Accurate expression of the mass of charged leptons and neutrinos

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Abstract

Through the long-term extensive research on the experimental data of the fundamental physical constants and the mass of elementary particles such as charged leptons and neutrinos, the present study defines the source of "generation" difference generated from leptons, which thereby allows the accurate expression of the lepton mass to be derived. It is particularly important to point out that the data at the best fitting point $\Delta m_{32}^2 = 1.59\cdot 10^{-3}$ eV^2 obtained in *"Study of the wave packet treatment of neutrino oscillation at Daya Bay"* reached an accuracy of 96%.

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It is known that physics is a highly quantitative discipline, for which any experimental data should be taken seriously, while the profound association behind various experimental data might be the scientific law itself. For instance, Newton's second law is found on the basis of summing up a large number of experimental data.

It is well-known that the mass of a charged lepton is a free parameter of the standard model of particle physics, which has to be obtained from experiments; while neutrino mass is beyond the standard model. There are more fundamental physical constants that need to be measured through experiments as well. Hence, what is the close relation among these constants? At present, a standard model cannot answer this. However, with the development of technology, the experimental accuracy for obtaining these constants has constantly been improved, which reasonably allowed the relation among these fundamental constants to be determined through methods such as dimensional analysis, orders of magnitude estimation, and series correction. Subsequently, a "standard model" free of free parameter could probably be "forced" out.

1 Accurate expression formula of the charged lepton mass

The latest (2016) experimental values[1] of charged leptons mass are:

m_e = 0.5109989461(31) MeV,

m_μ = 105.6583745(24) MeV,

m_τ = 1776.86(12) MeV.

Assuming that the electron is the ground state of a charged lepton, μ and τ are the conjugated excited states, Θ_μ is the difference between m_τ/m_e and m_μ/m_e, and Ξ_μ is the source of the "generation" difference generated from the charged leptons, therefore:

$$\Theta_\mu = \frac{m_\tau}{m_e} - \frac{m_\mu}{m_e} = \frac{\left(\sqrt{m_\tau}-\sqrt{m_\mu}\right)\left(\sqrt{m_\tau}+\sqrt{m_\mu}\right)}{m_e} = \frac{8\pi}{\sqrt{3}\alpha}\sum_{n=1}^{\infty}\frac{1}{n^2}, \tag{1}$$

$$\Xi_\mu = \frac{\sqrt{\frac{m_\tau}{m_e}} - \sqrt{\frac{m_\mu}{m_e}}}{\sqrt{\frac{m_\tau}{m_e}} + \sqrt{\frac{m_\mu}{m_e}}} = \frac{\sqrt{m_\tau}-\sqrt{m_\mu}}{\sqrt{m_\tau}+\sqrt{m_\mu}} = \left(\sum_{n=1}^{\infty}\frac{1}{n^2}\right)^{-1}. \tag{2}$$

In which α= 1/137.035999139(31)[2] is the fine structure constant;

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = \zeta(2) = \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} + \cdots. \tag{3}$$

Calculate Formulas (1) and (2) to obtain:

$$\sqrt{\frac{m_\tau}{m_e}} = \sqrt{\frac{2\pi}{\sqrt{3}\alpha}}\left(\frac{\pi^2}{6}+1\right), \tag{4}$$

$$\sqrt{\frac{m_\mu}{m_e}} = \sqrt{\frac{2\pi}{\sqrt{3}\alpha}}\left(\frac{\pi^2}{6}-1\right). \tag{5}$$

Obviously:

$$\frac{m_\tau}{m_\mu} = \frac{\left(\frac{\pi^2}{6}+1\right)^2}{\left(\frac{\pi^2}{6}-1\right)^2} = 16.818957\cdots. \tag{6}$$

