The Quantum Statistical Mechanics in the Analysis of the Context Dependence in Quantum Cognition Studies: may a quantum statistical analysis connect the science of complexity ?

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ABSTRACT

Quantum probability fields are involved during processing of human perception and cognition with a wave function that mathematically is articulated in a proper Hilbert space and conceptually represents the space of all the potential alternatives at a semantic and semiotic level. Each subject has developed a unique mind within his/her correspondent context, hence his/her own wave function requires to be displayed as a complex-valued probability amplitude. Reaction time as well as the time of observation / decision / measurement also represent important variables contributing to our cognitive response and having also proper neurological correlates. Mind is a so complex abstract entity so that we can certainly retain that, generally speaking, the observer and the observed system enter in the domain of what technically are considered to be un-complete specified systems . An elementary wave function remains insufficient to represent a mental state, whereas a quantum wave function, which exhibits intrinsic indetermination and fluctuation of the basic probability amplitudes, may be implemented. In this paper, along with von Neumann's postulate for non-existing dispersion-free ensembles, a new quantum statistical approach is elaborated to elucidate the peculiar quantum mechanical feature of the context dependent dynamics of human cognitive conceptual processing. In the framework of the present quantum statistical elaboration we introduce for the first time the possibility of using the Pareto distribution as a probability density function. This result links quantum statistical mechanics with the science of complexity .

1. INTRODUCTION

Let us follow some previous important elaborations (1). It is an idealization to retain that material entities possess intrinsic properties having an independent reality of their own. Quantum mechanics describes physical reality in a substantially context-dependent manner.Quantum entities cannot be conceived of as things-in-themselves, having intrinsic individuality and temporal identity. Instead, they are carriers of patterns and properties that arise in interaction with the experimental context/environment pertaining to their external reality. The existence of the state-property of material entities depends on the context into which they are embedded and on the subsequent abstraction of their entangled correlations with the chosen context of investigation. Consequently, the resulting contextual entities are interactive and dependent, meaning, that of a material entity exhibiting a particular property with respect to a certain experimental situation. The contextual character of property description implies that a state-dependent property of a quantum object is not a well-defined property that has been possessed a priori and that not all contextual properties can be ascribed to an object at once. One and the same quantum object does exhibit several possible contextual manifestations with several definite properties manifested only with respect to distinct experimental arrangements which mutually exclude each other. Thus, in contradistinction to a mechanistic or naive realistic perception, we arrive at the following general conception of an object in quantum mechanics: a quantum material entity constitutes an entity, a totality, defined by all the possible relations and thus alternatives in which this material entity may be involved. Quantum objects, therefore, are viewed as carriers of inherent dispositional properties. As consequence at the basis of our reality we have an ontic potentiality producing actual effects whenever it is embedded within an appropriate experimental context. As said,a quantum object is not an individual entity that possesses well-defined intrinsic properties at all times even beyond context interactions, it is not a well-localized entity in space and time that preserves deterministic causal connections with its previous and subsequent states. A quantum object exists, independently of any operational procedures, only in the sense of 'potentiality of its alternatives', namely, as being characterized by a set of potentially possible values for its various physical quantities that are actualized when the object is interacting with its pertinent experimental context. It is at this level that mind, perception and cognition enter with a fundamental role. Quantum mechanics induces a transition from the referent, classical point of view of a physics accepting a mind-independent reality,to that of a contextual approach. The classical approach of a mind-independent reality has no scientific support with relative demonstration. As a postulate it dogmatically admits the existence of things in themselves regardless of any act of empirical testing.

