An Interpretation for Quantum Mechanics

Tejas A. Chaudhari, University of Self Learning†

Email id: *tejastalk@gmail.com*
Linked In: *www.linkedin.com/in/tejas-chaudhari-b302b9110/*
Facebook: *www.facebook.com/tejas1804*

***Abstract*: This paper gives interpretation of Quantum Mechanics (QM) by redefining the theory using 3 new postulates. The first of these postulates specifies the underlying structure that every massive fundamental particle must possess. The mass-Energy equivalence and wave nature of matter emerge as a direct consequence. The second postulate describes the quantum state of particles. Wave function, its conjugate, Born interpretation and the Energy-momentum operators can be derived from these two postulates. The third postulate describes the effect of measurement and interaction on the wave function. The equations of QM starting from Schrödinger's equation are described. The phenomenon of Quantum entanglement and Schrödinger's cat thought experiment are described under this interpretation. Finally, the origin of spin resulting from the first postulate is discussed.**

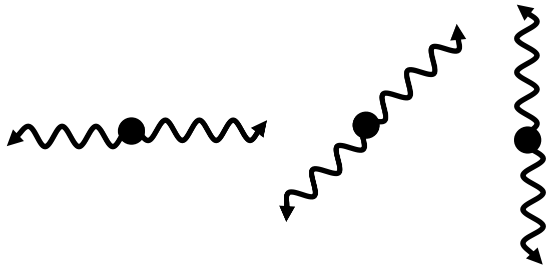
I. FIRST POSTULATE

The first postulate is the most consequential of the three and forms the backbone of this interpretation. It implies that the fundamental particles having mass have an internal structure. The statement of the postulate is given as

"Every massive fundamental particle is made up of vibrating pair/(s) of massless fundamental particles such that in each pair the massless particles have equal frequency but are vibrating in opposite direction when viewed from the classical state of rest frame. "

The massless fundamental particles should **at least** have the following properties.

i. The energy of massless particle must be given by $E = hf$ where h is the Planck's constant and f is the frequency of vibration.
ii. The momentum of the particles must be given by $\vec{p} = h/\lambda$ where λ is the wavelength.
iii. $c = \lambda f$ where c is the speed of light in vacuum



(a) (b) (c)

Fig 1(a),(b),(c) Illustrations of underlying structure of a massive fundamental particle made up of a single pair of massless fundamental particles as seen in classical state of rest frame. The three diagrams are shown to indicate that there is no preferred direction for the vibration.

Besides these, the massless particles can also have additional properties like spin etc. How the spin of massive particle is generated from the spin of its massless fundamental particles is described in the last section of the paper. Also, the pairing up of the massless particles can generate internal properties such as charge etc. Different pairs of the massless particles are separated by these different internal properties. Therefore, although a single pair can be split into many pairs, they in fact belong to the same pair as they have the same internal properties. Figure 1 gives the illustration of a particle made up of single pair of massless fundamental particle. The figure is just an illustration because the postulate does not concern itself with how the pairing occurs or the other details involved.

The classical state mentioned in definition of the postulate refers to the state in which particle's position, momentum and energy are accurately defined using a particular frame of reference. In QM, however a particle exits in what is called a quantum state. The quantum state of a particle is defined in the second postulate. The best example of massless fundamental particle is a photon. Therefore a pair of photons can make a massive particle under this postulate. Indeed the electron under this interpretation can be taken to be made up of a single pair of photons. A single pair because the spin of an electron is $\hbar/2$ (last section). Thus an electron can

†The author does not have any degree in the field of physics

emit a virtual photon because it itself is made up of pair of photons. Thus this postulate has fundamental consequences for the structure of matter itself.

However, the main reason for the proposition of this postulate is because the massive particle thus composed obeys the energy, momentum equations of the Einstein's special theory of relativity and mass comes out as an emergent property. To see this, consider a particle made up of single pair of photons such as in Fig 1. The analysis for particle made up of multiple pairs can be done similarly.

In the rest frame, the massless particles are vibrating in opposite direction. So there is no net momentum and particle is at rest. But the massless particles have same frequency, therefore the rest energy of particle is $E = 2hf_o$. Now let's consider the same particle in a frame of reference such that it has a velocity $\vec{v}$ to the right. Let θ_o and θ_v be the angles for each photon to the direction of $\vec{v}$ in the rest frame and the moving frame resp. Three cases arise which can be discussed.

