispute about differing principles. Rather—merging our hopes with those of EPR and Bell; and given ¶1.0—*we simply reject inferences that are false in quantum settings.* We thus show that Bell and d'Espagnat fail under (iii): ie, ignoring consistent observations re the validity of QM—and Bohr's insight—they draw conclusions that are false under both QM and experiment (eg, see Aspect 2004). Indeed, for us—readily accepting the commonsense in d'Espagnat's (i)-(iii) above; and succeeding with it (as we'll show)—QM seems to be better-founded than Bell imagined; eg, here's Bertlmann (2017:54) on Bell (with Bellian naivety, puzzlement and doubts that we do not share):

> "John was totally convinced that *realism* is the right position of a scientist. He believed that experimental results are predetermined and not just induced by the measurement process. Even more, in John's EPR analysis reality is not assumed but inferred! Otherwise (without realism), he said, *'It's a mystery if looking at one sock makes the sock pink and the other one not-pink at the same time.'* So he did hold on [to] the hidden variable program continuously, and was not discouraged by the outcome of EPR-Bell experiments but rather puzzled. For him: *'The situation was very intriguing that at the foundation of all that impressive success* [of QM] *there are these great doubts*,' as he once remarked."

1.3. In this way identifying the source of Bell's dilemma—¶1.6(i)—let's be clear about our own position: our core quantum-compatible principle is *true local realism* (TLR), the union of true locality (no influence propagates superluminally, after Einstein) and true realism (some beables change interactively, after Bohr). TLR is therefore consistent with most interpretations of QM—and with contextuality; thus bypassing the Kochen-Specker theorem—since interactions need not reveal preexisting properties. We then advance EPR's program by including every relevant beable in our EPRB analysis (see ¶2.1)—validating EPR's belief (see next)—but rejecting their famous criterion (see ¶¶1.4-1.5):

> "In a complete theory there is an element corresponding to each element of reality," EPR (1935:777). "While we have thus shown that the wavefunction does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible," EPR (1935:780). [Using every relevant EPRB beable at ¶2.1, we show that a more complete description is possible. Further (¶3.7), we show that *hidden dynamics* can be adequately treated by encoding *incomplete information* in probabilistic relations; eg, via Bayes' Law: $P(XY\,|\,Z) = P(X\,|\,Z)P(Y\,|\,ZX)$. Given our focus on truth, our consequent expectations are validated by QM and experimental facts.]

1.4. So, using vector-products and physical operators in 3-space (not wavefunctions, etc, in Hilbert space), we study the interaction of beables (particles) with other beables (polarizers). That is, taking particles and polarizers to be sensitive contributors to the *veiled reality* (d'Espagnat 1983:94) of our world, we allow: (i) any interactant may be transformed under TLR; (ii) any transformation may be subject to an uncertainty induced by Planck's action-constant; (iii) every less-than-certain probability distribution represents a veiled reality. (iv) Then, identifying every relevant beable under EPRB—¶¶2.1-2.7—we provide a 'complete description' of what can be inferred from incomplete information. (v) We also address Bell's dilemma—¶1.6(i)—by endorsing EPR's *next two* sentences; marked [i], [ii]: but amending—at ¶1.5—the famous [naive] EPR *criterion* [iii] that follows [ii] here:

> "[i] The elements of physical reality ... must be found by an appeal to the results of experiments and measurements. [ii] A comprehensive definition of reality is, however, unnecessary for our purpose. [iii] We [ie, EPR, but not us] shall be satisfied with the following criterion, which we regard as reasonable. *If, without any way disturbing a system,*

we can predict with certainty $[P = 1]$ *the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*," EPR (1935:777).

1.5. Departing subtly from EPR—but wholly compatible with EPRB, QM, the Bell/d'Espagnat 'local-realism' of ¶1.2, and TLR at ¶1.3—here's our sufficient condition for an element of physical reality (a beable), presented in the context of (8)-(9) below to be clear: 'If, without in any way disturbing a system $q(\boldsymbol{\mu}_i)$, we can predict with adequate accuracy the result $B_i = -1$ of the interaction $\delta_a^{\pm} q(\boldsymbol{\mu}_i)$—ie, an interaction that may disturb $q(\boldsymbol{\mu}_i)$—then local beables $q(\boldsymbol{\mu}_i)$, $\delta_a^{\pm}$ [and the consequent interaction-output $q(a^-)$] will mediate this result; ie, B_i will be a local function of $q(a^-)$ and $[a \cdot *]$,' after Watson (1998:417). [As to *sufficiency*: this condition delivers Bell's hope (2004:167) for 'a simple constructive model' of EPRB; see ¶2.15. As to our view of *adequacy*: here's Aspect's (2004:24) example against our predictions (when we too depart from idealization): $S_{\text{WM}} = S_{\text{QM}} = 2.70 \pm 0.05$; $S_{\text{Exp}} = 2.697 \pm 0.015$.]

1.6. All of which brings us to: (i) Bell's dilemma re action-at-a-distance (AAD hereafter); (ii) a clarifying expansion about the motivation we share with Bell (and with EPR) from ¶1.0:

> (i) 'I cannot say that AAD is required in physics. *I can say that you cannot get away with no* AAD. You cannot separate off what happens in one place and what happens in another. *Somehow they have to be described and explained jointly.* That's the fact of the situation; Einstein's program fails ... *Maybe we have to learn to accept not so much* AAD*, but the inadequacy of no* AAD*. ...* **That's the dilemma**. We are led by analyzing this situation to admit that, somehow, distant things are connected, or at least not disconnected. ... *I don't know any conception of locality that works with QM. So I think we're stuck with nonlocality ...* I step back from asserting that there is AAD and *I say only that you cannot get away with localit*y. You cannot explain things by events in their neighbourhood. But, I am careful not to assert that there is AAD,' after Bell (1990:5-13); *emphasis added.*
>
> (ii) "Now nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary. It is hard for me to envisage intelligible discourse about a world with no classical part—no base of given events, be they only mental events in a single consciousness, to be correlated. On the other hand, it is easy to imagine that the classical domain could be extended to cover the whole. The wavefunctions—[not beables in our terms; in agreement with Bell (2004:53)]—would prove to be a provisional or incomplete description of the quantum-mechanical part, of which an objective account would become possible. It is this possibility, of a homogeneous account of the world, which is for me the chief motivation of the study of the so-called 'hidden variable' possibility," Bell (2004:29-30). We agree and deliver. But.

1.7. Sharing this motivation, we deliver differently: for we prefer short-form expectations—like LHS (67) at ¶5.7—that at-once bypass the limitations in Bell's analyses. However, for now, to match the style of typical Bellian essays, we defer our use of short-forms until we've established their validity. Thus, *using our notation* per ¶2.1, we come to what we call (for convenience; it matters not), Bell's definition of *true local realism*: ie, we come to Bell's view (1990a:109) re 'locally explicable' correlations where factorizability is not taken to be the starting point of the analysis—nor the *formulation* of 'local causality'—but a consequence thereof. Thus, with past causes included in β (which defines EPRB):

$$P(AB\,|\,\beta, a, b, \boldsymbol{\lambda}) = P(A\,|\,\beta, a, \boldsymbol{\lambda})P(B\,|\,\beta, b, \boldsymbol{\lambda}) \text{ after Bell 1975a:(4)-(6), 1990a:(6.9.3); with} \tag{1}$$

$$P(AB\,|\,\beta, a, b, q(\boldsymbol{\lambda}), q(\boldsymbol{\mu})) = P(A\,|\,\beta, a, q(\boldsymbol{\lambda}))P(B\,|\,\beta, b, q(\boldsymbol{\mu})), \text{ which is (1) amended prudently:} \tag{2}$$

because (for us, under TLR) it's clearer to allow particle-variables $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ to be different—see ¶2.1—correlated via (6). Thus, using Bell's widely-accepted idea, (1), we move to show—via adequate and relevant classes of local beables—that our world is truly local and truly realistic.

1.8. So, under the principles in ¶¶1.2-1.5, eliminating false inferences and non-facts and resolving Bell's dilemma—via Analysis, Conclusions, Acknowledgment, Appendices, References—we move to defend the Abstract in line with our continuing respect for Oliver Heaviside and connected facts.

> "Facts are of not much use, considered as facts. They bewilder by their number and their apparent incoherency. Let them be digested into theory, however, and brought into mutual harmony, and it is another matter. Theory is of the essence of facts. Without theory scientific knowledge would be only worthy of the madhouse," Heaviside (1893:12).

1.9. In short: TLR links many facts to the Bell/d'Espagnat principles in ¶1.2: eg, (i) EPR-correlations in EPR-experiments; (ii) repeated validation of QM, Bohr's insight and special relativity; (iii) validation of (2) and famous (1) in theory and in practice *after* TLR-factoring at ¶3.6; (iv) retrodiction re spacelike-separated events via logical implication—nb: there is no backward or nonlocal causation here—but first we best establish TLR's credentials beyond dispute.

2 Analysis

> 2.0. Einstein "argued that the EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way," Bell (2004:86). EPR suggest that a state, 'richer in content than the quantum state, would provide a commonsense explanation of certain perfect correlations predicted by QM, which are otherwise baffling,' after GHSZ (1990:1131): though Bell (1990a:108)—discussing EPR—writes contrarily, "Commonsense does not work here." However, certain that it does work *here*—agreeing with Einstein and GHSZ; using Aspect (2004); and working through EPRB to CHSH and GHZ/Mermin, etc—we deliver commonsense TLR-based explanations that are true via Bohr's insight, local via Einstein locality, and realistic via Bell beables. Therefore, seeking to provide a helpful prelude to this analysis, we suggest that the key to the commonsense here is this (contra Bell): under TLR, we focus on an adequate specification of beables in 3-space, and not at all on QM formalisms. For we are of the same opinion as Einstein, and Bell (1990a:112): 'the new cookery of QM ... contains all the same a certain unpalatability.'