This is the keyword:*empirical testing*. Empirical testing implies a semantic act and a semantic act implies the mind, perception, cognition, a decision. The question that may be posed runs as follows: from one hand we have a dogmatic postulate of matter entities existing independently from mental reality. Have we a physics dismissing this prescientific position? The answer is positive. Quantum mechanics dismisses this standard approach. As its peculiar feature it introduces ab initio an abstract function. It is called the wave function of quantum mechanics.It enters in any description of a quantum system and, in addition, it delineates an ontic scheme in which it admits the ontic reality of the potentialities, of the alternatives existing for some property of a given material entity and at the same time indicates the transition occurring in our reality from potentiality to actualization. Any quantum system has its marked quantum wave function describing the ontic potential alternatives and this can not be dismissed. Still, it is connected to each kind of observable property that, by a semantic act, we decide to actualize. In addition, such a quantum wave function delineates in itself another peculiar entity that again we cannot escape in any manner of use of quantum theory. It is called the projector or, equivalently, the idempotents as we have considered in our Clifford algebraic bare bone skeleton of quantum mechanics in a number of papers (2-15;18-40). But there is still more, Von Neumann in 1936 has demonstrated that such projectors or idempotents are logic statements.A question arises : what is the reason for the existence of a theory such as quantum mechanics that requires these peculiar features: a)the existence of an abstract entity such as the quantum wave function operating in a proper cognitive-semantic space , (b) the need of context dependence,(c) the inescapable presence in the theory of variables representing logic statements connected to each constructed quantum wave function, (d) a demonstrated, as we have done algebraically (2,19,21,) , existing transition from an ontic potentiality to actualization as reported previously, the so called wave function collapse. We have sufficient indications that such an abstract entity as the quantum wave function, determining simultaneously connected projectors and/or idempotents (logic statements) , delineates for the first time a basic function of existence and of knowledge , and a new basic scheme of reality. The theory indicates that without the delineation of such a basic function indicating thus semantics, logic, and cognition, we cannot aim to describe reality. In conclusion, quantum mechanics evokes the two faces of the God Giano looking from one side to what we call matter and from the other to our mind, to our perception and cognition.

The familiar conclusion arises again, just as we have stated it in many papers reported in references :

> *There are stages of our reality in which we cannot separate logic (and thus cognition and thus the conceptual entity) from the features of "matter per se". In quantum mechanics, logic, and thus cognition and thus the conceptual entity of cognitive performance, assume the same importance as the features of what is being described. There are levels of our reality in which the truths of logical statements about dynamic variables become commuting dynamic variables themselves, and through commuting with the same observable, create a profound link which is established from the primary quantum theoretical foundations between physics and conceptual entities.*
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> We have given here only a condensed exposition of the basic conceptual foundations that characterize our studies on quantum cognition . The basic interest of this paper is to discuss in detail the role of the context in our mental dynamics by using quantum statistical mechanics . This is precisely what we will develop in the subsequent section .

2. A quantum statistical approach to quantum context dependence in cognition

Quantum mechanics provides a more realistic interpretation than classical physics does, by conceptualizing a non-separable and contextual nature of reality(1). In this holistic quantum system an ontic potentiality with intrinsic and superimposed properties resides in a state that through the act of measurement, evolves into an irreversible state and displays a property in the correspondent experimental arrangement .

We have conducted large number of theoretical and experimental studies in which we have demonstrated (2-15;18-40) that quantum probability fields are involved in processing of mental events . Recently we have also formulated a quantum neurological model of the relative brain dynamics (6, 11, 13, 14). In an experimental setting, an ambiguous figure shown to the participants, may assume a dichotomic variable form with either the value of -1 or the value +1, indicating that there is a difference between the figures or not, respectively, and representing the quantum superposition of two modes/interpretations. When the participants evaluate these two subsequent ambiguous figures , classical Bayesian approach fails and quantum probability rules are observed with the presence of the quantum interference term.The results of this study show that the two potential alternatives of the first ambiguous figure coexists in the conscious mind of the subjects and such initial superposition induces interference with the two modes pertaining to the second ambiguous figure (6). In another recent quantum cognition study, the case of the Dalmatian dog has been investigated(13), parts of which can be identified at a first look, and then the rest is inferred as a whole from these parts. Similar to mutually exclusive ambiguous figures, the figure of the Dalmatian dog allows us to determine whether one perception will predominate over the other, and the presence of one, once defined (a dog figure), and one undefined (spots) percepts, can then be determined. This study also confirms a quantum interference effect and hence permit us to deduce:consciousness operates according to the rules of quantum mechanics (14).

For the evaluation of ambiguous figures, each participant displays the uniqueness of his or her mind that includes his/her own set of data, like memories, emotions, learning and cognition through his/her conceptual network. Each participant will consequently exhibit his/her wave function as a complex-valued probability amplitude and his/her specific pool of data to give a response. Reaction time of each participant and the time of measurement also contribute to the response .In this sense, a macro-observer M is analyzing a large system, where neither the observer nor the observed system is complete. Therefore, a mental state, being a complex structure, cannot be represented by a simple and elementary wave function, but by a quantum wave function which displays intrinsic indetermination and fluctuation of the basic probability amplitudes while responding to an incompletely specified quantum system as first described by P. T. Landsberg (41,15,16,18).