Case I: The angle θ_o for the photons is 0 and π. Therefore in the moving frame one photon will be blue shifted and other will be red shifted by the Doppler Effect for light. The Doppler shift in frequency for the photon is given by

$$f_v = \frac{f_o}{\gamma\left(1-\frac{v}{c}cos\theta_v\right)} \qquad (1.1)$$

And θ_o and θ_v are related by

$$cos\theta_v = \frac{cos\theta_o+\frac{v}{c}}{1+\frac{v}{c}cos\theta_o} \qquad (1.2)$$

Where $\gamma = 1/\sqrt{1-\frac{v^2}{c^2}}$. Therefore the frequency of blue shifted photon and red shifted will be

$$f_v = \sqrt{\frac{c+v}{c-v}}\, f_o \text{ and } f_v = \sqrt{\frac{c-v}{c+v}}\, f_o \text{ resp.} \qquad (1.3)$$

Therefore, the energy of particle will be

$$E = \sqrt{\frac{c+v}{c-v}}\, hf_o + \sqrt{\frac{c-v}{c+v}}\, hf_o$$

$$= \gamma 2hf_o \qquad (1.4)$$

The momentum of the particle in the direction of $\vec{v}$ is

$$p = \sqrt{\frac{c+v}{c-v}}\,\frac{hf_o}{c} - \sqrt{\frac{c-v}{c+v}}\,\frac{hf_o}{c}$$

$$= \gamma\, 2hf_o v/c^2 \qquad (1.5)$$

The quantity $2hf_o/c^2$ has the dimension of mass. $2hf_o$ is the rest energy of the particle. Therefore by defining the rest mass as

$$m = 2hf_o/c^2 \qquad (1.6)$$

The energy of the particle (1.4) becomes $E = \gamma mc^2$ and the momentum of particle becomes $p = \gamma mv$ thus satisfying the energy momentum formulas of the special relativity

Case II: The angle θ_o for the photons is $\pi/2$ and $-\pi/2$. By using (1.2), we get for both photons $cos\theta_v = v/c$. Therefore the Doppler shift in frequency for both photons is given by

$$f_v = \frac{f_o}{\gamma\left(1-\frac{v^2}{c^2}\right)}$$

$$= \gamma f_o \qquad (1.7)$$

Therefore the energy of particle is given by

$E = \gamma 2hf_o = \gamma mc^2$ and the momentum of particle in the direction of $\vec{v}$ is given as

$$p = \gamma\frac{hf_o}{c}\frac{v}{c} + \gamma\frac{hf_o}{c}\frac{v}{c} = \gamma\frac{2hf_o v}{c^2} = \gamma mv \qquad (1.8)$$

Case III: The general case. The angle θ_o for the photons is θ_o and $\theta_o+\pi$ .Using (1.1) and (1.2), the Doppler shift in frequency is given by

$$f_v = \gamma f_o(1+\tfrac{v}{c}cos\theta_o) \qquad (1.9)$$

Therefore, the energy of particle will be

$$E = \gamma hf_o\left(1+\frac{v}{c}cos\theta_o\right) + \gamma hf_o\left(1+\frac{v}{c}cos(\theta_o+\pi)\right) = \gamma 2hf_o = \gamma mc^2 \qquad (1.10)$$

In the last step we made use of (1.6). The momentum for single photon in moving frame will be given by

$$p_{photon} = \frac{\gamma hf_o}{c}\left(1+\frac{v}{c}cos\theta_o\right) cos\theta_v \qquad (1.11)$$

By using (1.2) this becomes

$$p_{photon} = \frac{\gamma hf_o}{c}\left(\frac{v}{c}+cos\theta_o\right) \qquad (1.12)$$

Therefore, the momentum of particle will be given by

$$p = \frac{\gamma hf_o}{c}\left(\frac{v}{c}+cos\theta_o\right) + \frac{\gamma hf_o}{c}\left(\frac{v}{c}+\cos(\theta_o+\pi)\right)$$

$$= \frac{\gamma 2hf_o v}{c^2} = \gamma mv \qquad (1.13)$$

II. WAVE NATURE OF MATTER

The de Broglie hypothesis of the wave nature of matter can also be justified with the help of first postulate. As the massive fundamental particle itself is made up of pairs of massless fundamental particles having frequency and wavelength such as photons, the particle itself will also have frequency and wavelength as a result of it. Therefore when the particle is viewed in a classical state, the energy of the massive particle is given by

$$E = \sum_{pairs}(hf_{1pair} + hf_{2pair}) \quad (2.1)$$

Where f_{1pair} and f_{2pair} are the frequencies of the two massless fundamental particles in a pair and the summation is carried out for all the pairs in the particle. Therefore, the particle has an effective frequency f_{matter} such that the energy of particle

$$E = hf_{matter} \quad (2.2)$$

and $f_{matter} = \sum_{pairs}(f_{1pair} + f_{2pair})$ (2.3)