2.1. Under TLR-completeness, every relevant beable of the subject reality [per Bell (1964)] follows: including 3-space (since time and gravity are not essential to the analysis here). We let the beable $\boldsymbol{\lambda}$ in Bell's 1964:(1)—with its spin s implicit—denote a pristine particle's total angular momentum; and we allow that in the ith pair, $\boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0$ via the pairwise conservation of total angular momentum. Thus, under this relation, information about an associated property of one beable reveals information about the other beable that is similarly associated with the given relation:

$$.A_i \equiv +1 \leftarrow \Delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \triangleleft\beta\triangleright \ q(\boldsymbol{\mu}_i) \Rightarrow \Delta_b^{\pm} \rightarrow +1 \equiv B_i. \tag{3}$$

$$.A_i \equiv +1 = a \cdot a^+ \leftarrow [a \cdot *] \Leftarrow q(a^+) \leftarrow \delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \triangleleft\beta\triangleright \ q(\boldsymbol{\mu}_i) \Rightarrow \delta_b^{\pm} \rightarrow q(b^+) \Rightarrow [b \cdot *] \rightarrow b \cdot b^+ = +1 \equiv B_i. \tag{4}$$

$$\llbracket \quad \text{Alice's locale} \quad \rrbracket \quad \llbracket \text{Source} \rrbracket \quad \llbracket \quad \text{Bob's locale} \quad \rrbracket \tag{5}$$

$$\boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0;\ i = 1, 2, ..., n.\ A_i(a, \boldsymbol{\lambda}_i) = -B_i(a, \boldsymbol{\mu}_i);\ \text{etc.}\ P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_j \,|\, i \neq j) << 1. \tag{6}$$

$$.A_i \equiv +1 = a \cdot a^+ \leftarrow [a \cdot *] \Leftarrow q(a^+) \leftarrow \delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \triangleleft\beta\triangleright \ q(\boldsymbol{\mu}_i) \Rightarrow \delta_a^{\pm} \rightarrow q(a^-) \Rightarrow [a \cdot *] \rightarrow a \cdot a^- = -1 \equiv B_i. \tag{7}$$

2.2. (3) shows experiment β (EPRB, with β honoring Bohm) and a test on (a decoherent interaction with) each member of the ith particle-pair: thick arrows ($\Rightarrow$) denote movement toward an interaction, thin arrows ($\rightarrow$) point to the subsequent output (here, a transformation). With spin s implicit, and properties $\boldsymbol{\lambda}_i$ and $\boldsymbol{\mu}_i$, our pristine (pretest) spin-$\frac{1}{2}$ particles $q(\boldsymbol{\lambda}_i)$ and $q(\boldsymbol{\mu}_i)$ emerge from $\triangleleft\beta\triangleright$ (a decay conserving angular momentum) such that (6) holds. Each particle interacts with a dichotomic linear-polarizer-analyzer $\Delta_x^{\pm}$—freely and independently operated by Alice (with result A) and Bob (result B)—where x denotes any relevant orientation of its principal-axis. Under EPRB: $x^+ = +x$; $x^- = -x$.

2.2a. Given that A and B are discrete (± 1), and seeking generality, we employ variables like $\boldsymbol{\lambda}_i$, $\boldsymbol{\mu}_i$ (ordinary vectors with lengths in units of $\frac{\hbar}{2}$) so that our EPRB results are associated with $\frac{\hbar}{2}$. In this way linking to vector-magnitudes—eg, $\boldsymbol{\lambda}_i = |\boldsymbol{\lambda}_i|\hat{\boldsymbol{\lambda}}_i$, with $|\boldsymbol{\lambda}_i|$ in units of $\frac{\hbar}{2}$, and $\hat{\boldsymbol{\lambda}}_i$ the direction-vector—our variables may be continuous or discrete. This choice accords with the generality of our approach: and with Bell's (1964:195) indifference to whether such variables are discrete or continuous. [Then, in that we seek equivalence relations under orientations—taking Bell's (1964) a and b to be principal-axis direction-vectors—$\frac{\hbar}{2}$ is suppressed in (3)-(7). The more complete story under EPRB—eg, $\delta_a^{\pm} q(\boldsymbol{\lambda}_i) \to q(\frac{\hbar}{2}a^{\pm})$, with allied relations under magnitudes—is developed at ¶5.3.]

2.3. Identifier i is used generically: but each particle may be tested once only in (what we call) its *pristine* state, and thereafter until absorbed in an analyzer; nb, under TLR, we generally favor the term *test* over the term *measurement*. Then, since the tests are locally-causal and spacelike-separated, we hold fast to Einstein's principle of local causality: the real factual situation of $q(\boldsymbol{\mu}_i)$ is independent of what is done with $q(\boldsymbol{\lambda}_i)$ which is spatially separated from $q(\boldsymbol{\mu}_i)$—and vice-versa—per Bell (1964: endnote 2; citing Einstein). Consistent with this principle, paired test outcomes are correlated via (6).

2.4. (4) expands on (3) to show that each polarizer-analyzer $\Delta_x^{\pm}$ is built from a polarizer $\delta_x^{\pm}$ and a removable analyzer $[x \cdot *]$ that responds to the polarization-vector $(*)$ of each post-polarizer particle $q(*)$ upon receipt. We thus assign the correct polarization $x^{\pm}$ to $q(*)$ by observing the ± 1 output of the related analyzer; or by understanding the nature of particle/polarizer interactions. So experiment β is EPRB—per Bell (1964)—with this benign finesse: to facilitate additional analysis and experimental confirmations, we can employ additional polarizers ($\delta_y^{\pm}$) to test $q(x^{\pm})$, etc; and y may equal x.

> nb: as in (3)-(4), with $q(\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm}$ leading to $\delta_a^{\pm} q(\boldsymbol{\lambda}_i) \to q(a^{\pm})$—never requiring that $q(\boldsymbol{\lambda}_i) = q(a^{\pm})$ prior to the interaction $\delta_a^{\pm} q(\boldsymbol{\lambda}_i)$—we take *interaction* and *transformation* to be concepts more fundamental than *measurement*: "Doesn't any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts?" Bell (2004:118). We agree and deliver.

2.5. (5), with no symmetry requirements, shows the locales in (3)-(4) and (7) arbitrarily spacelike-separated from each other and from the source. Given that (3)-(7) hold over any spacelike separation, it follows that the relevant (pretest) particle properties are stable between emission and interaction with a polarizer. Further, our theory is locally-causal and Lorentz-invariant because A_i and B_i are locally-caused by precedent local events $\delta_a^{\pm} q(\boldsymbol{\lambda}_i)$ and $\delta_b^{\pm} q(\boldsymbol{\mu}_i)$ respectively, which are spacelike-separated.

2.6. (6) shows $\boldsymbol{\lambda}_i$ and $\boldsymbol{\mu}_i$ pairwise correlated via the conservation of total angular momentum; our use of ordinary vectors being prompted by Dirac (1982:149, eqn (48)) and geometric algebra as we seek a realistic replacement for Pauli's vector-of-matrices. (Note: via Fröhner (1998), we reject no tools of the quantum trade.) Motivated by Bell, these TLR-based variables provide a more complete specification of particle-pairs under β. Thus, for now, we allow these pristine spin-related variables to be ordinary vectors for which all magnitudes and orientations are equally probable. (New conventions begin when we integrate our approach with geometric algebra at ¶5.2-5.4.) Then, under our doctrine of cautious conservatism—and though particle responses to interactions may be similar—(6) allows it to be far less than certain that two pristine (ie, pretest) particle-pairs are physically the same.

2.7. (7) shows experiment β with Alice and Bob having the same polarizer setting a. (Per ¶2.2a, $\frac{\hbar}{2}$ is suppressed here.) Thus, as an *idealized* example—ie, by observing one result, we may predict the other (spacelike-separated) result *with certainty*—here's how Alice predicts Bob's result after observing $A_i = +1$; and vice-versa, with Bob observing $B_i = -1$ here:

$$A_i = +1 \because q(\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm} \to q(a^+) \Rightarrow [a \cdot *] \to [a \cdot a^+] = +1. \therefore - \text{ using (6) } - \tag{8}$$

$$q(\boldsymbol{\mu}_i) = q(-\boldsymbol{\lambda}_i) \Rightarrow \delta_a^{\pm} \to q(a^-) \Rightarrow [a \cdot *] \to [a \cdot a^-] = -1 = B_i.\ QED. \blacksquare \text{ And vice-versa.} \tag{9}$$

2.8. This is consistent with our sufficient condition for a beable [¶1.5]. For, without in any way disturbing $q(\boldsymbol{\mu}_i)$, Alice can predict with certainty that Bob's result will be $B_i = -1$ when he tests $q(\boldsymbol{\mu}_i)$ with $\delta_a^{\pm}$ (which may be a disturber). So beables $q(\boldsymbol{\mu}_i), \delta_a^{\pm}$ and $q(a^-)$ mediate Bob's result; ie, the interaction $\delta_a^{\pm} q(\boldsymbol{\mu}_i)$ will yield $q(a^-)$ and the interaction $[a \cdot a^-]$ will yield $B_i = -1$. Thus the particle corresponding to Bob's $B_i = -1$ result will be $q(a^-)$; a TLR outcome acceptable to EPR, but an important departure from Bell's position. For Bell endorses d'Espagnat's (1979:166) inference that the input to the polarizer equals the output from the polarizer—ie, $q(\boldsymbol{\mu}_i) = q(a^-)$—but we do not.