In conclusion, quantum mechanics, when investigated by the Clifford algebra, is a two-faced theory covering both objects, “matter per se”, but simultaneously also conceptual entities (19).Mind operates on the basis of the following very rough scheme :

UNCONSCIOUS (prespace : space of the algebraic Clifford A(Si) events, standard superposition of quantum states) <---> CONSCIOUSNESS (including PRECONSCIOUS (prespace: space of the algebraic Clifford A(Si) events, standard superposition of quantum states)<--->Conscious (ASi and Ni space events. transition from superposition to collapsed wave function)). This is a possible pattern. One of us (R.N) has deepened some peculiar features at psycological and neurological level (40).

Still, another of us (F.K) has developed studies at level of metacognition and Ying -Yang Theory (38,39).

Evaluating the current data available from quantum cognition studies, the following quantum statistical theory may be formulated.

Human beings are adept and drawing context-sensitive and cognitive associations.

Consider a given quantum system S. Physically speaking, the Hilbert space of the system S then contains wave functions belonging to different possible contexts. We may conceive such states as lying in a given energy shell in phase space. Let the considered dimensionality be (m). Each possible state of the system is then represented by a unit vector in $\Lambda_0(m)$.The wave function $\psi(t)$ of the system lies in the appropriate space $\Lambda_0(m)$ having dimension (m). Consider the orthonormal vectors $\omega_1, \omega_2, ..., \omega_m$ that are a basis for $\Lambda_0(m)$. The projection of $\psi(t)$ in a coordinate axis ω_j is

$$c_{t,j}(a) \equiv (\psi(t), \omega_j)\,; \quad j = 1,2,..., m \tag{1}$$

and it depends directly on the considered context (a). The probability of finding the system at time t in that state will be given by

$$p(t) = \sum_{j=1}^{m} \left| c_{t,j} \right|^2 \tag{2}$$

and depends on the context (a).

Consider now all the possible contexts. If such all contexts exist at all in principle, there will be many of them, and thus we may average over the different contexts. Consequently one may calculate the average probability of finding the system at a certain time in some subspace (n) of (m), and it will be given by

$$< p_1(t) >= \sum_{j=1}^{n} < \left| c_{t,j} \right|^2 > \tag{3}$$

The spread of the individual values of $p_1(t)$ due to the different contexts will be given by the second moment

$$Z = \frac{< (p_1(t) - < p_1(t) >)^2 >}{< p_1(t) >^2} = \frac{< p_1(t)^2 >}{< p_1(t) >^2} - 1 \tag{4}$$

and it may be estimated experimentally.

In detail, consider also the notion of dispersion free ensembles. According to von Neumann given the observable $\Re$, an ensemble is dispersion free if

$$< \Re^2 >=< \Re >^2 \tag{5}$$

We conclude that $Z \to 0$ in this case.

The applicability of different properties of one and the same concept vary. Let us take , as example, the mental construct Fishes. We have different typicality, for example many types of fishes (anchovy, bluefish, see bass, salmon, tuna, blue-fin tuna, shark, trout, turbot…) and we could continue with a very large number of other typicalities (2, 19-22).

Consider now the mental construct Tree. The wave function $\psi(t)$ of the entity $|tree\rangle$ lies in the appropriate space $\Lambda_0(m)$ having dimension (m). Suppose the orthonormal vectors $\omega_1, \omega_2, \ldots\ldots\ldots, \omega_m$ are a basis for $\Lambda_0(m)$, let us admit that

Olive $\rightarrow \omega_1$
Oleander $\rightarrow \omega_2$
Palm $\rightarrow \omega_3$
Cherry $\rightarrow \omega_4$
Apple $\rightarrow \omega_5$
Pear tuna $\rightarrow \omega_6$
Fraxinus $\rightarrow \omega_7$
Walnut $\rightarrow \omega_8$
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. $\rightarrow \omega_m$

Ignoring the time dependence, and according to the previous quantum mechanical exposition, we will consider to have in our mental profile a superposition of potential alternatives given in the following manner:

$$|Tree\rangle = c_1|\omega_1\rangle + c_2|\omega_2\rangle + c_3|\omega_3\rangle + c_4|\omega_4\rangle + c_5|\omega_5\rangle + c_6|\omega_6\rangle + c_7|\omega_7\rangle + \ldots\ldots\ldots\ldots\ldots\ldots + c_m|\omega_m\rangle$$

This the general structure of the quantum potential state discussed previously in detail.
Obviously it will be context dependent.