Similarly, the momentum of the particle is

$$\vec{p} = \sum_{pairs}(h/\vec{\lambda}_{1pair} + h/\vec{\lambda}_{2pair}) \quad (2.4)$$

$\vec{\lambda}_{1pair}$ and $\vec{\lambda}_{2pair}$ are the wavelengths of the two massless fundamental particles in a pair. So in terms of an effective wavelength $\vec{\lambda}_{matter}$, the momentum is

$$\vec{p} = h/\vec{\lambda}_{matter} \quad (2.5)$$

$$\frac{1}{\vec{\lambda}_{matter}} = \sum_{pairs}\left(\frac{1}{\vec{\lambda}_{1pair}} + \frac{1}{\vec{\lambda}_{2pair}}\right) \quad (2.6)$$

III. SECOND POSTULATE

In QM, a particle exists in what is known as a quantum state rather than a classical state. So it is important to understand what exactly a quantum state is and it's relation to the classical states so as to be able to define the concepts like the wave function used to describe particles in QM. The second postulate does exactly that. The statement of the postulate can be given as

"A quantum state is a collection of classical states with the probability attached to each classical state of the quantum state actually being that classical state."

A mathematical quantity that describes the quantum state is known as the wave function. Using first and second postulate we can define a wave function, its conjugate, Born interpretation and the operators of energy momentum. To do so, let's first imagine a function $\phi(\boldsymbol{x}, t)$ that describes a classical state and also the probability associated with it. In order to describe probability, $\phi(\boldsymbol{x}, t)$ need not give probability but a quantity related to it. As it will turn out, it will be the probability amplitude. Therefore $\phi(\boldsymbol{x_a}, t_a)$ describes that at time t_a the particle is at position $\boldsymbol{x_a}$ and also a measure related to the probability of classical state. Now let that classical state evolve into $\phi(\boldsymbol{x_b}, t_b)$ at time t_b. Now $\phi(\boldsymbol{x}, t)$ must satisfy certain conditions

i) $\phi(\boldsymbol{x}, t)$ must be periodic as the particle has a frequency associated with it

ii) $\phi(\boldsymbol{x}, t)$ must not depend upon the internal structure of the particle i.e. the number of pairs of massless fundamental particles contained in the particle.

iii) The evolution from $\phi(\boldsymbol{x_a}, t_a)$ to $\phi(\boldsymbol{x_b}, t_b)$ must be dependent on the path taken by the particle from $\boldsymbol{x_a}$ to $\boldsymbol{x_b}$.

iv) It must take into account Lorentz transformations so that it can be described in any inertial frame of reference.

In order to satisfy above conditions we can describe the evolution of $\phi(\boldsymbol{x_a}, t_a)$ into $\phi(\boldsymbol{x_b}, t_b)$ as

$$\phi(\boldsymbol{x_b}, t_b) = e^{\frac{i\int_{\boldsymbol{x_a},t_a}^{\boldsymbol{x_b},t_b} p_u dx^u}{\hbar}} \phi(\boldsymbol{x_a}, t_a) \quad (3.1)$$

Where the integral is carried out on the path traversed from $(\boldsymbol{x_a}, t_a)$ to$(\boldsymbol{x_b}, t_b)$. p^μ is energy momentum four vector $\{E/c, p_x, p_y, p_z\}$ and x^μ is position four vector $\{ct, x, y, z\}$. Two choices of metric signature (+,-,-,-) and (-,+,+,+) are available for use. The theory must be independent of a particular choice of metric signature. Let's assign a metric signature of (-,+,+,+) to $\phi(\boldsymbol{x}, t)$. Then by (3.1) we can see that if we use the other metric signature, the function associated with it will be complex conjugate i.e. $\phi^*(\boldsymbol{x}, t)$. In order to derive energy momentum operators we observe,

$$\phi(\boldsymbol{x} + \boldsymbol{\delta x}, t + \delta t) = e^{\frac{i p_\mu \delta x^\mu}{\hbar}} \phi(\boldsymbol{x}, t) \quad (3.2)$$