> We thus come to the issue of Bell's likely predilection for *preexisting* properties and the possibility of clarifying ¶1.1. Now the use of *induction*—drawing legitimate conclusions from consistent observations—was foreshadowed at ¶1.2. But we will next see that d'Espagnat (and thus, seemingly, Bell) ignore a long history of consistent observations/facts that support the validity of QM and Bohr's insight. This failing would be OK if Bell and d'Espagnat were merely out to rebut EPR—and thus (maybe) endorse our amendment at ¶1.5—but here, as in science generally: facts and subtle distinctions matter more than differing theorems based on different assumptions. And one fact is this: with Bell's (1980:7) endorsement, d'Espagnat (1979:166) uses the phrase *'definite spin components at all times'*—'definite at all times'—ie, *preexisting*. So the d'Espagnat/Bell approach here (unlike ours) is an HVT per ¶1.1; with related absurdities, see (28), (32), (40).
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> Here's Bell's (1980:7): "To explain this dénouement [of his (1964) theorem, say] without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."
>
> Here's d'Espagnat (1979:166), recast for EPRB (and our β) in our notation, *with added emphasis*: 'A physicist can infer that in every pair, one particle has the property a^+ [a positive spin-component along axis a] and the other has the property a^-. Similarly, he can conclude that in every pair one particle has the property b^+ and one b^-, and one has property c^+ and one c^-. *These conclusions require a subtle but important extension of the meaning assigned to our notation* a^+. Whereas previously a^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself. To be explicit, if some unmeasured particle has the property that a measurement along the axis a would give the definite result a^+, then that particle is said to have the property a^+. *In other words, the physicist has been led to the conclusion that both particles in each pair have definite spin components at all times.* ... This view is contrary to the conventional interpretation of QM, but it is not contradicted by any fact that has yet been introduced.' [nb: *definite spin components at all times* = preexisting.]

2.9. However, to the contrary under TLR, as we'll show: (i) d'Espagnat's inferences are false; (ii) weaker, more-general, inferences are available; (iii) there's no need to contravene known facts re QM; (iv) and no need to negate Bohr's insight: which—supported by Bell hereunder—bolsters our case against d'Espagnat's 'Bell-endorsed' inferences. [See also Kochen (2015:5): in QM, physicists 'do not believe that the value of the spin component (S_z) exists' prior to the (polarizer) interaction.]

> Here's Bell (2004: xi-xii): It's "Bohr's insight that the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'. It seems [to Bell] that full appreciation of [Bohr's insight] would have aborted most of the 'impossibility proofs' [like Bell's *impossibility* theorem, as we'll see], and most of 'quantum logic'." We agree, for in this way we reject the quantum/classical divide. Under true realism [¶1.3]*—some beables change interactively*—we do not assume that all 'measured' properties already exist prior to 'measurement' interactions. Thus, under TLR—and given our view that Malus' experiments involve disturbing interactions between polarizers and light-beams—we negate and reject the following assumption: "In classical physics, we assume that the measured properties of the system already exist prior

to the measurement. ... The basic [classical] assumption is that systems have intrinsic properties and the experiment measures the value of them," Kochen (2015:5); see ¶2.19a.

2.10. Thus, to be clear and consistent with Bohr's insight, TLR goes beyond the Bell-d'Espagnat inferences wherein the 'measured' property is *equated* to a pristine property. That is—going beyond d'Espagnat's *subtle extension* cited in ¶2.8—we instead infer here to *equivalence under a 'polarizing' operator*. For *equivalence*—a relation without which science would hardly be possible; a weaker, more general relation than *equality*—is here compatible with QM, Bohr's view, and the consequent need to recognize the effect of 'the means of observation' on EPRB inputs:

> "... the unavoidable interaction between the objects and the measuring instruments sets an absolute limit to the possibility of speaking of a behaviour of atomic objects which is independent of the means of observation," Bohr (1958:25).

2.11. So now, under TLR—via the known effect of linear-polarizer $\delta_x^{\pm}$ on polarized particles $q(x^+)$—we can match interactions like $\delta_x^{\pm} q(\boldsymbol{\lambda}_i) \rightarrow q(x^+)$ with ancillary interactions like $\delta_x^{\pm} q(x^+) \rightarrow q(x^+)$. Then, since $\delta_x^{\pm}$ is a dichotomic operator that dyadically partitions its binary domain, we let $\sim$ here denote the equivalence relation *has the same output under the same operator.* Thus, in the context of EPRB:

$$\text{If } \delta_a^{\pm} q(\boldsymbol{\lambda}_i) \rightarrow q(a^+) \text{ then } q(\boldsymbol{\lambda}_i) \sim q(a^+) \because \delta_a^{\pm} q(a^+) \rightarrow q(a^+) \text{ only.} \therefore q(\boldsymbol{\mu}_i) \sim q(a^-) \text{ via } q(-\boldsymbol{\lambda}_i) \sim q(a^-). \quad (10)$$

$$\text{If } \delta_b^{\pm} q(\boldsymbol{\mu}_i) \rightarrow q(b^+) \text{ then } q(\boldsymbol{\mu}_i) \sim q(b^+) \because \delta_b^{\pm} q(b^+) \rightarrow q(b^+) \text{ only.} \therefore q(\boldsymbol{\lambda}_i) \sim q(b^-) \text{ via } q(-\boldsymbol{\mu}_i) \sim q(b^-). \quad (11)$$

2.12. That is, in (10)—consistent with Alice's frame of reference wherein Alice observes $A_i = +1$, per $q(a^+)$—we confirm $\sim$ under $\delta_a^{\pm}$ as follows: (i) polarizing-operators $\delta_a^{\pm}$ deliver $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ to the same output; (ii) it is impossible (under idealization) that an interaction with a $\delta_a^{\pm}$ might to deliver $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ to two different outputs; (iii) an equivalence relation $\sim$ therefore holds between $q(\boldsymbol{\lambda}_i)$ and $q(a^+)$ under $\delta_a^{\pm}$. (11) similarly, via Bob's frame of reference, wherein Bob observes $B_i = +1$: the equivalence relation $\sim$ now holding between $q(\boldsymbol{\mu}_i)$ and $q(b^+)$ under $\delta_b^{\pm}$. [nb: further, at (20)-(21) and ¶¶2.17-2.19 below, we find that particles equivalent under $\delta_a^{\pm}$ are also equivalent under $\delta_b^{\pm}$ in probability functions; an important result because it licenses Malus' Law under TLR.]

2.13. Re our equivalence relations $\sim$ (using the format $\overset{\delta_x^{\pm}}{\sim}$ to include the operator when clarity requires):

$$Q \equiv \{q(\boldsymbol{\lambda}_i), q(\boldsymbol{\mu}_i); q(a^{\pm}) \,|\, \beta, \delta_a^{\pm}, \boldsymbol{\lambda}_i + \boldsymbol{\mu}_i = 0, i = 1, 2, ..., n\}, \text{ given (7)}; \quad (12)$$

$$[q(a^+)] \equiv \{q(\bullet_i) \in Q \mid \beta, q(\bullet_i) \overset{\delta_a^{\pm}}{\sim} q(a^+)\};\ [q(a^-)] \equiv \{q(\bullet_i) \in Q \mid \beta, q(\bullet_i) \overset{\delta_a^{\pm}}{\sim} q(a^-)\}; \quad (13)$$

$$Q/_{\sim} = \{[q(a^+)],\ [q(a^-)]\}; \quad (14)$$

where Q is the set of n particle-pairs under β and $\delta_a^{\pm}$: ie, $2n$ input particles $q(\bullet_i)$ and $2n$ output particles $q(a^{\pm})$ via $\delta_a^{\pm} q(\bullet_i) \rightarrow q(a^{\pm})$. In (13), equivalence classes $[q(a^+)]$ and $[q(a^-)]$ show Q partitioned dyadically under the mapping $\delta_a^{\pm} q(\bullet_i) \rightarrow q(a^{\pm})$. So, on the elements of $\delta_a^{\pm}$'s domain, $\sim$ denotes: *has the same output under* $\delta_a^{\pm}$. ($\delta_b^{\pm}$ similarly.) So the quotient set $Q/_{\sim}$ in (14)—the set of all equivalence classes under $\sim$—is a set of two diametrically-opposed extremes: a maximal antipodean discrimination; a powerful *deterministic* push-pull dynamic; a sound basis for determinism; see ¶¶2.16, 5.3.

2.13a. Consequently, in our terms: under β, the deterministic classes $[q(a^+)]$ and $[q(a^-)]$ in (13)-(14) are adequately concrete—ie, adequately informative—to adequately fulfill 'the more complete specification' that Bell (1964:195) wanted 'to be effected by means of $\boldsymbol{\lambda}$. It is a matter of indifference in the following whether $\boldsymbol{\lambda}$ denotes a single variable or a set ... .' And testing new pairs of particles (say $j = n + i$) under β and $\delta_b^{\pm}$ yields a similar deterministic dichotomy; eg, see ¶2.25 and ...

2.14. ... this. We now combine (3), (4), (10), (11) into a single test on the ith particle-pair from two perspectives: (15), which Alice reads from left-to-right; (16), which Bob reads from right-to-left:

$$A_i \equiv +1 \cdots q(a^+) \leftarrow \Delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i) \ \triangleleft\beta\triangleright \ q(\boldsymbol{\mu}_i) = q(-\boldsymbol{\lambda}_i) \sim q(a^-) \Rightarrow \delta_b^{\pm} \Rightarrow q(b^+) \rightarrow [b\cdot *] \rightarrow b\cdot b^+ = +1 \equiv B_i\,; \quad (15)$$

$$A_i \equiv +1 = a\cdot a^+ \leftarrow [a\cdot *] \Leftarrow q(a^+) \leftarrow \delta_a^{\pm} \Leftarrow q(b^-) \sim q(-\boldsymbol{\mu}_i) = q(\boldsymbol{\lambda}_i) \ \triangleleft\beta\triangleright \ q(\boldsymbol{\mu}_i) \Rightarrow \Delta_b^{\pm} \rightarrow q(b^+) \cdots +1 \equiv B_i : \quad (16)$$

> Thus, in line with Bell's (1964:196) specification for his $\boldsymbol{\lambda}$: (i) seeking a physical theory of the type envisioned by Einstein/EPR, our variables have dynamical significance and laws of motion; (ii) our pristine $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$—correlated under (6)—are the initial pretest values of such variables at some suitable instant; (iii) since different tests produce different disturbances, different properties may be pairwise revealed under $\sim$ without contradiction: ie, finding $q(\boldsymbol{\lambda}_i \sim a^+)$ experimentally, we learn $q(\boldsymbol{\mu}_i \sim a^-)$ relationally via (6); etc. *QED.* ■

