THE RUNWAY PROBLEM

by

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This paper relates simply to an imaginary off-hand conversation which the author heard about, but the numbers seem real. The conversation related to a un-named procurement official at some level of government who, along with other procurement officers in some un-named defense department of some un-named country, wanted a large runway built for experimental aircraft across a very large dry lake bed. These procurement personnel planned on having the project built by some un-named government contractor; however, this chosen government contractor would not be selected by the usual bid mechanism from a list of potential government contractors. Instead, they would choose the winning contractor by giving their engineers a bit of a mental exercise, because they wanted the most competent government contractor, irrespective of low bid, medium bid, or high bid. They therefore gave each potential contractor the dimensions of the proposed runway in the form of the total square footage of the runway. They gave the contractors only 6 hours to come up with the solution for the dimensions. The first to come up with the correct dimensions won the contract. There was never any indication as to the course of action should no one achieve the desired results. They gave the contractors only one hint, namely, that the total square footage was the product of two prime numbers.

My imaginary informant told me that one of the government contracting engineers solved the problem in the 6 hour time period allotted. As serendipity would have it, this engineer just happened to be using his computer with Quattro Pro Optimizer installed . *(I am simply relating the imaginary facts, not trying to extol the virtues of Quattro Pro.)* This engineer felt like he could solve the problem, but the Optimizer required a Target Value. He therefore set about to devise some sort of equation which would give a Target Value acceptable to the Optimizer program. My imaginary informant says that he took careful notes, in case he should have an occasion to use this method. I am also passing this method along to whoever can use it for whatever engineering purpose that it may happen to fit. I have attempted the method myself, but I had a bit of trouble with entering formulas with enough parentheses. The Solve button gave me incorrect results at first, until I put in parentheses properly and also used a hand held calculator to double check results that the Optimizer program gave me. When everything worked correctly, the program only took a few seconds to give the proper results.