$$\therefore\ \phi(\boldsymbol{x}, t) + \sum_{k=1}^{3} \delta x^k \frac{\partial \phi(\boldsymbol{x},t)}{\partial x^k} + \delta t \frac{\partial \phi(\boldsymbol{x},t)}{\partial t} = e^{\frac{i p_\mu \delta x^\mu}{\hbar}} \phi(\boldsymbol{x}, t) \quad (3.3)$$

repeated Greek indices are summed from 0 to 3 as per Einstein summation. Differentiating (3.3) by δx^k or δt we get,

$$\frac{\partial \phi(\boldsymbol{x}, t)}{\partial x^k} = \frac{i}{\hbar} p^k \phi(\boldsymbol{x}, t) \ and$$

$$\frac{\partial\phi(\boldsymbol{x},t)}{\partial t} = \frac{-i}{\hbar}E\phi(\boldsymbol{x},t)$$

$$\therefore p^k\phi(\boldsymbol{x},t) = -i\hbar\frac{\partial\phi(\boldsymbol{x},t)}{\partial x^k}\ and$$

$$E\phi(\boldsymbol{x},t) = i\hbar\frac{\partial\phi(\boldsymbol{x},t)}{\partial t} \tag{3.4}$$

If we use $\phi^*(\boldsymbol{x},t)$, we get

$$p^k\phi^*(\boldsymbol{x},t) = i\hbar\frac{\partial\phi^*(\boldsymbol{x},t)}{\partial x^k}\ and$$

$$E\phi^*(\boldsymbol{x},t) = -i\hbar\frac{\partial\phi^*(\boldsymbol{x},t)}{\partial t} \tag{3.5}$$

By using the second postulate we can define the wave function $\psi(\boldsymbol{x},t)$ associated with the quantum state as

$$\psi(\boldsymbol{x},t) = \sum\nolimits_{classical\ states}\phi(\boldsymbol{x},t) \tag{3.6}$$

and complex conjugate of wave function $\psi^*(\boldsymbol{x},t)$ as

$$\psi^*(\boldsymbol{x},t) = \sum\nolimits_{classical\ states}\phi^*(\boldsymbol{x},t) \tag{3.7}$$

The summation is carried out for all classical states in which the particle is at position $\boldsymbol{x}$ at time t. For multiple component wave function such as spinor etc. the complex conjugate of wave function is replaced by its Hermitian conjugate $\psi^\dagger(\boldsymbol{x},t)$. By (3.4) we can conclude that the operator for momentum is

$$\hat{p}^k = -i\hbar\frac{\partial}{\partial x^k} \tag{3.8}$$

and operator for energy is

$$\hat{E} = i\hbar\frac{\partial}{\partial t}. \tag{3.9}$$

If we had associated metric signature (+,-,-,-) with $\phi(\boldsymbol{x},t)$ instead, we would have had operator for momentum as $\hat{p}^k = i\hbar\frac{\partial}{\partial x^k}$ and operator for energy as $\hat{E} = -i\hbar\frac{\partial}{\partial t}$. Both the choices for operators are equivalent but we will use the first one as it has been used conventionally. We also observe from (3.4), (3.8) & (3.9) that operators operating on classical state function give eigenvalue as the result with the classical state function being the eigenfunction. The eigenvalue obtained has to be real because it is associated with a physical quantity such as momentum or energy. We also notice from (3.4) and (3.5) that, for an operator $\hat{A}$

$$\phi^*(\boldsymbol{x},t)\big(\hat{A}\phi(\boldsymbol{x},t)\big) = \left(\hat{A}^*\phi^*(\boldsymbol{x},t)\right)\phi(\boldsymbol{x},t) \tag{3.10}$$

$$\therefore \sum_{classical\ states}\phi^*(\boldsymbol{x},t)\big(\hat{A}\phi(\boldsymbol{x},t)\big) = \sum_{classical\ states}\left(\hat{A}^*\phi^*(\boldsymbol{x},t)\right)\phi(\boldsymbol{x},t)$$

$$\therefore\ \psi^*(\boldsymbol{x},t)\left(\hat{A}\psi(\boldsymbol{x},t)\right) = \left(\hat{A}^*\psi^*(\boldsymbol{x},t)\right)\psi(\boldsymbol{x},t) \tag{3.11}$$