The ratio to the experimental value is 0.999885. Formulas (4) and (5) are the roots of the quadratic equation

$$m_i - \frac{\pi^2}{6}\sqrt{\frac{8\pi m_e}{\sqrt{3}\alpha}}\sqrt{m_i} + \frac{2\pi}{\sqrt{3}\alpha}\left[\left(\frac{\pi^2}{6}\right)^2 - 1\right]m_e = 0, \quad (i = \mu, \tau), \tag{7}$$

which can be substituted into Koide formula[3] to obtain:

$$\frac{m_e+m_\mu+m_\tau}{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}=\frac{1+\frac{m_\mu}{m_e}+\frac{m_\tau}{m_e}}{\left(1+\sqrt{\frac{m_\mu}{m_e}}+\sqrt{\frac{m_\tau}{m_e}}\right)^2}=\frac{\frac{4\pi}{\sqrt{3}\alpha}\left[\left(\frac{\pi^2}{6}\right)^2+1\right]+1}{\left(\sqrt{\frac{2\pi^5}{9\sqrt{3}\alpha}}+1\right)^2}=\frac{2}{2.99997589469}. \tag{8}$$

However, if according to the Koide formula:

$$\frac{m_e+m_\mu+m_\tau}{\left(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau}\right)^2}=\frac{2}{3}, \tag{9}$$

to obtain:

$$\sqrt{\alpha}=\sqrt{\frac{\pi}{\sqrt{3}}\left(\frac{4\pi^2}{3\sqrt{2}}-\sqrt{\pi^4-12}\right)}=\frac{1}{\sqrt{136.952483\cdots}}. \tag{10}$$

Obviously, the deviation from the experimental value of the fine structure constant α is too much. Calculate Formulas (4) and (5) to obtain:

$$\frac{m_\tau}{m_e}=3477.63191135,$$

$$\frac{m_\mu}{m_e}=206.768581089.$$

The ratios to the experimental values are 0.999884 and 0.9999985565, respectively.

2 Accurate expression formula of the neutrino mass

At present, the experimental values of neutrino oscillations are measured through the solar neutrino experiment[1]:

$$\Delta m_{21}^2=m_2^2-m_1^2=(7.53\pm0.18)\times10^{-5}\,\mathrm{eV}^2 \tag{11}$$

and through the atmospheric neutrino experiment[1]:

$$\Delta m_{32}^2=m_3^2-m_2^2=(2.44\pm0.06)\times10^{-3}\,\mathrm{eV}^2 \quad \text{(normal mass hierarchy)} \tag{12-a}$$

$$\Delta m_{23}^2=m_2^2-m_3^2=(2.51\pm0.06)\times10^{-3}\,\mathrm{eV}^2 \quad \text{(inverted mass hierarchy)} \tag{12-b}$$

However, the atmospheric neutrino experiment cannot determine which is heavier between m_2 and m_3 . Hence, it is called the mass ordering problem (neutrino mass hierarchy problem).

It is known that the mass eigenstates of neutrino oscillation m_1 , m_2 and m_3 are not one-to-one corresponding to the flavor eigenstates of neutrino v_e , v_μ and v_τ , but they contain each other similar to "you have me, I have you". However, in the present study, it is regarded that the mass eigenstates of neutrino oscillation m_1 , m_2 and m_3 gave the possible mass range (magnitude) of three neutrinos v_e , v_μ and v_τ . In the revelation of Formulas (1) and (2), it should be possible to determine the mass expression formula of neutrino.

Therefore, in the present study, it is assumed that if the mass ordering is normal, v_e will be corresponding to m_1 ; if it is in the inverted ordering, v_e will be correspond to m_3 . Based on the long-term extensive research and calculation of fundamental physical constants, the author has obtained the mass of the electron neutrino ν_e in advance (given in another study): m_{ν_e} = 0.00249988 eV. When this value is substituted into Formulas (11) and (12) in normal and inverted ordering, respectively, it can be calculated as:

Normal ordering:

m_2 = 0.00903 eV,

m_3 = 0.0502 eV;

Inverted ordering:

m_2 = 0.0502 eV,

m_1 = 0.0494 eV.