2.15. So, from (6) and (10)-(16), with $A^{\pm}$ ($B^{\pm}$) denoting Alice's (Bob's) results (± 1), we can now provide (under β per Bell 1964): (i) functions that satisfy Bell 1964:(1); (ii) valid *correlated* EPRB probabilities and expectations; (iii) our rejection of the generality of Bell's 1964 theorem; (iv) the whole followed by explanatory comments. [nb: here we use Malus' classical Law at (20-(21) and Bayes' Law at (22). At (52)-(54) and (78)-(80) we use Malus' Law to deliver Bayes' Law. We thus show (contra the views of many) that both laws—one from classical physics; one from classical logic—are valid under TLR. To be clear: there are many logical implications here, but no backward or nonlocal causation.]

$$\Delta_a^{\pm} q(\boldsymbol{\lambda}) \rightarrow A(a, \boldsymbol{\lambda}) = \cos(a, \boldsymbol{\lambda} \,|\, q(\boldsymbol{\lambda}) \sim q(a^{\pm})) = \pm 1 \equiv A^{\pm};\ \langle A \,|\, \beta \rangle = 0 \because P(A^+ \,|\, \beta) - P(A^- \,|\, \beta) = 0. \quad (17)$$

$$\Delta_b^{\pm} q(\boldsymbol{\mu}) \rightarrow B(b, \boldsymbol{\mu}) = \cos(b, \boldsymbol{\mu} \,|\, q(\boldsymbol{\mu}) \sim q(b^{\pm})) = \pm 1 \equiv B^{\pm};\ \langle B \,|\, \beta \rangle = 0 \because P(B^+ \,|\, \beta) - P(B^- \,|\, \beta) = 0. \quad (18)$$

$$P(A^+ \,|\, \beta) = P(A^- \,|\, \beta) = P(B^+ \,|\, \beta) = P(B^- \,|\, \beta) = \tfrac{1}{2} \ \because \ \boldsymbol{\lambda} \text{ and } \boldsymbol{\mu} \text{ are random variables here.} \quad (19)$$

$$P(A^+ \,|\, \beta B^+) = P(q(\boldsymbol{\lambda} \sim a^+) \,|\, \beta, q(\boldsymbol{\mu} \sim b^+)) = P(\delta_a^{\pm} q(b^-) \rightarrow q(a^+) \,|\, \beta) = \cos^2 s\,(a^+, b^-) = \sin^2 \tfrac{1}{2}(a, b). \quad (20)$$

$$P(B^+ \,|\, \beta A^+) = P(q(\boldsymbol{\mu} \sim b^+) \,|\, \beta, q(\boldsymbol{\lambda} \sim a^+)) = P(\delta_b^{\pm} q(a^-) \rightarrow q(b^+) \,|\, \beta) = \cos^2 s\,(a^-, b^+) = \sin^2 \tfrac{1}{2}(a, b). \quad (21)$$

$$\therefore P(A^+ B^+ \,|\, \beta) = P(A^+ \,|\, \beta) P(B^+ \,|\, \beta A^+) = P(B^+ \,|\, \beta) P(A^+ \,|\, \beta B^+) = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(a, b).\ QED. \blacksquare \quad (22)$$

$$\therefore \left\langle A^+ B^+ \,|\, \beta \right\rangle = \langle A^- B^- \,|\, \beta \rangle = \tfrac{1}{2} \sin^2 \tfrac{1}{2}(a, b);\ \left\langle A^+ B^- \,|\, \beta \right\rangle = \left\langle A^- B^+ \,|\, \beta \right\rangle = -\tfrac{1}{2} \cos^2 \tfrac{1}{2}(a, b). \quad (23)$$

$$\therefore \langle AB \,|\, \beta \rangle \equiv \left\langle A^+ B^+ \,|\, \beta \right\rangle + \left\langle A^+ B^- \,|\, \beta \right\rangle + \left\langle A^- B^+ \,|\, \beta \right\rangle + \langle A^- B^- \,|\, \beta \rangle = -a\cdot b.\ QED. \blacksquare \quad (24)$$

2.16. That is. Given (15), the cosine function in (17) reads: with $q(\boldsymbol{\lambda})$ equivalent to $q(a^+)$ under $\sim$, $\cos(a, \boldsymbol{\lambda} \,|\, q(\boldsymbol{\lambda}) \sim q(a^+))$ denotes the cosine of the angle (a, a^+): ie—under the deterministic push-pull dynamic identified in ¶2.13; with $q(\boldsymbol{\lambda}) \sim q(a^+)$—the outcome is $+1 = A^+$ here. (18) similarly, given (16); etc. Thus, under $\sim$, we could embrace Bell-d'Espagnat inferences [¶2.8] to equality, but: (i) the probability that such inferences are valid is negligible; (ii) their theory does not embrace ours; (iii) under our safe conservatism—allowing $P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_j \,|\, \beta, i \neq j) << 1$, per (6)—we get the right results.

2.17. Next in the logic-flow, (19) is self-explanatory. Then, re (20)—and (21) similarly—via standard probability theory and Bayes' Law ¶1.3: (i) the correlation of $A^{\pm}$ and $B^{\pm}$ via (6) induces the probability relation at LHS (20); (ii) such correlation is recognized by Bell (in our favor) as follows:

> Recasting Bell (2004:208) in line with EPRB: "There are no 'messages' in one system from the other. The inexplicable [sic] correlations of quantum mechanics do not give rise to signalling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system [eg, from Alice's A^+] to events in the other [eg, to Bob's B^+] are possible." [All consistent with our use of Bayes' Law in (22). The use of Malus' Law in (20)-(21) is discussed next.]

2.18. Continuing the logic flow: in (20) under $\sim$, the LHS probability relation is—from the middle term in (20)—equivalent to a test on spin-$\frac{1}{2}$ particles of known polarization. So we derive RHS (20) by extending Malus' $\cos^2 s\,(a^+, b^-)$ Law (c.1808)—re the relative intensity of beams of polarized photons ($s = 1$)—to spin-$\frac{1}{2}$ particles ($s = \frac{1}{2}$). (21) similarly. Then, since our equivalence relations hold under probability functions P, P is well-defined under $\sim$ and is that same law—Malus' Law—now TLR-compatible by extension; validated under $s = \frac{1}{2}$ in (20)-(21), under $s = 1$ in Aspect (2004).

> Thus: re Aspect's (2004:5-7) 'concerned' discussion of Malus' Law, our trigonometric arguments represent clear law-based dynamical processes under (10)-(11) and ¶¶2.3, 2.13: eg, $(q(\boldsymbol{\lambda}_i) \sim q(a^+)) \equiv (\delta_a^{\pm} q(\boldsymbol{\lambda}_i) \rightarrow q(a^+))$; the a in $\delta_a^{\pm}$ denotes the orientation of a non-uniform field with which $q(\boldsymbol{\lambda}_i)$ interacts; superscript $\pm$ denotes two output ports. A wire-grid microwave-polarizer provides a macroscopic analogy. With its conducting wires represented by a direction-vector in 3-space, an impinging unpolarized beam of microwaves drives electrons within the wires, thereby generating an alternating current (Hecht 1975:104). So the wires become polarizing-operators (in our terms), for the transmitted beam is strongly linearly polarized. (Polaroid®-sheet is a molecular equivalent for photons.) This suggests (see ¶5.3), that the micro-dynamics of particle/polarizer interactions may be represented by a suitable vector-product with two boundary-conditions: (i) the remnant angular momentum finally aligned ($\pm$) with the field is *typically* the spin $s\hbar$; (ii) *each pairwise EPRB correlation* arises from the pairwise-dynamics associated with the conservation of total angular momentum in (6).

2.19. Thus, from (21), $P(B^+ \,|\, \beta A^+)$ under $\sim$ is given by Malus' Law under TLR. And Malus' Law applies to the properties of beables—ie, the polarization of a Malusian light-beam or an equivalence relation related to the angular momentum of an EPRB particle—defined to the point of adequacy, as at ¶3.6. So, using (10), (21) expands to:

$$P(B^+ \,|\, \beta A^+) \equiv P(\delta_b^{\pm} q(\boldsymbol{\mu}) \rightarrow q(b^+) \,|\, \beta, \delta_a^{\pm} q(\boldsymbol{\lambda}) \rightarrow q(a^+)) = P(\delta_b^{\pm} q(\boldsymbol{\mu}) \rightarrow q(b^+) \,|\, \beta, q(\boldsymbol{\lambda}) \sim q(a^+))$$

$$= P(\delta_b^{\pm} q(\boldsymbol{\mu}) \rightarrow q(b^+) \,|\, \beta, q(-\boldsymbol{\mu}) \sim q(a^+)) = P(\delta_b^{\pm} q(\boldsymbol{\mu} \sim a^-) \rightarrow q(b^+) \,|\, \beta) = \cos^2 s\,(a^-, b^+) = \sin^2 \tfrac{1}{2}(a, b). \quad (25)$$

$$\therefore \text{[using LHS (65)]: } \langle AB \,|\, \beta \rangle = 2P(B^+ \,|\, \beta A^+) - 1 = 2\sin^2 \tfrac{1}{2}(a, b) - 1 = -a \cdot b.\ QED. \blacksquare \quad (26)$$

2.19a. Given (25)-(26), we're in good company: "Nobody knows just where the boundary between the classical and quantum domain is situated. ... More plausible to me is that we will find that there is no boundary," Bell (2004:29-30). QM 'can be understood as a powerful extension of ordinary probability theory,' Fröhner (1998:652). "The major transformation from classical to quantum physics lies not in modifying the basic classical concepts ... but rather in the shift from intrinsic to extrinsic properties," Kochen (2015:26). But our strategy differs. Under TLR, we adequately predict the probabilities of interaction outcomes (including internal interactions in composite systems), via relevant classes of beables. Thus, from (25), interactions $\delta_b^{\pm} q(\boldsymbol{\mu} \sim a^-) \rightarrow q(b^{\pm})$ proceed probabilistically to a $\cos^2$ Malusian distribution: $q(b^+)$ proportional to $q(b^-)$ as $\cos^2 \frac{1}{2}(a^-, b^+)$ is to $\cos^2 \frac{1}{2}(a^-, b^-)$.