$|c_1|^2$ will represent the probability that, given the context, the Olive tree will be activated. In the same manner $|c_2|^2$ will represent the probability that, given the context, the Oleander will be activated, $|c_3|^2$ will represent the probability that, given the context, the Palm tree will be activated, $|c_4|^2$ will represent the probability that, given the context, the Cherry tree will be activated, $|c_5|^2$ will represent the probability that, given the context, the Apple tree will be activated, $|c_6|^2$ will represent the probability that, given the context, the Pear tree will be activated, $|c_7|^2$ will represent the probability that, given the context, the Fraxinus will be activated, $|c_8|^2$ will represent the probability that, given the context, the Walnut tree will be activated, and so on until we arrive at $|c_m|^2$ that will give the probability for activation (under context) for the m^{th} selected typicality.

Obviously we will have that

$|c_1|^2 + |c_2|^2 + |c_3|^2 + |c_4|^2 + |c_5|^2 + |c_6|^2 + |c_7|^2 + |c_8|^2 + \ldots\ldots\ldots\ldots + |c_m|^2 = 1$.

Our elaboration enables us to introduce the notion of context dependence in concepts and to experience it.

The $|c_1|^2, |c_2|^2, |c_3|^2, |c_4|^2, |c_5|^2, |c_6|^2, |c_7|^2, |c_8|^2, \ldots\ldots\ldots\ldots, |c_m|^2$

vary in function of the context and obviously characterize it. Let us admit as example that we consider the Context –Desert. Obviously the tree that in this particular context has a great probability to be activated is Palm – tree to which it is connected ω_3, and probability to be activated, given by $|c_3|^2$. Thus we possibly will have that

$|c_3|^2 >> |c_1|^2, |c_2|^2, |c_4|^2, |c_5|^2, |c_6|^2, |c_7|^2, |c_8|^2, \ldots\ldots\ldots\ldots, |c_m|^2$

The

$|Tree\rangle = c_1|\omega_1\rangle + c_2|\omega_2\rangle + c_3|\omega_3\rangle + c_4|\omega_4\rangle + c_5|\omega_5\rangle + c_6|\omega_6\rangle + c_7|\omega_7\rangle + \ldots\ldots\ldots\ldots\ldots\ldots + c_m|\omega_m\rangle$

will be arranged so to give

$|c_3|^2 >> |c_1|^2, |c_2|^2, |c_4|^2, |c_5|^2, |c_6|^2, |c_7|^2, |c_8|^2, \ldots\ldots\ldots\ldots, |c_m|^2$

Thus it will have potentially the characterization of the context Desert with proper values for the complex coefficients $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, \ldots\ldots\ldots\ldots, c_m$.

Let us admit that now the context changes. This time it is Olive-Oil. The tree that in this particular context has a great probability to be activated is the Olive –tree, to which it is connected ω_1, and probability to be activated, given by $|c_1|^2$. Thus we possibly will have that

$|c_1|^2 >> |c_2|^2, |c_3|^2, |c_4|^2, |c_5|^2, |c_6|^2, |c_7|^2, |c_8|^2, \ldots\ldots\ldots\ldots, |c_m|^2$

The

$|Tree\rangle = c_1|\omega_1\rangle + c_2|\omega_2\rangle + c_3|\omega_3\rangle + c_4|\omega_4\rangle + c_5|\omega_5\rangle + c_6|\omega_6\rangle + c_7|\omega_7\rangle + \ldots\ldots\ldots\ldots\ldots\ldots + c_m|\omega_m\rangle$

will be arranged so to give

$|c_1|^2 >> |c_2|^2, |c_3|^2, |c_4|^2, |c_5|^2, |c_6|^2, |c_7|^2, |c_8|^2, \ldots\ldots\ldots\ldots, |c_m|^2$

Thus it will have potentially the characterization of the context Olive-oil with proper values for the complex coefficients $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, \ldots\ldots\ldots\ldots, c_m$ that will be totally different from the previous values had in the case of context given from the notion of the Desert.

