An approach for analyzing time dilation in the TSR (v7. 2018-01-22)

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Abstract. We present an approach to analyze time dilation in the theory of special relativity, starting out from a variant of the Lorentz transformation. The concepts of symmetry and simultaneity are essential in these investigations. We also stress the importance of the observational principle, *i.e.*, the location of clocks used for the clock comparisons of the two reference frames (RFs) moving relative to each other. For a specific RF we may follow just a single clock (SC), or we can use multiple clocks (MC) to follow a single clock on the other RF. In addition to these standard cases, we consider an approach, utilizing an auxiliary RF, which – in combination with symmetry considerations – provides a consistent definition of 'simultaneity at a distance'. We use the overall approach to provide a thorough discussion of the travelling twin paradox, and arrive at a conclusion regarding the twins' ages, which deviates from the prevalent view regarding this example.

Key words: Lorentz transformation, symmetry, simultaneity, auxiliary reference frame, travelling twin.

1 Introduction

The present work explores some basic concepts related to time dilation. We start out by providing a reformulation of the Lorentz transformation (LT), also being suitable for a graphical illustration. The required specification of which clocks to apply for the clock comparisons between the two inertial reference frames (RFs) is facilitated by this new version of the LT and the corresponding illustration. We refer to the specification of clocks as the observational principle.

Symmetry is important when we discuss time dilation within special relativity. It may appear paradoxical that- at the same time as we have complete symmetry between the two RFs – we will also 'take the perspective' of one of them, apparently destroying symmetry. For instance the common statement that the 'moving clock goes slower' represent such an apparent paradox, which is handled somewhat differently in the literature. Some authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality. Actually, Giulini, [1] in his Section 3.3 states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of special relativity, (*i.e.* no gravitation *etc.*) However, for instance Pössel [2] points out that it is the procedure related to clock comparison ('observational principle') that decides which reference system has the time which is 'moving faster', resp. 'slower'.

Simultaneity at a distance is another important but problematic issue in special relativity. Using synchronized clocks of a specific reference frame we can define simultaneity of events 'in the perspective' of any RF, but the various RFs will give different specifications of simultaneity. However, we will introduce an *auxiliary reference frame,* which – in combination with symmetry requirements – provides a useful tool for specifying simultaneity at a distance.

We present an approach to meet these challenges in the analysis of time dilation, and also apply the suggested approach to provide a rather lengthy discussion of the 'travelling twin' example (under the strict conditions of special relativity). Our solution deviates from the one usually given in the literature.

Actually, some authors also question the validity of the theory of special relativity (TSR) and the LT, (*e.g.* see McCausland [3], Phipps [4], Robbins [5]); and perhaps we should include Serret [6]. In particular Ref. [3] reviews various controversies on the topic (related to H. Dingle) during several decades, and gives many references. Ref. [5] also treats the Bergson-Einstein controversy, dating back to 1922. The scope of the present work, however, is more restricted, accepting the validity of the TSR as a premise. Our objective is mainly to investigate the logical implications of the Lorentz transformation and thereby provide an approach for analyzing relative time and simultaneity within the framework of the TSR. However, the suggested approach will challenge some aspects of the current narrative on time dilation and simultaneity in the TSR.

2 The Lorentz transformation and some special cases

We first specify some basic notation and assumptions. Then we provide a variant of the Lorentz transformation (LT) as the basis for our investigations.