2.20. That is—allowing that every relevant beable here can be classified under an equivalence relation—Malus' Law applies generally. To put it another way, in Malus' 19th-century context, consider two photons: (i) under the format in (4), $(\delta_x^{\pm} q(\boldsymbol{\lambda}_j) \rightarrow q(x^+)) \equiv q(\boldsymbol{\lambda}_j \overset{\delta_x^{\pm}}{\sim} x^+)$ is a defining relation in our terms; (ii) $(\delta_x^{\pm} q(\boldsymbol{\lambda}_k = x^+) \rightarrow q(x^+))$ is our notation for $\delta_x^{\pm}$ interacting with an x^+-polarized photon in a Malusian x^+-polarized beam . We then say that (x^+) is a defining property under $\sim$. For—with P well-defined under $\sim$ from ¶2.17—they yield identical/valid results; ie, with $s = 1$ here:

$$P(\delta_a^{\pm} q(\boldsymbol{\lambda}_j \overset{\delta_x^{\pm}}{\sim} x^+) \rightarrow q(a^+)) \;=\; P(\delta_a^{\pm} q(\boldsymbol{\lambda}_k = x^+) \rightarrow q(a^+)) = \cos^2 s(a^+, x^+) = \cos^2(a, x). \quad (27)$$

"It is not easy [maybe] to identify precisely which physical processes are to be given the status of 'observations' and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision [as is our aim here] might be possible by concentration on the *be*ables, which can be described 'in classical terms', because they are there [like our $q(\boldsymbol{\lambda}_j)$, with $q(\boldsymbol{\lambda}_j \sim x^+)$ under $\delta_x^\pm$; and $q(\boldsymbol{\lambda}_k = x^+)$]. ... 'Observables' [like A_j and A_k in our notation] must be *made*, somehow, out of beables [as our results are; eg, in (27)]. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables [as TLR does]," Bell (2004:52).

2.21. Returning to the logic-flow in (17)-(24): (22) follows from (19)-(21) via Bayes' Law; which is applicable here—and thus applicable to EPR studies generally—since $A^\pm$ and $B^\pm$ are correlated via (6). The expectations in (23) follow from (22) via the definition of an expectation. Then, with (24) from (23) via the definition of the overall expectation, we have the expectation $\langle AB\,|\,\beta\rangle$. *Thus*—despite Bell's claim in the line below his 1964:(3) that (24) is impossible—*the generality of Bell's theorem is constrained by the limited generality of his inferences.* [See Appendix B for a consequential refutation of Bell's 1964 *impossibility* claim.] With ▲ denoting absurdity, the source of that 'impossibility theorem'—ie, the mathematical consequence of Bell's EPR-inspired false inference [¶¶2.8-2.9]—follows:

2.22. Under '**Contradiction**: The main result will now be proved', Bell (1964:197) takes us via his 1964:(14), direction-vector c, and three unnumbered equations—say, (14a)-(14c)—to his 1964:(15); ie:

$$|\,\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle\,| \leq 1 + \langle BC\,|\,\beta\rangle\,;\ \text{ie, using our (24): } |\,(a\cdot c) - (a\cdot b)\,| \leq 1 - (b\cdot c);\ ▲ \tag{28}$$

$$\text{ie, Bell 1964:(15) is absurd under TLR, mathematics and QM} \because |\,(a\cdot c) - (a\cdot b)\,| \leq \tfrac{3}{2} - (b\cdot c). \tag{29}$$

2.23. To pinpoint the source of this absurdity (and avoid any defective intermediaries), we now link LHS Bell 1964:(14a) directly to LHS Bell 1964:(15). Using illustrative angles, Bell's 1964:(15) allows:

$$0 \leq \langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle \leq 1 + \langle BC\,|\,\beta\rangle\,; \tag{30}$$

$$\text{ie, using our (24), } 0 \leq (a\cdot c) - (a\cdot b) \leq 1 - (b\cdot c); \tag{31}$$

$$\text{so, if } (a,b) = \tfrac{\pi}{4} \text{ and } (a,c) = (c,b) = \tfrac{\pi}{8}, \text{ then } 0 \leq 0.217 \leq 0.076 \text{ (conservatively); } ▲ \tag{32}$$

$$\text{ie, Bell 1964:(14a)} \neq \text{Bell 1964:(14b)} = \text{Bell 1964:(14c)} = \text{Bell 1964:(15)}.\ QED.\ ■ \tag{33}$$

2.24. Thus, under EPRB and TLR: Bell's theorem (and related inequalities) stem from the $\neq$ in (33); ie, they begin with Bell's move from his *valid* (14a) to his *invalid* (14b). Now, via Bell's note at 1964:(14b), we find that Bell moves from (14a) to (14b) via the generalization $(A(b,\boldsymbol{\lambda}))^2 = 1$. But if $i \neq j$, $A(b,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_j) = \pm 1$; ie, the product of uncorrelated scalars—[each of which may take the value ± 1]—is ± 1. So, as we'll show, Bell's generalization—ie, his set here of $\boldsymbol{\lambda}$ that allows $(A(b,\boldsymbol{\lambda}))^2 = 1$ to go through—is invalid under EPRB, with the following consequences: (i) absurdities—like (28), (32), (40)—flow from the likes of Bell's limiting generalization $(A(b,\boldsymbol{\lambda}))^2 = 1$; (ii) Bell's theorem is limited to systems for which his limited generalization holds; (iii) EPRB-based settings are not such systems; (iv) Bell's generalization has nothing to do with local causality; (v) based on such a constrained 'realism', Bell's ambit claims are misleading. So let's find the source of his problem:

2.25. Under TLR *we distinguish between relevant classes of beables.* Using our (3)-(7) and a particle-by-particle analysis of β: let $3n$ random particle-pairs be equally distributed over three randomized polarizer-pairings $(a,b), (b,c)$, (c,a). We allow each particle-pair to be unique, and thus uniquely indexed [$i = 1, 2, ..., 3n$] for identification purposes. [This conservative unrestricted generalization under TLR is consistent with our incomplete knowledge in (6).] Let n be such that (for convenience in presentation and to an adequate accuracy hereafter):

$$\text{Bell 1964:(14a)} = \langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i) - A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i})] \tag{34}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)[A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}) - 1] \tag{35}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)[A(b,\boldsymbol{\lambda}_i)A(c,\boldsymbol{\lambda}_i) - 1] \textbf{ (after using } \boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i} \textbf{ [sic])} = \text{Bell 1964:(14b)}: \blacktriangle \tag{36}$$

absurd, for under β, and TLR per (6): $P(\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{n+i} \,|\, \beta) << 1$. So (36) joins (28) under ▲. (37)

2.26. Thus, via $(A(b,\boldsymbol{\lambda}))^2 = 1$ at ¶2.24, Bell makes a *quantum-incompatible* move akin to using an ordered sample of n objects subject to repetitive non-destructive testing, with $\boldsymbol{\lambda}_i \equiv \boldsymbol{\lambda}_{n+i}$ per (36). Allowing that *adequate concreteness* will eliminate such absurdities, we now derive the consequences. Since the average of $|\,A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)\,|$ is ≤ 1, *valid* (35) reduces to *valid* (38):

$$\text{Bell 1964:(14a)} = |\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle| \leq 1 - \frac{1}{n}\sum_{i=1}^{n} A(a,\boldsymbol{\lambda}_i)A(b,\boldsymbol{\lambda}_i)A(a,\boldsymbol{\lambda}_{n+i})A(c,\boldsymbol{\lambda}_{n+i}). \tag{38}$$

2.27. Now, under TLR: (i) the independent and uncorrelated random variables $\boldsymbol{\lambda}_i$ and $\boldsymbol{\lambda}_{n+i}$ generate independent and uncorrelated random variables (ie, the binary outputs ± 1), per (6); (ii) the expectation over the product of two independent and uncorrelated random variables is the product of their individual expectations; (iii) so valid (38) reduces to valid (39), a mathematical fact:

$$\text{Bell 1964:(14a)} = |\,\langle AB\,|\,\beta\rangle - \langle AC\,|\,\beta\rangle\,| \leq 1 - \langle AB\,|\,\beta\rangle\langle AC\,|\,\beta\rangle \neq \text{Bell 1964:(14b)}; \tag{39}$$

ie, $|\,(a \cdot b) - (a \cdot c)\,| \leq 1 - (a \cdot b)(a \cdot c) \neq$ RHS Bell 1964:(15) unless $a = b \vee c$, which is absurd. ▲ (40)

2.28. In short: since LHS (40) is a mathematical fact, Bell's 1964:(15) is absurd and false. In passing, the CHSH (1969) inequality—eg, Peres (1995:164)—falls to a similar mathematical fact. To wit:

$$|\,(a{\cdot}b) + (b{\cdot}c) + (c{\cdot}d) - (d{\cdot}a)\,| \leq 2\sqrt{2}.\ \therefore\ |\,(a{\cdot}b) + (b{\cdot}c) + (c{\cdot}d) - (d{\cdot}a)\,| \leq 2 \text{ is absurd. } \blacktriangle \tag{41}$$

2.29. Finally, furthering our analysis, we consider experiment γ, Mermin's (1990) 3-particle variant of GHZ (1989); often regarded as the best variant of Bell's theorem. *Respectively, hereafter*: three spin-$\frac{1}{2}$ particles with properties $\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}$ emerge from an angular-momentum conserving decay such that

$$\boldsymbol{\lambda} + \boldsymbol{\mu} + \boldsymbol{\nu} = \pi.\ \therefore \boldsymbol{\nu} = \pi - \boldsymbol{\lambda} - \boldsymbol{\mu} \text{ (for convenience; the choice matters not).} \tag{42}$$