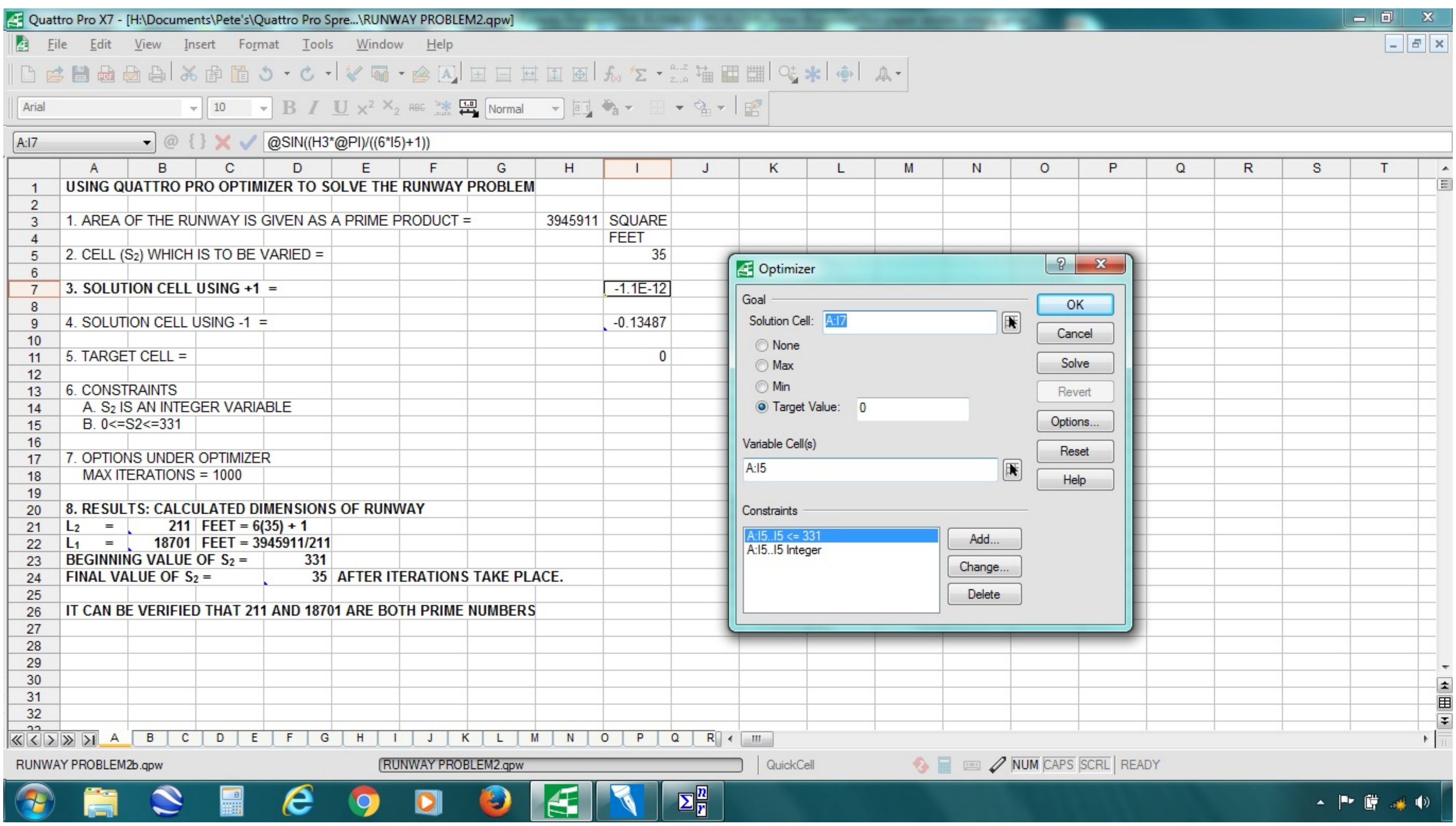
This is apparently how the engineer did it.
The procurement officials gave the area of the runway as
A = 3,945,911 square feet with the clue that it was a prime product. The engineer used a formula which gave only the smaller of the two dimensions of this area, and that this smaller dimension had an upper bound to it. He defined the smaller dimension as $L_2 = 6S_2 + \alpha$, where $\alpha = +1$ or -1. He gave the limits of S_2 as