Similarly, the electron neutrino is assumed to be the ground state, the μ neutrino and τ neutrino are the conjugated excited states, Θ_ν is the difference between m_{ν_τ}/m_{ν_e} and m_{ν_μ}/m_{ν_e}, and Ξ_ν is the source of the "generation" difference generated from the neutrinos. By analogy with the accurate expression of charged lepton mass Formulas (1) and (2), the normal ordering can be easily obtained (while the inverted ordering is not the case):

$$\Theta_\nu = \frac{m_{\nu_\tau}}{m_{\nu_e}} - \frac{m_{\nu_\mu}}{m_{\nu_e}} = \frac{\left(\sqrt{m_{\nu_\tau}} - \sqrt{m_{\nu_\mu}}\right)\left(\sqrt{m_{\nu_\tau}} + \sqrt{m_{\nu_\mu}}\right)}{m_{\nu_e}} = \frac{8\pi}{108\alpha}\frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2}, \tag{13}$$

$$\Xi_\nu = \frac{\sqrt{\frac{m_{\nu_\tau}}{m_{\nu_e}}} - \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_e}}}}{\sqrt{\frac{m_{\nu_\tau}}{m_{\nu_e}}} + \sqrt{\frac{m_{\nu_\mu}}{m_{\nu_e}}}} = \frac{\sqrt{m_{\nu_\tau}} - \sqrt{m_{\nu_\mu}}}{\sqrt{m_{\nu_\tau}} + \sqrt{m_{\nu_\mu}}} = \frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2}. \tag{14}$$

In which

$$\frac{1}{4}\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{1}{4}\zeta(2) = \frac{\pi^2}{24} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots + \frac{1}{(2n)^2} + \cdots. \tag{15}$$

Calculate Formulas (13) and (14) to obtain:

$$\sqrt{\frac{m_{\nu_\tau}}{m_{\nu_e}}} = \sqrt{\frac{2\pi}{108\alpha}}\left(1 + \frac{\pi^2}{24}\right), \tag{16}$$

$$\sqrt{\frac{m_{\nu_\mu}}{m_{\nu_e}}} = \sqrt{\frac{2\pi}{108\alpha}}\left(1 - \frac{\pi^2}{24}\right). \tag{17}$$

Obviously:

$$\frac{m_{\nu_\tau}}{m_{\nu_\mu}} = \frac{\left(1 + \frac{\pi^2}{24}\right)^2}{\left(1 - \frac{\pi^2}{24}\right)^2} = 5.74528539\cdots. \tag{18}$$

Formulas (16) and (17) are the roots of the quadratic equation

$$m_j-\sqrt{\frac{8\pi m_{\nu_e}}{108\alpha}}\sqrt{m_j}+\frac{2\pi}{108\alpha}\left[1-\left(\frac{\pi^2}{24}\right)^2\right]m_{\nu_e}=0,\quad (j=\nu_\mu,\nu_\tau), \tag{19}$$

which can be compared with the Koide formula to obtain:

$$\frac{m_{\nu_e}+m_{\nu_\mu}+m_{\nu_\tau}}{\left(\sqrt{m_{\nu_e}}+\sqrt{m_{\nu_\mu}}+\sqrt{m_{\nu_\tau}}\right)^2}=\frac{1+\frac{m_{\nu_\mu}}{m_{\nu_e}}+\frac{m_{\nu_\tau}}{m_{\nu_e}}}{\left(1+\sqrt{\frac{m_{\nu_\mu}}{m_{\nu_e}}}+\sqrt{\frac{m_{\nu_\tau}}{m_{\nu_e}}}\right)^2}=\frac{\frac{\pi}{27\alpha}\left[1+\left(\frac{\pi^2}{24}\right)^2\right]+1}{\left(\sqrt{\frac{2\pi}{27\alpha}}+1\right)^2}=\frac{4}{8.99814551561}. \tag{20}$$