In conclusion for each selected context, we will have a different set of values of complex coefficients $c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, \ldots\ldots\ldots\ldots, c_m$, and since we may consider a very large number of possible contexts, we will have consequently a very large number of different superpositions

$|Tree\rangle = c_1|\omega_1\rangle + c_2|\omega_2\rangle + c_3|\omega_3\rangle + c_4|\omega_4\rangle + c_5|\omega_5\rangle + c_6|\omega_6\rangle + c_7|\omega_7\rangle + \ldots\ldots\ldots\ldots\ldots\ldots + c_m|\omega_m\rangle$.

If such all contexts exist at all in principle potentially, there will be many of them, and thus we may average over the different contexts. Consequently one may calculate the average probability of finding the system at a certain time, and it will be given by

$$< p(t) > = \sum_{j=1}^{m} < |c_{t,j}|^2 >$$

The spread of the individual values of $p(t)$ due to the different contexts will be given by the second moment

$$Z = \frac{< (p_1(t) - < p_1(t) >)^2 >}{< p_1(t) >^2} = \frac{< p_1(t)^2 >}{< p_1(t) >^2} - 1$$

and verify the existence or absence of a dispersion free ensemble.

In this manner we have given an interesting quantum mechanical statistical approach to the problem of context dependence in concept elaboration. It is of interest not only under the generic theoretical profile of the basic elaboration but also as it relates to possible applications and experimental verifications. We have given here the simple example of the *Tree* that of course we considered also in (23) and references therein of other authors. It is obvious that we could find other similar examples. *House* , as example . In this case it could be cottage -> ω_1 , palace -> ω_2 ,skyscraper --> ω_3 , ... ω_m . *Animal* : Cat -> ω_1 , dog -> ω_2 , horse -> ω_3 ,............ ω_m . *Flower* : orchid-> ω_1 , tulip-> ω_2 , daisy -> ω_3 ,...... ω_m For each word the reader may arrange typicality - context schemes.

Let us take now a little step on.

We will not specify here the details of the very specialized field concerning the mental lexicon that of course is strongly linked to mental constructs as concepts. It represents a well advanced field of research and applications that we cannot consider here for brevity. We will assume that it is well known to the reader. In any case, to concern this theme and the previous relative features about typicality we quote papers of some authors as in particular Kitto and Bruza , Nelson Mc Evoy,Ramma, Sitbona, Blombergc, Songb suggesting to the reader to read such papers (42, 43).

We will limit our efforts here to offering proof that language is inherently contextual. According to the previously mentioned authors, consider the word 'bat'. This word has obviously many senses but at least two of them will be considered here in their standard form. It might refer to a flying mammal that lives in caves, or alternatively it might refer to a sporting implement. This is only a very restricted example, and actually we could relate the word "bat" to a very large number of contexts. However, we use here only such two senses in order to make explicit once again the meaning of the term context. Of course, the different senses of a word can be explored via word association experiments as actually has been done.

In free association, the words are presented to large samples of participants who produce the first associated word to come to their mind. The probability or strength of a pre-existing link between words is computed by dividing the production frequency of a response word by its sample size. We can also find out which words are likely to produce the word 'bat'. Call it the target. One way of achieving this involves a process known as extra-listcuing. Here, subjects typically study a list of to-be-recalled target words. For greater scope and detail concerning these experiments, we again require the reader reading of the work in (42,43) and references therein.

The thesis repeatedly discussed previously, appears once again of fundamental importance. The basic notion is that of context. The basic principle is that the words take different senses depending upon the context in which they occur. It remains in fact the problem to estimate the probability of recalling 'bat' when some context is present. Also here we will not enter into the detailed theory and deepening that this notion of recalling requires in psychology. We will assume that the reader is familiar with these matters of association, repression and meaning so familiar to depth psychology. Let us instead, restrict our "semantic" Hilbert space to $m = 2$, and identify by ω_1 the cognitive state of recalling and by ω_2 the complementary state of not recalling.

The recall (or not) of a word can be represented using a superposition state,

$$\psi(t) = c_{1,t}\omega_1 + c_{2,t}\omega_2 \qquad (6)$$

with specific notations given in the (2).