The summation is carried out for all classical states in which the particle is at position $\boldsymbol{x}$ at time t. Again for a multicomponent wave function $\psi(\boldsymbol{x},t)$, (3.11) becomes

$$\psi^\dagger(\boldsymbol{x},t)\big(\hat{A}\psi(\boldsymbol{x},t)\big) = \left(\psi^\dagger(\boldsymbol{x},t)\hat{A}^\dagger\right)\psi(\boldsymbol{x},t) \tag{3.12}$$

Putting (3.12) in modified bra-ket notation

$$< \psi(\boldsymbol{x},t)|\hat{A}\psi(\boldsymbol{x},t) >=< \hat{A}\psi(\boldsymbol{x},t)|\psi(\boldsymbol{x},t) > \tag{3.13}$$

Thus the operators must be self adjoint. For operators being matrices of finite dimensions, this reduces to the condition that operators must be Hermitian. The average value for an operator operating on wave function will be the summation of all eigenvalues for all classical states throughout the space with probability density for each classical state. Probability density because the summation is carried out for all points in the space. The probability density at any point $(\boldsymbol{x},t)$ has to be positive, therefore cannot be given by $\psi(\boldsymbol{x},t)\, or\, \psi^\dagger(\boldsymbol{x},t)$. But the quantity $\psi^\dagger(\boldsymbol{x},t)\,\psi(\boldsymbol{x},t)$ is positive for every point in space and therefore must be the probability density of finding particle at $(\boldsymbol{x},t)$. Therefore the average value for an operator $\hat{A}$ is given as

$$\int\psi^\dagger(\boldsymbol{x},t)(\hat{A}\,\psi(\boldsymbol{x},t))d^3x \equiv \int(\psi^\dagger(\boldsymbol{x},t)\hat{A}^\dagger)\,\psi(\boldsymbol{x},t)d^3x \tag{3.14}$$

with probability of finding particle in all of space 1.

$$\int\psi^\dagger(\boldsymbol{x},t)\psi(\boldsymbol{x},t)d^3x = 1 \tag{3.15}$$

Thus the Born probability interpretation is also obtained. In the presence of electromagnetic field, the momentum of the particle p_i in (3.1) must be replaced by canonical conjugate momentum $p_i + \frac{q}{c}A_i(\boldsymbol{x},t)$ and correspondingly p_0 must be replaced by $p_0 + \frac{q}{c}V(\boldsymbol{x},\mathrm{t})$. Therefore, in the presence of EM field, (3.1) becomes

$$\phi(\boldsymbol{x_b}, t_b) = e^{\frac{i\int_{x_a,t_a}^{x_b,t_b}(p_u+\frac{q}{c}A_u)dx^u}{\hbar}}\,\phi(\boldsymbol{x_a}, t_a) \tag{3.16}$$

where $A^\mu = \{V, A_x, A_y, A_z\}$. Therefore, in such a case the operator for momentum of the particle is

$$\hat{p}^k - \frac{q}{c}A^k = -i\hbar\frac{\partial}{\partial x^k} - \frac{q}{c}A^k(\boldsymbol{x}, t) \tag{3.17}$$

and the operator for energy of the particle becomes

$$\hat{E} - qV = i\hbar\frac{\partial}{\partial t} - qV(\boldsymbol{x}, t) \tag{3.18}$$

In (3.16), if $\int(p_u + \frac{q}{c}A_u)dx^u = nh$, then in such a case $\phi(\boldsymbol{x_b}, t_b) = \phi(\boldsymbol{x_a}, t_a)$. Here n can be any integer. This result may be used to find connection between the old quantum theory and modern quantum mechanics. Further analysis is needed in this regard.

IV. EQUATIONS OF QM

Before discussing the third postulate, the equations of QM are discussed in this section starting from the Schrödinger's equation. Taking (3.17) and (3.18) into account, the Schrödinger's equation becomes in operator form

$$\left[\frac{1}{2m}\left(\hat{p} - \frac{q}{c}\hat{A}\right)^2 + q\hat{V}\right]\psi = \hat{E}\psi \tag{4.1}$$

Explicitly writing out the operators, the equation becomes

$$\left[\frac{1}{2m}\left(-i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x}, t)\right)^2 + qV(\boldsymbol{x}, t)\right]\psi(\boldsymbol{x}, t) = i\hbar\frac{\partial}{\partial t}\,\psi(\boldsymbol{x}, t) \tag{4.2}$$