2.30. The particles separate in the y-z plane and interact with spin-$\frac{1}{2}$ polarizers that are orthogonal to the related line of flight. Let a, b, c here [nb: elsewhere, they are direction-vectors] be the angle of each polarizer's principal-axis relative to the positive x-axis; and let the equivalence relations for $\boldsymbol{\lambda}, \boldsymbol{\mu}, \boldsymbol{\nu}$ be expressed in similar terms. Finally, let the test results be A, B, C. Then, based on LHS (17)-(18) in short-form—ie, $A^+ = \cos(a, \boldsymbol{\lambda}\,|\,q(\boldsymbol{\lambda}) \sim q(a^+)) = \cos(a - \boldsymbol{\lambda}\,|\,\boldsymbol{\lambda} \sim a^+) = 1$; etc—let

$$A^+ = \cos(a - \boldsymbol{\lambda}\,|\,\boldsymbol{\lambda} \sim a) = 1;\ B^+ = \cos(b - \boldsymbol{\mu}\,|\,\boldsymbol{\mu} \sim b) = 1;\ C^+ = \cos(c - \boldsymbol{\nu}\,|\,\boldsymbol{\nu} \sim c) = 1. \tag{43}$$

2.31. Via the principles in (3)-(24)—and nothing more—we now derive $\langle ABC\,|\,\gamma\rangle$, the expectation for the Mermin/GHZ experiment γ. (Explanatory notes follow the derivation.)

$$\left\langle A^+B^+C^+\,|\,\gamma\right\rangle \equiv$$

$$P(\boldsymbol{\lambda} \sim a\,|\,\gamma)\cos(a - \boldsymbol{\lambda}\,|\,\boldsymbol{\lambda} \sim a) \cdot P(\boldsymbol{\mu} \sim b\,|\,\gamma)\cos(b - \boldsymbol{\mu}\,|\,\boldsymbol{\mu} \sim b) \cdot P(\boldsymbol{\nu} \sim c\,|\,\gamma, \boldsymbol{\lambda} \sim a, \boldsymbol{\mu} \sim b)\cos(c - \boldsymbol{\nu}\,|\,\boldsymbol{\nu} \sim c) \tag{44}$$

$$= \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot P(\boldsymbol{\nu} \sim c\,|\,\gamma, \boldsymbol{\lambda} \sim a, \boldsymbol{\mu} \sim b) = \tfrac{1}{4}P((\pi - \boldsymbol{\lambda} - \boldsymbol{\mu}) \sim c\,|\,\gamma, \boldsymbol{\lambda} \sim a, \boldsymbol{\mu} \sim b) \tag{45}$$

$$= \tfrac{1}{4}P((\pi - a - b) \sim c\,|\,\gamma) = \tfrac{1}{4}\cos^2 \tfrac{1}{2}(\pi - a - b - c) = \tfrac{1}{4}\sin^2 \tfrac{1}{2}(a + b + c). \tag{46}$$

$$\text{Similarly: } \left\langle A^+B^-C^-\,|\,\gamma\right\rangle = \left\langle A^-B^+C^-\,|\,\gamma\right\rangle = \left\langle A^-B^-C^+\,|\,\gamma\right\rangle = \tfrac{1}{4}\sin^2 \tfrac{1}{2}(a + b + c), \text{ and} \tag{47}$$

$$\left\langle A^+B^+C^-\,|\,\gamma\right\rangle = \left\langle A^+B^-C^+\,|\,\gamma\right\rangle = \left\langle A^-B^+C^+\,|\,\gamma\right\rangle = \langle A^-B^-C^-\,|\,\gamma\rangle = -\tfrac{1}{4}\cos^2\tfrac{1}{2}(a+b+c). \quad (48)$$

$$\therefore\ \langle ABC\,|\,\gamma\rangle \equiv \Sigma\,\langle A^\pm B^\pm C^\pm\,|\,\gamma\rangle = \sin^2\tfrac{1}{2}(a+b+c) - \cos^2\tfrac{1}{2}(a+b+c) = -\cos(a+b+c).\ QED.\blacksquare \quad (49)$$

2.32. Here's the logic-flow: (44) defines the required expectation. (45) follows (44) by reduction using (17)-(19). (46) follows from (45) by allocating the equivalence relations in the conditioning space to the related variables. Thus, in words, LHS (46) is one-quarter the probability that $\boldsymbol{\nu}$ — ie, $\boldsymbol{\nu} \sim (\pi - a^+ - b^+)$ — will be equivalent to c^+ under $\delta_c^\pm$. In other words: LHS (46) $= \frac{1}{4}P(\delta_c^\pm q(\boldsymbol{\nu} \sim \pi - a^+ - b^+) \to q(c^+)\,|\,\gamma) =$ RHS (46) via Malus' Law. So (46) is the three-particle variant of (23) in the two-particle EPRB experiment sketched in (3)-(6). (47)-(49) then follow naturally.

2.33. Thus, delivering Mermin's (1990:11) *crucial minus sign*, (49) is the correct result for γ: for when $(a+b+c) = 0$, $\langle ABC\,|\,\gamma\rangle = -1$. So—using TLR and our rules for physical operators and EPRB-based interactions in 3-space—we again deliver intelligible EPR/QM correlations. [nb: our use of a, b, c as the angle of a polarizer's principal-axis relative to the positive x-axis ends here.]

2.34. Via TLR's valid results for EPRB at (24), CHSH at (41), Mermin/GHZ at (49), Aspect (2004) at (68)—and such results so clearly in conflict with Bellian conclusions—we rest our case. With TLR's credentials established—contra Bell—ours is a valid general theory; eg, see how we factor (1) at ¶3.6.

3 Conclusions

3.0. TLR resolves Bell's dilemma re AAD and fulfills his hope: 'Let us hope that these analyses [local-causality 'impossibility' proofs] also may one day be illuminated, perhaps harshly, by a simple constructive model. However long that may be, long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination,' Bell (2004:167). For Bellian difficulties arise from inadequately imagining the nature of micro-reality: ie, missing true (classical/quantum) realism at ¶0.2, they champion nonlocality at ¶0.1(ix) against true (relativistic) locality [¶0.4)], ¶0.1(viii) notwithstanding.

3.1. To be clear: via the Bell-endorsed d'Espagnat-principles at ¶1.2—deriving the correct results for EPRB at (24), CHSH at (41), Mermin (1990) at (49), Aspect (2004) at (68); GHSZ and GHZ similarly—TLR resolves Bell's AAD/locality dilemma in line with his hope for a simple constructive model of EPRB. And though we reject and amend EPR's 'realism' at ¶1.5, we still justify their belief that additional variables would bring locality and causality to QM. We conclude that we rightly reject 'nonlocal' claims—like those at ¶0.1(ix)—for, as demonstrated via our simple constructive models: the world (with no quantum/classical divide) is governed by true local realism, etc.

3.2. Further, under true realism: against false Bell/d'Espagnat inferences to equality—¶¶2.8-2.10—our weaker more-general equivalence relations ($\sim$) in (10)-(11) correctly relate beables like $q(\boldsymbol{\lambda})$ to more familiar beables like $q(a^\pm)$; etc. So Bellian absurdities arise under equality relations while (as in TLR), science is hardly possible without equivalence relations under operators. Nevertheless, in and from Bellian studies—and honoring Bohr; though we learnt it from Malus's work—we conclude that Bohr's oft-neglected insight into true realism should henceforth rank equally with Einstein's well-known insight into true locality. The more so since it is this neglect that leads to the naivety of Bell's realism—¶1.1—and the rejection of locality in many Bellian studies. Here's a wiser Bell in 1989:

> "When it is said that something is 'measured' it is difficult not to think of the result as referring to some pre-existing property of the object in question. This is to disregard Bohr's insistence that in quantum phenomena the apparatus as well as the system is essentially involved. If it were not so, how could we understand, for example, that 'measurement'

of a component of ‘angular momentum’ – in an arbitrarily chosen direction – yields one of a discrete set of values? When one forgets the role of the apparatus, as the word ‘measurement’ makes all too likely, one despairs of ordinary logic – hence ‘quantum logic’. When one remembers the role of the apparatus, ordinary logic is just fine,” Bell (2004:216).

3.3. Bringing logic to Bell’s equation at (1), TLR: (i) amends EPR’s sufficient condition for a beable; (ii) corrects Bell/d’Espagnat inferences; (iii) negates the quantum/classical divide; (iv) distinguishes our approach to realism—with no hint of, nor need for, nonlocal or backwards causation.

3.4. Thus, under causal and logical independence—given the outputs A and B in (1)—we should find $\langle AB\,|\,\beta\rangle = \langle A\,|\,\beta\rangle\langle B\,|\,\beta\rangle = 0$. However,

$$\text{from (17)-(18), } \langle A\,|\,\beta\rangle = \langle B\,|\,\beta\rangle = 0; \text{but from (24), } \langle AB\,|\,\beta\rangle \neq 0: \tag{50}$$

so, with A and B causally independent (via true locality) *but* correlated, we conclude that our simple departure from Bell’s naive position—via logical implication—makes all the difference.