2.1 Basic notation and assumptions

We start out from the standard theoretical experiment: Two co-ordinate systems (inertial reference frames), pointing in the same direction are moving relative to each other at speed, v. We consider just one space co-ordinate, (x-axis), and investigate the relation between space and time parameters, (x, t) on one RF and the corresponding parameters (x_v, t_v) on the other. Thus, we have the following *basic simultaneity*: At time (*i.e. clock reading*), t and position, x on one system, we observe that time equals t_v and position equals x_v on the other. We will base the discussions on the LT, including the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems.
- On both RFs there is an arbitrary number of identical, synchronized clocks, located at any positions where it is required.
- We will choose the *perspective* of one of the RFs, denoted, K, and will refer to this as the *primary* system. Simultaneity in the perspective of this RF means that all clocks on this RF show the same value, t. We refer to the other RF as the 'secondary' system.
- Throughout we let SC refer to a RF utilizing a 'single clock' (or the 'same clock'), for the time comparisons with other RFs. Similarly, MC will refer to a reference frame, which utilizes 'multiple clocks' (at various locations) for time comparisons.

2.2 The standard formulation of the Lorentz transformation

In the above notation the LT takes the form

$$t_v = \frac{t-(vx)/c^2}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$x_v = \frac{x-vt}{\sqrt{1-(v/c)^2}} \tag{2}$$

These formulas include the length contraction along the x-axis (inverse Lorentz factor):

$$k_x = \sqrt{1-(v/c)^2} \tag{3}$$

2.3 An alternative formulation

At any time, t and position, x we now introduce w equal to $w = x/t$. By inserting $x = wt$ in (1) we directly get that time on the secondary RF at this position equals:

$$t_v = t_v(w) = \frac{1-vw/c^2}{\sqrt{1-(v/c)^2}}\,t \tag{4}$$

Note the change in notation. In *eq.* (1) we suppressed the dependence of t_v on x Now, however, we pinpoint its dependence on w, and will – when appropriate – write $t_v(w)$ rather than t_v.

The new time dilation formula (4) will – for a given time, t on the primary system, K - give the time, $t_v(w)$ on the secondary system, K_v, as a linear, decreasing function of w. The important thing is that we take out t as a separate factor. The relation $t_v(w)/t$ is given as the 'general time dilation factor':

$$\gamma_v(w) = (1-\frac{vw}{c^2})\Big/\sqrt{1-(\frac{v}{c})^2}$$

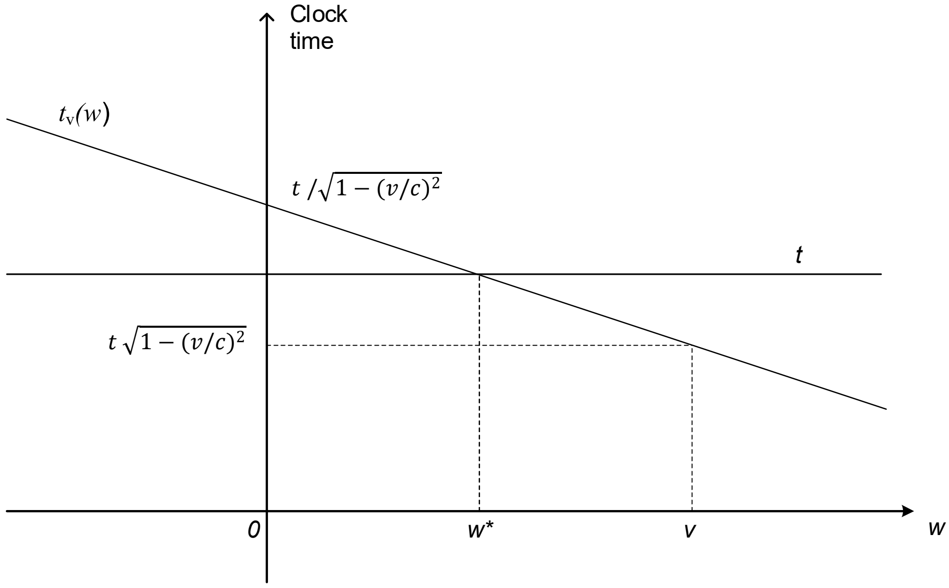

Figure 1. Clock readings in the perspective of *K.* Thus, 'time' all over *K* equals, *t.* while clock readings, $t_v(w)$ on the other RF is given as a function of *w, cf.* (4); where *w=x/t* provides the 'position' on *K.*