The equation gives out the classical relation of kinetic energy plus the potential energy is equal to the total energy of the particle in classical state. This relation is satisfied for every classical state at any point in space and therefore for the wave function $\psi(\boldsymbol{x}, t)$ as well using the second postulate. When solving this equation for an electron in the hydrogen atom, one must keep in mind that the vector potential $A(\boldsymbol{x}, t)$ cannot be ignored. This will become more apparent later when relativistic spin ½ equation other than the Dirac equation is discussed. Therefore, using the reduced mass for the electron and neglecting the vector potential $A(\boldsymbol{x}, t)$ for solving the hydrogen atom is an approximation at best. The Schrödinger's equation does not take into account the spin of the electron or special relativity. The equation that takes into account the spin of electron while not taking into account special relativity is the Pauli equation which is given in operator form as

$$\left[\frac{1}{2m}\left(\boldsymbol{\sigma}.\left(\hat{p} - \frac{q}{c}\hat{A}\right)\right)^2 + q\hat{V}\right]\psi = \hat{E}\psi \tag{4.3}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices. Explicitly writing out the operators, the equation becomes

$$\left[\frac{1}{2m}\left(\boldsymbol{\sigma}.\left(-i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x}, t)\right)\right)^2 + qV(\boldsymbol{x}, t)\right]\psi(\boldsymbol{x}, t) = i\hbar\frac{\partial}{\partial t}\,\psi(\boldsymbol{x}, t) \tag{4.4}$$

Here $\psi(\boldsymbol{x}, t)$ has two components and hence is a spinor. The equation works because for every classical state $\boldsymbol{\sigma}.\left(i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x}, t)\right)$ gives magnitude of momentum along with + or – sign depending upon the spin in the direction of momentum. This is squared and thus gives again the classical relation of kinetic energy + potential energy = total energy for the classical state. This is valid for every classical state and thus for the wave function $\psi(\boldsymbol{x}, t)$. Again while solving this equation for an electron in hydrogen atom, the vector potential $A(\boldsymbol{x}, t)$ cannot be ignored.

The equation that takes into account special relativity but not the spin of an electron is the Klein-Gordon equation which in operator form is

$$\left[\left(\hat{E} - q\hat{V}\right)^2 - \left(\hat{p} - \frac{q}{c}\hat{A}\right)^2 c^2\right]\psi = m^2c^4\psi \tag{4.5}$$

Explicitly writing out the operators, the equation becomes

$$\left[\frac{1}{c^2}\left(i\hbar\frac{\partial}{\partial t} - qV(\boldsymbol{x}, t)\right)^2 - \left(-i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x}, t)\right)^2\right]\psi(\boldsymbol{x}, t) = m^2c^2\,\psi(\boldsymbol{x}, t) \tag{4.6}$$

The equation gives out the relation of special relativity $E^2 - p^2c^2 = m^2c^4$ for every classical state and thus is valid for the wave function as well. The equation does not take into account the spin of electron. Building upon these equations we can describe an equation which takes into account, the spin of electron and special relativity. This equation is different from the Dirac equation and in operator form can be given as

$$\left[\left(\hat{E} - q\hat{V}\right)^2 - \left(c\boldsymbol{\sigma}.\left(\hat{p} - \frac{q}{c}\hat{A}\right)\right)^2\right]\psi = m^2c^4\psi \tag{4.7}$$

Explicitly writing out the operators, the equation becomes

$$\left[\frac{1}{c^2}\left(i\hbar\frac{\partial}{\partial t} - qV(\boldsymbol{x},t)\right)^2 - \left(\boldsymbol{\sigma}.\left(-i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x},t)\right)\right)^2\right]\psi(\boldsymbol{x},t) = m^2c^2\,\psi(\boldsymbol{x},t) \tag{4.8}$$

Here $\psi(\boldsymbol{x},t)$ has two components and hence is a spinor. The equation works because for every classical state $\boldsymbol{\sigma}.\left(i\hbar\nabla - \frac{q}{c}A(\boldsymbol{x},t)\right)$ gives magnitude of momentum along with + or – sign depending upon the spin in the direction of momentum. This is squared and thus again the relation $E^2 - p^2c^2 = m^2c^4$ is obtained for every classical state and thus is valid for the wave function $\psi(\boldsymbol{x},t)$. Here it is extremely important to note that while solving this equation for an electron in hydrogen atom, the vector potential $A(\boldsymbol{x},t)$ cannot be ignored and it is obvious that ignoring it will not yield the fine structure of hydrogen atom.