3.5. For—(i) replacing Bell’s (1990a:106) “*full specification* of *all* local beables in a given space-time region” (*our emphasis*) with TLR’s *adequate specification* of local micro-beables, foreshadowed at ¶1.7; (ii) given Bell’s (1990a:109) reference to logically independent correlations which permit symmetric factorizations as *locally explicable*; (iii) taking such factorizations to be a consequence of local causality and not a *formulation* thereof; (iv) and using (6) and (19)—we conclude that TLR’s adequacy goes beyond Bell to deliver a rudimentary factorization of (1); like this [but also see what follows at ¶3.6]:

$$P(A^+B^+\,|\,\beta, a, q(\boldsymbol{\lambda}{\sim}a^+), b, q(\boldsymbol{\mu}{\sim}b^+)) = P(A^+\,|\,\beta, a, q(\boldsymbol{\lambda}{\sim}a^+))P(B^+\,|\,\beta, b, q(\boldsymbol{\mu}{\sim}b^+)) = 1. \tag{51}$$

3.6. We therefore conclude that Bell’s focus on (an improbable) *full* specification (¶3.5)—in typical unrealistic HVT fashion—prevents him from deriving the result that follows next via TLR’s *adequate* specification. For, more prudent and conservative, TLR allows us to complete (1)—which is often called Bell’s locality hypothesis—via (2) like this [with $\wedge$ denoting *and*; using (19)-(22) at the end]:

$$P(A^+B^+\,|\,\beta, a, q(\boldsymbol{\lambda}), b, q(\boldsymbol{\mu})) = P(A^+\,|\,\beta, a, q(\boldsymbol{\lambda}))P(B^+\,|\,\beta, b, q(\boldsymbol{\mu})) \tag{52}$$

$$= P(q(\boldsymbol{\lambda}) \overset{\delta_a^\pm}{\sim} q(a^+)\,|\,\beta)P(q(\boldsymbol{\mu}) \overset{\delta_b^\pm}{\sim} q(b^+)\,|\,\beta) = \tfrac{1}{2}P(q(a^-) \overset{\delta_b^\pm}{\sim} q(b^+)\,|\,\beta) \wedge \tfrac{1}{2}P(q(b^-) \overset{\delta_a^\pm}{\sim} q(a^+)\,|\,\beta) \tag{53}$$

$$= \tfrac{1}{2}\sin^2\tfrac{1}{2}(a, b) = P(A^+|\beta)P(B^+|\beta A^+) \wedge P(B^+|\beta)P(A^+|\beta B^+) = P(A^+B^+\,|\,\beta).\ QED.\ \blacksquare \tag{54}$$

3.7. (51)-(54) shows that logical independence at the micro-level—in (51), with 1x1 = 1; or in (52)—may lead to Malus’ Law at the macro-level, per (54); and vice-versa. Moreover, against Aspect (2004:9 with that *hopeless* search) and Bell generally, our TLR factorings under Bayes’ Law are licensed by the experimentally-verified generality of Malus’ Law; and vice-versa: note the link between (53) and (54) under our equivalence relations. (Moreover, contra Bell and his dilemma at ¶1.6(i), TLR explains events via local interactions.) In passing: the symmetry associated with $\wedge$ in (53)-(54) shows that Alice’s factoring is—of course—similar to Bob’s. Importantly, wrt Bayes’ Law at ¶1.3: valid *equivalence relations* allow us the encode *better information* about *random beables* and their *hidden dynamics* in our probability relations; thus (52) leads to RHS (54), and vice-versa.

3.8. Per du Sautoy (2016:170), “Bell’s theorem is as mathematically robust as they come.” But Bell’s use of $[A(b, \boldsymbol{\lambda})]^2 = 1$ (see ¶2.24), renders his theorem *unphysical* under EPRB, *physically* false at (24), absurd at (28) and (32), refuted at (80), etc. For, per ¶2.25, $[A(b, \boldsymbol{\lambda})]^2 = 1$ is invalid under EPRB due to matching problems; ie, under $i \neq j$, the product of uncorrelated outcomes is: $A(b, \boldsymbol{\lambda}_i)A(b, \boldsymbol{\lambda}_j) = \pm 1$. [nb: macro-pairing (eg, $B^\pm$ with A^+ and A^- via Malus’ Law) yields valid results; see (54).] We conclude: Bayes’ Law is never false here (neither mathematically nor experimentally). We thus confirm

Bell's (1990a:106) *utmost suspicion*: he did throw the baby—baby Bayes—out with the 'macro' bath-water. For, since A and B are 'macro' and independent—but correlated per ¶3.4 and (50)—Bayes' Law (and thus Malus' Law), is central to a commonsense understanding of EPRB. And that understanding leads back from RHS (54) to (52): delivering the local explicitness that Bell sought.

3.9. Re ¶¶2.19-20, we conclude that opportunities for a wholesale reconstruction of QM remain: 'collapse' as the Bayesian updating of an equivalence class via prior correlations; 'states' as states of information about multivectors; 'measurements' as the outcomes of interactions involving physical operators; more physically-significant TLR-style approaches, like that at (56) re Pauli's vector-of-matrices. For: (i) our Lorentz-invariant analysis resolves Bell's AAD/locality dilemma; (ii) we dispense with AAD; (iii) we validate Einstein's program; (iv) we do get away with locality; (v) we thus justify Bell's motivation and validate our common enterprise; based on ¶¶1.0, 1.6-1.7.

3.10. Finally, re our position at ¶1.2—concerned re the meaning of *generic* realism; *taking QM to be better-founded than Bell imagined; correcting Bellian naiveties, puzzlements and doubts that we do not share*—we've justified our concern re the content of Bell's remarks (in Bertlmann 2017:54) at ¶1.2.

3.11. In sum, consistent with Einstein's locally-causal Lorentz-invariant worldview: (i) Bell's theorem is bypassed; (ii) its unphysical restriction—via (36)—leads to its consequent lack of generality; (iii) Bell's dilemma at ¶1.6(i) is resolved; (v) Bell's chief motivation via ¶1.6(ii) is justified; (vi) his locality -causality at ¶1.7 is developed; (vii) his questions answered via (22), (24), (33), (49), (51)-(54), etc. We thus conclude that—at peace with QM and relativity—a truly realistic account of the world beckons: TLR—true local realism—via interactions/transitions/transformations per (52)-(54), etc.

TLR: true via Bohr's insight, local via Einstein's locality, realistic via Bell's beables.

4 Acknowledgment

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5 Appendix A: A new vector product for Geometric Algebra (GA)

5.1. Our TLR analysis—via equivalence relations under orientations; consistent with EPRB, QM and experiment—resolves the Bellian dilemma defined at ¶1.6(i). So, from ¶2.2a and ¶2.6, we now show TLR's accord with QM via relations under magnitudes. To this end: (i) from Bell 1964:(1) and ¶2.1, we let the beable $\boldsymbol{\lambda}$ denote a pristine particle's total angular momentum; (ii) from ¶2.15 we have the relationships missing from Bell 1964:(1); (iii) from ¶2.34, and the likes of Aspect's experiments, such relationships are experimentally-validated; (iv) new relationships may be validated similarly.

5.2. The link between TLR and GA) follows: (i) let a_1, a_2, a_3 be a right-handed set of orthonormal basis vectors; (ii) let our $a \equiv a_3$; (iii) let a be our preferred term. As the original identifier of the principal axis of Alice's polarizer (from ¶2.1), a is the unit-vector denoting the key variable of polarizing-operator $\delta_a^\pm$ with respect to spin-$\frac{1}{2}$ particles $q(\boldsymbol{\lambda})$ under EPRB. Then, in conventional short-form notation under GA—eg, Chappell *et al* (2011:3)—with ϵ_{ijk} the Levi-Civita symbol:

$$a_i a_j = a_i \cdot a_j + a_i \wedge a_j = \delta_{ij} + \imath\epsilon_{ijk} a_k; \imath \equiv a_i a_j a_k; \imath^2 = (a_i a_j a_k)^2 = -1; a_1 a_2 = a_1 \wedge a_2 = \imath a_3. \quad (55)$$

So our real vectors satisfy the defining relation of the Pauli matrices: $\sigma_i\sigma_j = \delta_{ij} + \imath\epsilon_{ijk}\sigma_k.$ (56)

The equiprobable spin-bivectors under the interaction $\delta_a^{\pm} q(\boldsymbol{\lambda})$ are then: $\pm|\mathbf{s}|\mathrm{a}_1\mathrm{a}_2 = \pm|\mathbf{s}|\imath\mathrm{a}_3$; (57)

where the spin-vector is: $\mathbf{s} = \pm\frac{\hbar}{2}a_3 = \pm\frac{\hbar}{2}a$; where $+$ denotes spin-up wrt a; etc. (58)

5.3. Based on ¶2.18, we now represent particle/polarizer interactions by a new vector-product. Symmetrically, under the deterministic push-pull dynamics of ¶2.13, let a_- be appropriately orthogonal to a^+ as determined by the relevant spin; see (59). Then—with $\approx$ denoting equiprobability; $\oplus \equiv$ xor; a^- antiparallel to a^+; $a^{\perp}$ perpendicular to a^+—we define the spin-product $a\{s\hbar\}\boldsymbol{\lambda}$, a fair-coin:

$$a\{s\hbar\}\boldsymbol{\lambda} \approx s\hbar a_-^+ \text{: if } s = \tfrac{1}{2}, a_-^+ \approx a^+ \oplus a^-; \text{ if } s = 1, a_-^+ \approx a^+ \oplus a^{\perp}. \tag{59}$$

5.4. For digital outputs, eg Bell 1964:(1), here's the reduced spin-product $a\{s\}\boldsymbol{\lambda}$, another fair-coin:

$$a\{s\}\boldsymbol{\lambda} \approx \cos 2s(a, a_-^+) = \pm 1 = A^{\pm}; \text{ with } a_-^+ \text{ defined in (59).} \tag{60}$$

5.5. Thus, using (4) to create two examples, we have for Alice under β (where $s = \frac{1}{2}$):

$$(\Delta_a^{\pm} q(\boldsymbol{\lambda}_i) \to A(a, \boldsymbol{\lambda}_i)\,|\,\beta) = +1 \equiv A^+ = \cos 2s(a, \boldsymbol{\lambda}_i\,|\,q(\boldsymbol{\lambda}_i) \sim q(a^+)) \equiv a\{s\}\boldsymbol{\lambda}_i = a\cdot a^+ = +1; \tag{61}$$

$$(\Delta_a^{\pm} q(\boldsymbol{\lambda}_j) \to A(a, \boldsymbol{\lambda}_j)\,|\,\beta) = -1 \equiv A^- = \cos 2s(a, \boldsymbol{\lambda}_j\,|\,q(\boldsymbol{\lambda}_j) \sim q(a^-)) \equiv a\{s\}\boldsymbol{\lambda}_j = a\cdot a^- = -1. \tag{62}$$