This is the word "bat", represented in some context (a), as a superposition of recalled, and not recalled. Thus, the word 'bat' is a target word, expected to be recalled in an extra-list cueing experiment upon presentation of the cue word 'cave' which in this case acts as the context (a_1).We have

$$\psi(t) = c_{1,t}(a_1)\omega_1 + c_{2,t}(a_1)\omega_2 \qquad (7)$$

The probability of 'bat' being recalled in this context is represented by $|c_{1,t}(a_1)|^2$, and the probability of not being recalled is represented by $|c_{2,t}(a_1)|^2$,

When given the cue word 'ball' we represent 'bat' as
the new superposition

$$\psi(t) = c_{1,t}(a_2)\omega_1 + c_{2,t}(a_2)\omega_2 \qquad (8)$$

where (a_2) represents the new context "ball" and the new probabilities result now modified as $|c_{1,t}(a_2)|^2$, and $|c_{2,t}(a_2)|^2$, respectively, and assuming obviously totally different values in respect to the previous case just as they may be retrieved from memory when a subject is presented with the cue 'ball' or the cue word 'cave'.

We would evidence here the importance of our new approach, previously articulated in detail . We have here a general formalism to delineate a very natural representation of contextual effects as they actually occur in language.

It is evident that we may continue with the word “bat”, considering each time a different context ($a_1, a_2, ..., a_n$), and thus obtaining each time a different representation of the assumed quantum superposition (recalling-not recalling) with different values of the coefficients $c_{1,t}(a_i)$ and $c_{2,t}(a_i)$ $(i = 1,2,.........., n)$, and each time different values of probabilities. The basic assumption that we make, is that we have a very large number of alternative possibilities, and that, in order to properly characterize such a situation, we must use a continuous distribution. Without loss of generality we may consider

$c_1 = \cos\alpha$ and $c_2 = sen\alpha$; $p_{recalling} = \cos^2\alpha$; $p_{not-recalling} = sen^2\alpha$; $p_{+,-}$ being the corresponding probabilities where any one value of α $(0 \le \alpha < 2\pi)$ now characterizes a different and possible context.

Let us consider the case of a strictly uniform distribution . Generally speaking , the probability of finding an angle in the range $(\alpha, \alpha + \delta\alpha)$ will be given by

$$f(\alpha) = A sen^b(2\alpha) d\alpha \tag{9}$$

where A is a normalizing constant and for a strictly uniform distribution we have $(b = 0)$, while for a weakly uniform distribution we have possible values $(b = 2, or\ b = 4)$ and so on. Under the different theoretical as well as experimental situations, we may also consider more restricted range of possible values for α as $(0 \le \alpha < \pi)$ or $(0 \le \alpha < \pi/2)$, and so on.

First consider the very interesting case in which the possible contexts obey a law of strictly uniform distribution.

Generally speaking, we know that, given the density function of probability $f(x)$, it must be

$$\int_a^b f(x)dx = 1, \; < x >= \int_a^b x\, f(x)dx \;, \quad < x^2 >= \int_a^b x^2 f(x)dx \tag{10}$$

In the case of strictly uniform distribution $(b = 0)$, we obtain that for $(0 \le \alpha < 2\pi)$

$$A = 1/2\pi, \; < p_+ >=< p_- >= 1/2; \; < p_+^2 >=< p_-^2 >= 3/8 \tag{11}$$

In this case, under experimentation, we expect:

a) The (5) is violated (not existing dispersion free ensembles)
b) The Z – value, given in (4) furnishes $Z = 0.5$
c) Finally, $< (p_{+,-}(specific\, \alpha) - 0.5)^2 >= 1/8$.

Let us examine now the case of a weakly uniform distribution in $(0 \le \alpha < 2\pi)$, $(b = 2)$ in the case of (9). It shows that

$$A = 1/\pi \;, \; < p_+ >=< p_- >= 1/2; \; < p_+^2 >=< p_-^2 >= 5/16$$

Under experimentation, we expect:

d) The (5) is violated (not existing dispersion free ensembles)

e) The Z – value, given in (4) furnishes $Z = 0.25$

f) Finally, $<(p_{+,-}(generic\,\alpha)-0.5)^2>=1/16$.

For $b = 4$, we have that

$A = 4/3\pi$, $<p_+>=<p_->=1/2$; $<p_+^2>=<p_-^2>=7/24$

Under experimentation we should find that

g) The relation specified in (5) is violated (not existing dispersion free ensembles)

h) The Z – value, given in (4) furnishes $Z = 0.16$

i) Finally, $<(p_{+,-}(generic\,\alpha)-0.5)^2>=0.04$.