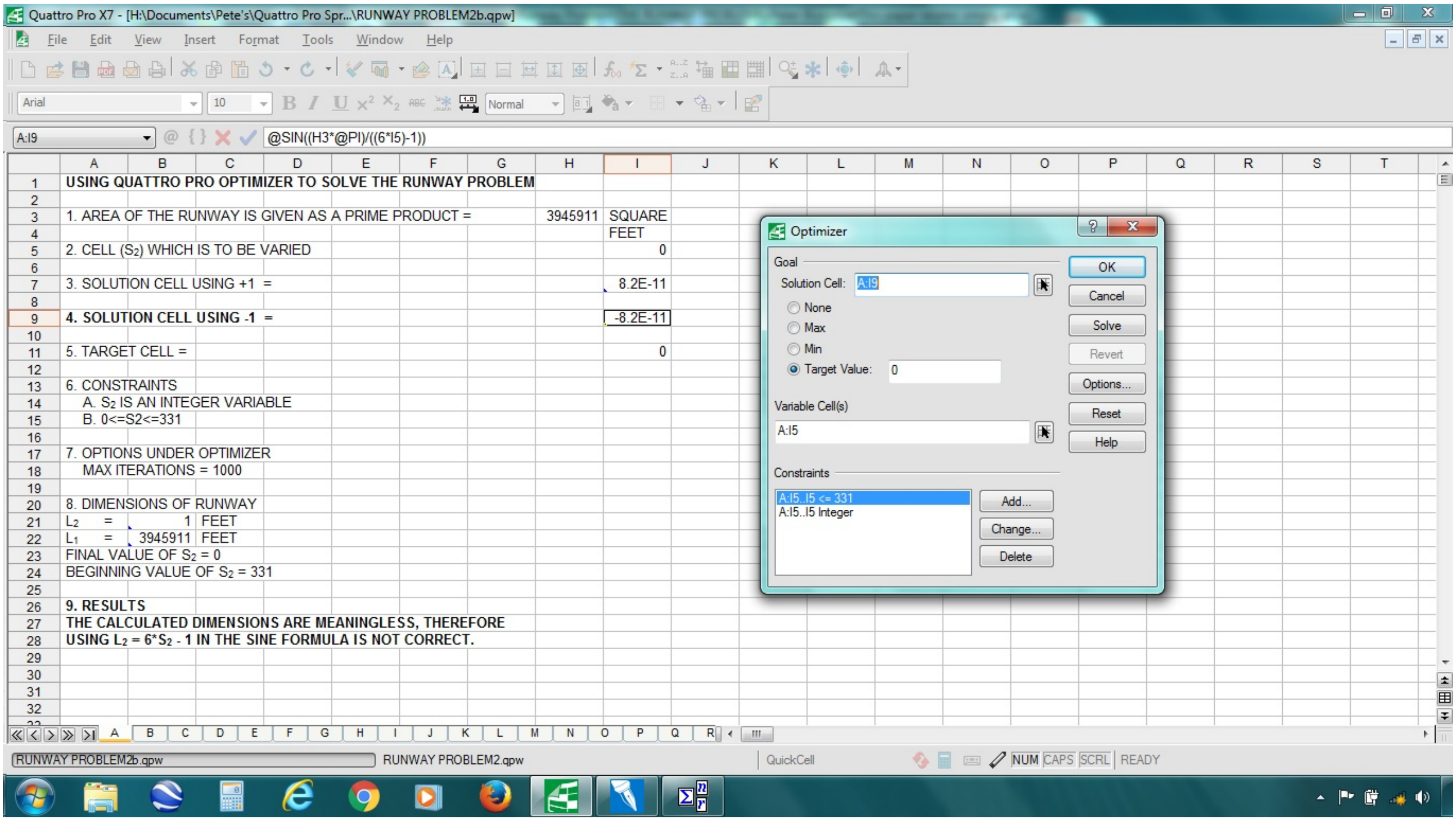
$$0 < S_2 \le \frac{\sqrt{A}+1}{6}$$

The functions which he used in order to achieve a Target Value, required for the program are as follows:

$$SIN\left(\frac{A\pi}{6S_2+1}\right) \text{ and } SIN\left(\frac{A\pi}{6S_2-1}\right)$$

When the correct value of S_2 has been reached, the Sine functions go to zero, which is the sought after Target Value. The Solution Cell in Optimizer is where the Sine functions are entered. The Variable Cell is where the upper bound of S_2 is entered. The Constraints indicate this upper bound, which is 331, and the fact that S_2 is an integer, so that Optimizer begins at the upper bound of 331 and works down only using sequential integers, with the final value of S_2 = 35, which then yields the value of L_2 = 211. The long side or length of the runway is just the area divided by 211, which yields a length of 18, 701 feet or about 3.54 miles. The author has included screen shots of the Quattro Pro spreadsheet and the Optimizer dialogue box for both of the Sine functions. Incidentally, I have checked and both 211 and 18,701 are both prime numbers.





I also did only one single quick check of the method itself by using prime numbers of 109 and 163 with their product being 17,767. I calculated the upper bound of $S_2 = 22$, using the engineers method. There is also an advantage using the Optimizer, as one can change values fairly quickly. Using 17,767 as the area and the upper bound of $S_2 = 22$, it was easy to obtain the calculated value of $S_2 = 18$, from which follows $L_2 = 6(18) + 1 = 109$.

Maybe all of this will be of help to some other engineer.