Modelling Ideal MHD as a System of Coupled Quaternionic Riccati Equations and Its Application for MHD Power Generator

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Abstract

In recent years, there are several proposals of using MHD theory for clean power generators on top of coal plant. But the theory involved appears too complicated, so in this paper we will use a simpler approach using ideal MHD equations which then they can be reduced to a system of coupled quaternionic Riccati equations. Further numerical and experimental investigations are advisable.

Key Words: ideal MHD, coupled Riccati equations, quaternionic algebra, PDE, Maxwell electromagnetic theory.

1. Introduction

Modern society requires a variety of goods and services which require energy as the diversity of range of services increases so is the demand for energy. Electrical energy because of its versatility takes major share. Coal has to be transported to thermal stations located away from coalfields by railways and power has to be transmitted over large distances from pithead stations. These problems can be eliminated or reduced by converting coal into SNG (synthetic natural gas) at pithead and transporting the gas by pipe-grid to all thermal stations. The efficiency of power station can be increased by adopting combined cycle. Eco-friendly MHD (Thermal Cell) generator is now suggested for development as a topping addition for combined cycle to further improve the efficiency.[1]

Magneto-hydrodynamic (MHD) power generation has been studied as a novel commercial power plant due to its inherent advantage of high-efficiency with high-working temperatures.

MHD equations are a set of equations to extend Maxwell equations with fluid dynamics. Analytical solution of MHD problem is difficult but remains possible to obtain, and such a solution will be discussed in section 2. In section 3, we discuss how ideal MHD reduces to a system of coupled quaternionic Riccati equations.

2. MHD equations and their analytical solution

This section presents an analytical solution obtained by Ligere et al. [3] of the MHD problem on a fully developed flow of a conducting fluid in a duct with the rectangular cross-section, located in a uniform external magnetic field, and under a slip boundary condition on side walls of the duct. The flow is driven by a constant pressure gradient. The case of perfectly conducting Hartmann walls and insulating side walls is considered. The solution is derived by using integral transforms.

The dimensionless MHD equations, describing the problem, have the form:[3]

$$\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial y^2} + 1 + Ha\frac{\partial b_x}{\partial z} = 0, \tag{1}$$

$$\frac{\partial^2 b_x}{\partial z^2} + \frac{\partial^2 b_x}{\partial y^2} + 1 + Ha\frac{\partial U}{\partial z} = 0, \tag{2}$$

Where:

$Ha = B_0 h\sqrt{\sigma / \rho\nu}$ is the Hartmann number, which characterizes the ratio of electromagnetic force to viscous force; σ, ρ, ν are the conductivity, the density and the viscosity of the fluid, respectively.

After some steps and integral transforms, the solution is given by:[3]

$$\bar{u}(\lambda_n, y) = i\big(A\cosh(k_1 y) - B\cosh(k_2 y)\big) - \frac{2(-1)^{n+1} Ha}{\lambda_n(\lambda_n^2 + Ha^2)}, \tag{3}$$

where the coefficients A and B are determined from the boundary conditions.

3. Ideal MHD reduces to a system of coupled quaternionic Riccati equations

Quaternions are 4-vectors whose multiplication rules are governed by a simple non-commutative division algebra. The concept was originally invented by Hamilton to generalize complex numbers to R^4.[4]

The purpose of this section is to show that the equations of ideal incompressible MHD have a quaternionic structure in Elsasser variables but in a non-degenerate form. The equations of ideal incompressible MHD couple the inviscid fluid to a magnetic field B:[4]

$$\frac{Du}{Dt} = B.\nabla B - \nabla p, \tag{4}$$

$$\frac{DB}{Dt} = B \cdot \nabla u, \tag{5}$$

Together with div u=0 and div B=0. It can be shown that the above system of differential equations can be reduced to a system of coupled quaternionic Riccati equations. The 4-vectors $\zeta^{\pm}$ satisfy

$$\frac{D^{\mp}\zeta^{\pm}}{Dt} + \zeta^{\pm} \otimes \zeta^{\mp} + \zeta_p^{(m)} = 0, \tag{6}$$

subject to the Poisson relation.[4]

It is worth remarking here that the above system of coupled quaternionic Riccati equations are very difficult to solve analytically, but considering our paper elsewhere suggesting that a system of coupled Riccati equations can be solved numerically using computer algebra packages such as Mathematica [6] Therefore, we are optimistic that it is also possible to find numerical solution of equation (6), although it will be much more complicated than what we obtained in [6]. Nonetheless, we do not discuss it yet in this paper.

Concluding remarks

In recent years, there are several proposals of using MHD theory for clean power generators on top of coal plant. But the theory involved appears too complicated, so in this paper we will use a simpler approach using ideal MHD equations which then they can be reduced to a system of coupled quaternionic Riccati equations.
It is worth remarking here that the above system of coupled quaternionic Riccati equations are very difficult to solve analytically, but considering our paper elsewhere suggesting that a system of coupled Riccati equations can be solved numerically using computer algebra packages such as Mathematica, then we are optimistic that it is also possible to find numerical solution of equation (6). Nonetheless, we do not discuss it yet in this paper.
Further numerical and experimental investigations are advisable.

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