We can examine also selected ranges of context with respect to the whole normalized range (as example, $0 \le \alpha < \pi/2$).

The most promising evidence is that by this methodology we may analyze and find the absence or presence of dispersion free ensembles. We remember here a datum that may be of basic interest when exploring quantum cognition. Von Neumann in 1932 was the first to outline the possible non-existence of dispersion free ensembles in quantum mechanics, and he used also such basic evidence to give a preliminary proof of the incompatibility between quantum mechanics and local hidden variable theory.

The results of von Neumann have been largely debated in these last years. However they provide us a large profile covering quantum cognition studies in the fields of mental constructs as concepts, and concerning language and the mental lexicon. Non-existent dispersion free ensembles are commonly retained as a peculiar feature in quantum mechanics.

There is still another basic feature to be described.

Accounting for quantum statistical effects in the description of context dependence in language, we have used here classical statistical distributions as the strictly uniform distribution and the weakly uniform distribution. Instead rather recently we have demonstrated that states of our consciousness may be described in their subjectivity by the science of complexity, and, in particular, through chaos, fractal and multifractal theories (24,25). For this reason, we suggest that the more appropriate probability distribution is the well known Pareto distribution (44).

Given X as random variable

The Pareto function.

$$F(x) = P(X > x) = \left(\frac{x_m}{x}\right)^{\alpha} \quad \text{for } x \ge x_m$$

and

$F(x) = P(X > x) = 1 \quad \text{for } x < x_m$

or equivalently

the probability density function is given in the following manner

$$f_X(x) = \frac{\alpha x_m^{\alpha}}{x^{\alpha+1}} \quad \text{for } x \geq x_m$$

and

$$f_X(x) = 0 \quad \text{for } x < x_m$$

where x_m is the (necessarily positive) minimum possible value of X, and α is a positive parameter. The Pareto distribution is characterized by a scale parameter x_m and a shape parameter α, which is known as the *tail index*. It may be inserted in the equation (10) and the possible experimental estimation of x_m and the parameter α, then characterizing the *subjective basic feature of each subject,* may be possible. Of course, the link between the Pareto distribution with processes of high complexity, chaos and fractals is well known. We studies it in an our previous work (..........).

Generally speaking we may consider n random variables $X_1, X_2, ..., X_n$. They may have an n-dimensional self-affine distribution. This happens if their probability density function assumes the following form

$$f_{X_1, X_2, ..., X_n}(x_1, x_2, ..., x_n) = \prod_{j=1}^{n} a_j k_j^{a_j} x_j^{-a_j - 1}$$

with $a_j > 0$, $x_j > k_j > 0$, $j = 1,2,...,n$,

a_j shape parameters and k_j scale parameters.

The considered Pareto probability density function is

$f(x) = ak^a x^{-a-1}$

with $a > 0$ -shape parameter , $x \geq k > 0$ (a – shape parameter, k -scale parameter) .

We have that

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx = 1 - (x/k)^{-a}$$

All the details and the relative references are given in (45,46).

Consequently, the present paper draws a link between quantum statistical mechanics and the science of complexity.

3. Conclusion.

In the present note we have given a summary indication of our current studies on quantum cognition. First of all we have given primary importance to the concept of context dependence in perception and cognitive dynamics. Of course, this field of study was pioneered by us years ago giving for the fist time experimental confirmations of the role of quantum mechanics in cognition and consequently, our results have correlated mentation to quantum theoretical constructs, and have also produced in the last years advances in scientific directions that cause us concern as well. We do not agree as to the use of this theory in cognitive studies intending quantum mechanics as an instrumental method only.Our approach avoids empirical methods and such dependence in studies and research. Quantum mechanics has peculiar features including its basic foundations in the analysis of mental entities, and,in particular,pertaining to perception and cognition. Therefore, those studies not articulating the basic foundations of the theory as they interpretively support experimental results, do not advance this burgeoning new discipline.We do not support empirical exclusivity in the methods of quantum mechanics. Of course we are certainly aware that quantum theory is physics and as such it involves mathematics, physics, and conceptual foundations that often may cause difficulties in researchers who may not have solid skills in these fields.It is also true that over the course of years a new branch of psychology has been introducedwhich is known as"mathematical psychology" with several journals dedicated to this specialization. Hence, it is hoped that a suitably well guided and foundationally supported effort may provide a valid strategy for facilitating everyone's engagement.

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