Otherwise:

$$\frac{m_{\nu_e}+m_{\nu_\mu}+m_{\nu_\tau}}{\left(\sqrt{m_{\nu_e}}+\sqrt{m_{\nu_\mu}}+\sqrt{m_{\nu_\tau}}\right)^2}=\frac{4}{9}, \tag{21}$$

to obtain:

$$\sqrt{\alpha}=\sqrt{\frac{\pi}{25}}\left(\sqrt{\frac{32}{27}}-\sqrt{1-\frac{5\pi^4}{3\cdot 24^2}}\right)=\frac{1}{\sqrt{136.752603\cdots}}. \tag{22}$$

Similarly, it deviates too much from the experimental value of the fine structure constant α, and is not the same as Formula (10). Therefore, if it is not a coincidence, the only possibility is that some symmetry is broken in both Koide Formula (9) and Formula (21). Calculate Formulas (16) and (17) to obtain:

$$\frac{m_{\nu_\tau}}{m_{\nu_e}}=15.8777349621,$$

$$\frac{m_{\nu_\mu}}{m_{\nu_e}}=2.76361118293.$$

When substituted with m_{ν_e} = 0.00249988 eV, it becomes:

$$m_{\nu_\mu}=0.00690870\,\text{eV},$$

$$m_{\nu_\tau}=0.0396925\,\text{eV}.$$

And

$$m_{\nu_\mu}{}^2-m_{\nu_e}{}^2=4.14808\times10^{-5}\,\text{eV}^2, \tag{23}$$

$$m_{\nu_\tau}{}^2-m_{\nu_\mu}{}^2=1.52776\times10^{-3}\,\text{eV}^2. \tag{24}$$

Hence, it fits well for the order of magnitude with both the solar neutrino experiment and the normal mass ordering of the atmospheric neutrino experiment. Meanwhile, it is also within the range of Ref. [4], and reached 96% compliance with the data at the best fitting point of $\Delta m_{32}^2 = 1.59\cdot10^{-3}$ eV^2 [5].

However, there is a problem here: Formulas (4) and (5) are the determined mass expression of flavor of charged leptons, while Formulas (16) and (17) are obtained by analogy with Formulas (1), (2), (4) and (5) after assuming that the mass eigenstate of neutrino oscillation corresponds to the flavor eigenstate of neutrinos. There are two hypotheses involved, for which, in the author's opinion, the former just uses the possible mass range (magnitude) of neutrinos for reference, while the analogy with Formulas (1), (2), (4) and (5) is the core. Hence, in the present study, Formulas (16) and (17) are regarded as the accurate expression of the neutrino mass. Meanwhile, according to this, normal ordering can be determined as the mass ordering of the neutrino oscillation, and there is no other type of neutrino, for instance, the sterile neutrino.

At this point, the accurate expression of the lepton mass in Formulas (4), (5), (16) and (17) are given in this paper. Nevertheless, the accurate expression of the mass of the electron and electron neutrino, which involve some other fundamental physical constants, will be presented in another study. In addition, it seems to be able to see whether the neutrino is a Majorana fermion or not, and the trace of CP violation from the expansions of Formulas (1), (2) and Formulas (13), (14).

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$$2.4 \times 10^{-5} < \Delta m_{21}^2 /\mathrm{eV}^2 < 2.4 \times 10^{-4}, \text{ (LMA)},$$

$$1.4 \times 10^{-3} < \Delta m_{32}^2 /\mathrm{eV}^2 < 6.0 \times 10^{-3};$$

$$1.4 \times 10^{-3}\ \mathrm{eV}^2 < \Delta m_{\mathrm{atm}}^2 < 6 \times 10^{-3}\ \mathrm{eV}^2,$$

$$2.4 \times 10^{-5}\ \mathrm{eV}^2 < \Delta m_{\odot}^2 < 2.4 \times 10^{-4}\ \mathrm{eV}^2, \text{ (LMA)}.$$

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