5.6. Then, given (60), Bob's corresponding results $B^{\pm}$ are correlated with Alice's $A^{\pm}$ via (6). So, using the most basic (ie, a probability-based) definition of an expectation—eg, Whittle (1976:20)—we take the expectation $\langle X\,|\,\beta\rangle$ to be the conventional arithmetic mean of X under the conditional β:

$$\therefore\ \langle X\,|\,\beta\rangle \equiv \sum_{i=1}^{n} P_i\, x_i\ :\ \text{given } P_i \equiv P(X = x_i\,|\,\beta);\ \sum_{i=1}^{n} P_i = 1. \tag{63}$$

$$\therefore\ \langle AB\,|\,\beta\rangle = P(AB = +1\,|\,\beta) - P(AB = -1\,|\,\beta) = 2P(AB = 1\,|\,\beta) - 1 = 4P(A^+B^+\,|\,\beta) - 1 \tag{64}$$

$$= 2P(B^+\,|\,\beta A^+) - 1 = 2P(b\{s\}\boldsymbol{\mu} = 1\,|\,\beta, a\{s\}\boldsymbol{\lambda} = 1) - 1, \text{ using Bayes' Law and (60),} \tag{65}$$

$$= 2\sin^2\tfrac{1}{2}(a, b) - 1 = -a\cdot b, \text{ using Malus' Law as in (20). QED.} \blacksquare \tag{66}$$

5.7. Note that the short-form representation of the expectation on LHS (65) is our preferred format. [Earlier (per ¶1.7), to be more in line with typical Bell essays, we refrained from using it.] By way of experimental confirmation—using α—the experiment in Aspect (2004) with photons ($s = 1$):

$$\langle AB\,|\,\alpha\rangle = 2P(B^+\,|\,\alpha A^+) - 1 = 2P(b\{s\}\boldsymbol{\mu} = 1\,|\,\alpha, a\{s\}\boldsymbol{\lambda} = 1) - 1, \text{ using (65),} \tag{67}$$

$$= 2\cos^2(a, b) - 1 = \cos 2(a, b), \text{ using Malus' Law as in (27). QED.} \blacksquare \tag{68}$$

5.8. Thus, with Bayes' Law and Malus' Law to the fore here in our short-form expressions, and in the light of TLR, we now analyze Fröhner 1998:(75). There we see the inner products of the polarizer direction-vectors a and b with 'the spin $\boldsymbol{\sigma}_1 = -\boldsymbol{\sigma}_2$ taken to be an ordinary vector for which all orientations are equally probable'. Fröhner is thus able to 'equal the QM result' (in his terms):

$$\text{Fröhner 1998:(75): } \langle (a\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_2\cdot b)\rangle = -\langle (a\cdot\boldsymbol{\sigma}_1)(\boldsymbol{\sigma}_1\cdot b)\rangle = -\tfrac{\langle\sigma_1^2\rangle}{3}(a\cdot b). \tag{69}$$

5.9. Thus, in our terms, and to match Bell 1964:(1), (69) needs to be solved for:

$$(a\cdot\boldsymbol{\sigma}_1) = \pm 1; (\boldsymbol{\sigma}_2\cdot b) = \pm 1; \tfrac{\langle\sigma_1^2\rangle}{3} = 1. \tag{70}$$

5.10. Fröhner 1998:(70)-(73) does this by describing the spin-coordinates via Pauli matrices and using EPR's criterion at ¶1.4 [that we reject and amend at ¶1.5]. In that our method is coordinate-free, we now show our resolution of (69)-(70). Under TLR—using the statistical terms variance (var), covariance (cov) and statistical-correlation (cor); with $\langle A\,|\,\beta\rangle = \langle B\,|\,\beta\rangle = 0$ from (17)-(18)—we have:

$$\text{cov}\,(A, B\,|\,\beta) \equiv \langle (A - \langle A\rangle)(B - \langle B\rangle)\,|\,\beta\rangle = (AB\,|\,\beta) = -a\cdot b\text{: from (24) or (66);} \tag{71}$$

$$\text{var}\,(A\,|\,\beta) \equiv \Big\langle (A - \langle A\rangle)^2\,|\,\beta \Big\rangle = \Big\langle A^2\,|\,\beta \Big\rangle = 1; \tag{72}$$

$$\text{var}\,(B\,|\,\beta) \equiv \Big\langle (B - \langle B\rangle)^2\,|\,\beta \Big\rangle = \Big\langle B^2\,|\,\beta \Big\rangle = 1. \tag{73}$$

$$\therefore \text{cor}\,(A, B\,|\,\beta) \equiv \frac{\text{cov}\,(A, B\,|\,\beta)}{\sqrt{\text{var}\,(A\,|\,\beta)}\sqrt{\text{var}\,(B\,|\,\beta)}} = \langle AB\,|\,\beta\rangle = -a{\cdot}b.\, QED. \blacksquare \tag{74}$$

5.11. Thus, independent of (71)-(74): our spin-products in (59)-(60), with their fair-coin outputs, deliver the correct (ie, QM/TLR-compatible) results.

5.12. In relation to EPRB and Bell (1964)—more particularly to EPR-completeness at ¶1.3 and our EPR-amendment at ¶1.5—TLR leads us to conclude that $\boldsymbol{\lambda}$ represents the total angular momentum of a particle in units of $s\hbar$; ie, in units of spin (the intrinsic angular momentum). It follows that our spin-product [¶5.3] represents the reduction of $\boldsymbol{\lambda}$ and the collateral rotation of the remnant angular momentum—ie, per ¶2.18, the rotation of the irreducible spin $s\hbar$—onto a relevant axis via each particle/polarizer interaction. With $\boldsymbol{\mu}$ similarly, under its pairwise correlation with $\boldsymbol{\lambda}$ at (6): ie, via the centrality and validated generality of Bayes' Law and Malus' Law to EPRB, Aspect (2004), etc.

6 Appendix B: Bell's (1964) impossibility claim refuted

6.1. In our terms, Bell's (1964) impossibility claim—stated in the line below his 1964:(3)—is:

$$A(a, \boldsymbol{\lambda}) = \pm 1 = \cos(a, \boldsymbol{\lambda}\,|\,q(\boldsymbol{\lambda}) \sim q(a^{\pm})); B(b, \boldsymbol{\mu}) = \pm 1 = \cos(b, \boldsymbol{\mu}\,|\,q(\boldsymbol{\mu}) \sim q(b^{\pm})); \tag{75}$$

$$\boldsymbol{\lambda} + \boldsymbol{\mu} = 0;\ 0 \leq \rho(\boldsymbol{\lambda});\ \textstyle\int d\boldsymbol{\lambda}\, \rho(\boldsymbol{\lambda}) = 1 \colon \langle AB\,|\,\beta\rangle = \int d\lambda\, \rho(\boldsymbol{\lambda}) A(a, \boldsymbol{\lambda}) B(b, \boldsymbol{\mu}) \neq -a{\cdot}b. \tag{76}$$

6.2. (i) (75) follows from Bell 1964:(1) and its completion via the functions that we introduced in (17)-(18); (ii) LHS (76) follows from our specification of EPRB (β)—(3)-(6)—and Bell (1964) generally; (iii) RHS (76) is our representation of Bell's claim. Our refutation of Bell's claim follows:

$$.A_i \equiv +1 = a{\cdot}a^+ \leftarrow [a{\cdot}*] \Leftarrow q(a^+) \leftarrow \delta_a^{\pm} \Leftarrow q(\boldsymbol{\lambda}_i)\ \lhd\beta\rhd\ q(\boldsymbol{\mu}_i) \Rightarrow \delta_b^{\pm} \rightarrow q(b^+) \Rightarrow [b{\cdot}*] \rightarrow b{\cdot}b^+ = +1 \equiv B_i. \tag{77}$$

$$\langle AB\,|\,\beta\rangle = \textstyle\int d\lambda\, \rho(\boldsymbol{\lambda}) A(a, \boldsymbol{\lambda}) B(b, \boldsymbol{\mu}) = \int d\lambda\, \rho(\boldsymbol{\lambda}) \cos(a, \boldsymbol{\lambda}\,|\,q(\boldsymbol{\lambda}) \sim q(a^{\pm}) \cos(b, \boldsymbol{\mu}\,|\,q(\boldsymbol{\mu}) \sim q(b^{\pm})) \tag{78}$$

$$\begin{aligned} &= \tfrac{1}{2}(1)[P(q(\boldsymbol{\mu}) \sim q(b^+)\,|\,\beta, q(\boldsymbol{\lambda}) \sim q(a^+)) - P(q(\boldsymbol{\mu}) \sim q(b^-)\,|\,\beta, q(\boldsymbol{\lambda}) \sim q(a^+)) \\ &- \tfrac{1}{2}(1)[P(q(\boldsymbol{\mu}) \sim q(b^+)\,|\,\beta, q(\boldsymbol{\lambda}) \sim q(a^-)) - P(q(\boldsymbol{\mu}) \sim q(b^-)\,|\,\beta, q(\boldsymbol{\lambda}) \sim q(a^-)) \end{aligned} \tag{79}$$

$$= \tfrac{1}{2}[\sin^2 \tfrac{1}{2}(b^+, a^+) - \cos^2 \tfrac{1}{2}(b^-, a^+) - \cos^2 \tfrac{1}{2}(b^+, a^-) + \sin^2 \tfrac{1}{2}(b^-, a^-)] = -a{\cdot}b.\, QED\colon \blacksquare \tag{80}$$

6.3. Bell's claim, RHS (76), is refuted. In the context of our (4)—for the $i-$th particle-pair; reproduced here as (77)—and via (78), a progressive denouement from Alice's point-of-view follows: (i) integrating over the space of $\boldsymbol{\lambda}$, particles from the equivalence classes $[q(a^+)]$ and $[q(a^-)]$ in (13) interact with Alice's polarizer $\delta_a^{\pm}$ equiprobably; (ii) via the corresponding Malusian distribution (21)—see ¶¶2.16-2.17—each clearly-separated twin [a member of the opposite class] interacts with Bob's polarizer $\delta_b^{\pm}$. (iii) (79) shows the related outcomes (± 1) and probabilities delivering the QM expectation. (iv) Importantly, (78)-(80) is consistent with the most basic definition of an expectation; see ¶5.6. (v) (80) follows similarly, from Bob's point-of-view, via (20) and the particle classes $[q(b^+)]$, $[q(b^-)]$.

6.4. We conclude: Bell's defective analyses start where our valid analyses begin—Bell 1964:(14a) $\neq$ Bell 1964:(14b)—see ¶¶2.26-2.28 and (40). And we stress: under TLR, experimentally-confirmed logical implications give no license to nonlocal or backwards causation.

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