Thus, we can write (4) as

$$t_v = t_v(w) = \gamma_v(w)\ t.$$

Fig.1 provides an illustration of this time dilation formula. Here we give clock reading ('time') both on *K* and K_v *in the perspective of K*. So the figure illustrates an instant when time equals *t* all over this reference frame. The horizontal axis gives the 'position' $w = x/t$ on *K* at which the clock measurements are carried out. The vertical axis gives the actual clock readings. So as time on *K* equals *t* at any 'position', *w*, the clock readings on K_v at this instant, $t_v(w)$, depend on *w*; see decreasing straight line.

Now, in analogy to letting $x = wt$, we also define a w_v so that $x_v = w_v \cdot t_v = w_v \cdot t_v(w)$. By inserting both $x = wt$ and $x_v = w_v \cdot t_v$, in (2), we will obtain

$$w_v = \frac{x_v}{t_v(w)} = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{5}$$

Now equations (4), (5) represent an alternative version of Lorentz transformation, here expressed by parameters (*t, w*) rather than (*t, x*). The equation (5) has a direct interpretation. According to standard results of TSR, *e.g.* Refs. [7]-[9], the velocities v_1 and v_2 sums up to *v,* given by the formula

$$v = v_1 \oplus v_2 \overset{\text{def}}{=} \frac{v_1+v_2}{1+\frac{v_1}{c}\cdot\frac{v_2}{c}} \tag{6}$$

So by defining the operator ⊕ this way, eq. (5) actually says that $w_v = w \oplus (-v)$, implying $w_v \oplus v = w$; thus, clearly interpreting *w* and w_v as velocities along the *x* - axis. That is we have a moving position along the *x*-axis for clock comparisons. So this *w* specifies what we refer to as the *observational principle,* pinpointing that this is an essential factor for the resulting observed time dilation.

Note that we do not have to think of *w* as a velocity; rather as a way to specify a certain position $x = wt$ on *K*; representing the location of clocks – being at rest on a specific RF – and being applied at time *t*. However, we will later see that it can also be fruitful to interpret *w* as the velocity of a third observational RF.

A final comment. *Eqs*. (4), (5) give the alternative LT choosing the RF with parameters (*t, w*) as the primary. We could of course solve these with respect to (*t, w*), and get an identical expression with the other RF as the primary, (just replacing *v* with –*v*).

2.4 Standard special cases (observational principles)

Now focusing on time dilation, *cf. eq.* (4), there are various interesting special cases (*observational principles*). First, if a specific clock located at the origin $x_v = 0$ on K_v is compared with the passing clocks on K. These clocks on K must have position $x = vt$, and thus we choose $w = v$ and directly get the relation

$$t_v(v) = t\,\sqrt{1-(v/c)^2} \tag{7}$$

which equals the 'standard' time dilation formula. Further, when a specific clock at the origin, $x = 0$, on K is used for comparisons with various passing clocks on K_v, we must choose $w = 0$ and thus get

$$t_v(0) = t\,/\sqrt{1-(v/c)^2} \tag{8}$$

as the relation between t and t_v. We specify the two special cases (7), (8) in Fig. 1, and will also discuss these in Ch. 3. Two other standard cases are obtained by inserting $w = \pm\, c$.

2.5 The symmetric case

There is another interesting special case of the LT, (4), (5). We can ask which value of w (and thus w_v) will result in $t_v(w) \equiv t$. We easily find that this equality is obtained by choosing $w = w^*$, where

$$w^* = \frac{c^2}{v}\left(1-\sqrt{1-(v/c)^2}\right) = \frac{v}{1+\sqrt{1-(v/c)^2}} \tag{9}$$

Further, by this choice of w we also get $w_v = -\,w^*$. This means that if we consistently consider the positions where simultaneously $x = w^*t$ and $x_v = -\,w^*t_v = -\,w^*t$, then no time dilation will be observed at these positions. In other words (*cf.* Fig. 1):

$$t_v(w^*) = t \tag{10}$$

At this position we find $x_v = -\,x$, and so we see this as the midpoint between the origins of the two reference frames; thus, providing a nice symmetry. Note that when we choose the observational principle, (9), then absolutely everything is symmetric, and it should be no surprise that we get $t_v = t$.