V. THIRD POSTULATE

The third postulate deals with the change in quantum state of the particle as a result of an external interaction with the particle. External interaction is a broad term and measurements made on the particle are in fact a subset of this term. The statement of the postulate is given as

"Any external interaction with the particle changes instantaneously its quantum state by addition or removal of its classical states and then by redistributing the probabilities to the classical states based on boundary conditions."

Thus all the three postulates necessary for the interpretation have been defined. A measurement carried out on the quantum system can be regarded as a type of external interaction usually associated with reduction in the number of classical states so as to yield a certain value for physical quantity associated with the measurement. This interpretation does away with the concept of objective reality which is central to the Copenhagen interpretation and thus gives a rational and consistent explanation for phenomenon related to QM. This will become clearer, when the phenomenon of Quantum Entanglement and the thought experiment of Schrödinger's cat are discussed under this interpretation. The third postulate does not violate the special theory of relativity because the classical states of a quantum state follow the rules of special relativity. In the classical state, the particle cannot travel faster than light. However there is no limitation by the special theory of relativity that the quantum state cannot add or remove classical states instantaneously in case of an external interaction and then redistribute the probabilities among the classical states.

A. Quantum Entanglement

The phenomenon of entanglement of spin of the two particles such that, the spin of the particles when measured in a particular direction have opposite values is discussed here under this interpretation. Let each of the particle have two possible spin states up $|\uparrow\,>$ and down $|\downarrow\,>$. Now after the entanglement of the two particles, the possible observable states are spin up for the first particle and spin down for the second $|\uparrow,\downarrow\,>$ and vice versa $|\downarrow,\uparrow\,>$. When the spin of the particles is not measured, the quantum state of the two particles contain two classical state $|\uparrow,\downarrow\,>$ and $|\downarrow,\uparrow\,>$ each having ½ probability. When the measurement of spin is carried out on let's say the first particle, then by the third postulate, only that classical state remains in which the second particle has spin opposite to the measured spin of the first particle. The other classical state is removed immediately and the probability of one gets assigned to the remaining classical state. Therefore, if the first particle is measured out to be $|\uparrow\,>$, then the quantum state immediately becomes $|\uparrow,\downarrow\,>$ and if the first particle is measured to be $|\downarrow\,>$, then the quantum state immediately becomes $|\downarrow,\uparrow\,>$. Thus the phenomenon of entanglement in which the measurement of spin of one particle causes the spin of the other particle to be in opposite direction instantaneously is explained. The particles may be separated by large distances. This does not change the outcome.

B. Schrödinger's cat

Schrödinger's cat is a thought experiment in which a cat, a radioactive source and a vial of poison are placed inside a closed box. Also present is a detector which detects radioactivity and a hammer. So even if a single nucleus decays, the detector detects it, causing the hammer to break the vial, thus releasing the poison and killing the cat. Let's say over the period of an hour, there is a ½ probability that one of the atoms decays and therefore ½ that no decay takes place. Thus under the Copenhagen interpretation when the box is closed, the radioactive source exits in a superposition of two states, one in which decay takes place and other in which no decay takes place. Therefore the cat exits in a superposition state of living and dead when the box is closed. Only when the box is opened, the wave function of radioactive source collapses and the cat becomes alive or dead. This is a paradox as the cat cannot be simultaneously alive and dead at the same time inside the box. This paradox is a result of objective reality of the Copenhagen interpretation in which an observation causes the wave function to collapse. The interpretation given in this paper does away with the concept of objective reality. Under this interpretation, a radioactive source is indeed initially in a quantum state with two classical states, one in which decay takes place and other in which no decay takes place. However, the radioactive detector plays the role of an external interaction on the quantum state. The radioactive detector is continuously detecting for the decay. So at any instant, when the detector does not detect a decay, then by the third postulate the classical state in which decay takes place is removed from quantum state and the cat remains alive for that instant. Due to the radioactivity of the source, again in the next instant the quantum state for the source starts as superposition of two classical states until again being modified by the interaction with the detector. Therefore, if a decay is detected by the detector then by the third postulate, the classical state in which no decay takes place is removed immediately from the quantum state. The cat dies in this scenario and exists only as a dead cat. Otherwise, the cat remains alive. Thus, when the box is opened, the observer observes the dead cat if the cat is dead or alive cat if the cat is alive. The outcome does not depend upon the act of conscious observation by the observer, thus eliminating the concept of objective reality. Before opening the box, it can be said that there is a ½ probability for cat being dead or cat being alive. But the cat is never in superposition of being alive and dead at the same time because of the interaction with radioactive detector changing the quantum state of the radioactive source continuously and thus resolving the paradox.