Note that w^* has a simple interpretation. Recalling the definition of the operator $\oplus$ in eq. (6) for adding velocities in TSR, ($v = v_1 \oplus v_2$), it is easily verified that when w^* is given by (9), then we get $w^* \oplus w^* = v$. So this confirms that when our point of observation 'moves' with velocity w^* relative to K and $-\,w^*$, relative to K_v, it corresponds exactly to the case that the relative speed between K and K_v equals v.

3 "The moving clock": SC *vs.* MC

We now take a closer look at the observational principles given by (7) and (8). These relate clock readings at a location where one of the clocks are positioned *at the origin* of a RF. Thus, we can combine *eqs.* (7) and (8) into one single formula:

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2} \tag{11}$$

Here t^{SC} is the clock reading of the specific clock at the origin (of either K or K_v). Further t^{MC} is the clock reading of the clock at the same location, but on the other frame. So, for instance, t^{SC} replaces $t_v(v)$ in *eq.* (7) and t in *eq.* (8); while t^{MC} replaces t in *eq.* (7) and $t_v(0)$ in *eq.* (8), This is clearly demonstrated in Fig. 1. At both these locations it is the SC that gives the lower value.

Note: Extended notation

Note that the notation of *eq.* (11), using t^{SC} and t^{MC}, shall not replace the more general notation, $t_v = t_v(w)$ and t, as used in (4). The new terms t^{SC} and t^{MC} shall just help us to realize the symmetry of two specific positions, also being marked out in Fig. 1. Thus, we will still use the general notation, but at the two locations, $w = 0$ and $w = v$, we may in addition apply the essential result, (11).

Also, observe that we in (11) have dropped the subscript, v in both time parameters. This just means that (11) is valid irrespective of which RF is chosen as the primary. However, we could (and will later) add a subscript v to either t^{SC} or t^{MC}, to indicate which of the systems we choose as the primary/secondary RF. Thus, using parameters (t_v^{SC} , t^{MC}) means that we 'follow' a fixed SC at the origin of the 'secondary' RF, and using (t^{SC}, t_v^{MC}) means that we 'follow' a SC on the primary RF.

We may consider this relation (11) as the 'essential Lorentz transformation', and is in my opinion more useful that the rather ambiguous (7). Actually, (11) is much more than an efficient way to write the two *eqs.* (7) and (8). By *eq.* (11) we stress that (7) and (8) actually represent the same result, and is thus more informative than (7) and (8). Actually, this choice of which reference frame shall apply a single clock is crucial, and it introduces an asymmetry between the two RFs.

Before we leave (11) some further comments are relevant. First, observers on both reference frames will agree on this result (11). Thus, it is somewhat misleading to apply the phrase 'as seen' regarding any clock reading. All time readings are objective, and all observers (observational equipment) on the location in question will 'see' the same thing. The main point is rather that observers at *different reference frames* will not agree regarding simultaneity of events.

Secondly, we have the formulation 'moving clock goes slower'. It is true that an observer on one RF, observing a *specific clock* (on the other RF) passing by, will see this clock going slower, when it is compared to his own clocks. So in a way this confirms the standard phrase 'moving clock goes slower'. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on the other RF are moving. The point is definitely not that clock(s) on one RF are moving and clocks on the other are not. It is the observational principle that decides which of the two clocks initially at the origin, which we observe to move slower. Therefore, I find the talk about the 'moving clock' rather misleading.

Note that the insight provided by *eq.* (11) is in no way new. Our concepts SC and MC correspond to the concepts 'proper' and 'improper' time used *e.g.* by Smith [10]. In particular, *eq*. (11) equals *eq*. (3-1) of that book. However, it seems this relation has not received the attention it deserves.