VI. SPIN

This section establishes a procedure by which a massive fundamental particle composed of pairs of massless fundamental particles according to the first postulate acquires the value of its spin. For the massive particle to have spin, its constituent massless particles must have their own spin. In the discussion carried out in this section, we will use photons having spin of $\hbar$ for the massless particles. First, the resulting spin states with their values have to be calculated for each pair of photons in the massive particle. Each photon has a spin of $+\hbar$ represented here with $\hbar|\uparrow>$ or $-\hbar$ represented here with $\hbar|\downarrow>$ in the direction of their momentum. Each spin state has probability ½ with it. For a pair of photons, the resulting spin states are given as

i) $\frac{1}{4}.(\hbar|\uparrow> \oplus \hbar|\uparrow>) = \hbar/2\,|\uparrow>$. The probability of both photons having spin $+\hbar$ multiplied with the special addition of those two states. This results in state having spin $+\hbar/2$.

ii) $\frac{1}{4}.(\hbar|\downarrow> \oplus \hbar|\downarrow>) = \hbar/2\,|\downarrow>$. The probability of both photons having spin $-\hbar$ multiplied with the addition of those two states. This results in state having spin $-\hbar/2$

iii) The null state produced by two other possibilities is not counted as a state. $\frac{1}{4}.(\hbar|\uparrow> \oplus \hbar|\downarrow>) = 0$ and $\frac{1}{4}.(\hbar|\downarrow> \oplus \hbar|\uparrow>) = 0$.

Thus only two resulting states are produced + $\hbar/2$ and $-\hbar/2$.The photon pair can also have entanglement condition in which spin of both photons is always opposite. The resulting pair will have no spin state because

$\frac{1}{2}.(\hbar|\uparrow> \oplus \hbar|\downarrow>) = 0$ & $\frac{1}{2}.(\hbar|\downarrow> \oplus \hbar|\uparrow>) = 0$. The photon pair can also have entanglement condition in which spin of both photons is always same. The resulting pair will have two spin states

i) $\frac{1}{2}.(\hbar|\uparrow> \oplus \hbar|\uparrow>) = \hbar\,|\uparrow>$. A spin state of $+\hbar$

ii) $\frac{1}{2}.(\hbar|\downarrow> \oplus \hbar|\downarrow>) = \hbar\,|\downarrow>$. A spin state of $-\hbar$

Thus the resulting spin states for each photon pair are calculated. Then the vector addition is carried out for spin states of different pairs taking one spin state from each pair. The resulting spin states from all the possible combinations are the spin states of the massive particle. As vector addition is carried out, the zero spin state $|0>$ is also included here. Then equal probability is assigned to each resulting spin state of the particle, resulting in the final spin states along with their probability of the particle.

For example, a particle with single pair with no entanglement condition will have resulting states $+\hbar/2$ and $-\hbar/2$. As there is no other pair, the states are assigned probability of 1/2 each and particle also has spin states $+\hbar/2$ and $-\hbar/2$ with probability of 1/2 each. For a particle with two pairs each having spin states $+\hbar/2$ and $-\hbar/2$, the resulting spin states for the particle will be $+\hbar$, 0, $-\hbar$ with probability 1/3 for each. For a particle with single pair of photons with entanglement condition with states $+\hbar$ and $-\hbar$, the resulting spin states for massive particle will be $+\hbar$ and $-\hbar$ with 1/2 probability for each. There can also be entanglement conditions between spin states of different pairs. For example, consider 3 pairs each having spin states $+\hbar/2$ & $-\hbar/2$.There can be entanglement conditions such that two of the three pairs always have opposite spin states. In such a case when the vector addition for the states will be carried out the result will be only two spin states $+\hbar/2$ & $-\hbar/2$ with 1/2 probability for each. Thus many different conditions are possible for the spin even with same number of pairs in the massive particle. The spin of any massive fundamental particle can be calculated once its internal composition and the conditions associated are known. From this discussion and by postulating that each pair of photons in a massive particle gives rise to an elementary charge of either +e or –e, it can be said that the electron having a charge of –e and spin of $\hbar/2$ is made out of single pair of photons under this interpretation. Photons because the electron interacts via the electromagnetic force by emission of virtual photons in the standard model. The same cannot be said for muon or tau lepton because they disintegrate into other fundamental particles by means of weak interaction.