We further stress that it is not required to point at one reference frame to be SC (having 'proper' time), and the other to be MC (having 'improper' time). We may at the same time have clock(s) on *both* RFs observed to 'go slower'. The equation (11) just says that if we follow a specific clock (here located at the origin), we will observe that this goes slower than the passing clocks on the other RF.

This point in my opinion also gives an answer to 'Dingle's question'. Dingle [11] raises the question of symmetry regarding the travelling twin paradox: "Which of the two clocks in uniform motion does the special theory require to work more slowly? This is an important question, which according to the discussion in McCausland [12] so far has not been given a satisfactory answer.

However, from the above discussion it is not the case that the clock(s) on *one* of the two reference frames go(es) slower than the clock(s) on the other, (as indicated by the Dingle's question). We could very well choose to follow *both* the two clocks being at the origin at time 0; which will give that both reference frames have a clock 'going slower'. So, the result on time dilation is actually fully symmetric with respect to the two reference frames! The question is not which reference frame has a clock that 'goes slower'; it is rather which observational principle we have chosen. This fully demonstrates that it is rather inappropriate to apply the statement 'moving clock goes slower'.

Actually, Professor Dingle in his later work claimed that the TSR itself was inconsistent; see thorough discussion by ref. [3]. However, according to [3], Dingle again seems to have focused on the apparent inconsistency of our *eqs.* (7), (8), rather than discussing the interpretation of the more relevant *eq*. (11).

4 Using an auxiliary reference frame of symmetry

We proceed to investigate the important question of simultaneity at a distance. We primarily elaborate on the fundamental result (11). However, we will now treat the two reference frames in a symmetric way, and denote them K_1 and K_2. In addition, we introduce an auxiliary RF, *K*. We chose this as our primary RF, and so we make our observations 'in the perspective' of this auxiliary *K*.

To get a completely symmetric situation we let K_1 having speed – w^* with respect to *K,* and K_2 having speed w^* with respect to *K.* As the speed between K_1 and K_2 shall equal *v*, it follows that we define w^* by (9); (*cf.* discussion at end of Section 2.5.)

Next we specify the observational principle. We choose to operate the auxiliary reference frame as MC, and so both K_1 and K_2 are SC.[1] Then single clocks at the origins of K_1 and K_2 are at any time compared with various clocks along *K*. Now we can apply the relation (11) between the auxiliary RF, *K* and the two RFs K_1 and K_2, giving, (also see the Note, *Alternative notation* in Chapter 3):

$$t_v^{SC} = t^{MC}\sqrt{1-(v/c)^2}, \quad \text{both for } v = -w^* \text{ and } v = w^* \tag{12}$$

It directly follows that

$$t_{-w^*}^{SC} = t_{w^*}^{SC} = t^{MC}\sqrt{1-(w^*/c)^2} \tag{13}$$

So here $t_{-w^*}^{SC}$ is the clock reading of the clock at the origin of K_1, and $t_{w^*}^{SC}$ is the reading of the clock at the origin of K_2. The essential result in (13) is that $t_{-w^*}^{SC} = t_{w^*}^{SC}$. So in the perspective of *K* these are now the simultaneous clock readings at the origins of the two 'main' reference frames, K_1 and K_2, moving relative to each other at speed, *v*.

We illustrate these results in Fig. 2, which provides an analogy to Fig. 1. While Fig. 1 presented time dilation between two RFs, taking the perspective of one of them, Fig. 2 gives a symmetric picture with respect to two RFs, also introducing a third RF, *K,* and taking the perspective of this new one. Fig. 2 gives a snapshot of the clock measurements at an instant when all clocks on *K* read time *t*; *cf.* horizontal line marked *t*.

The parameter, *w* (horizontal axis) refers to the 'positions' ($w = x/t$) on the auxiliary reference frame, *K*. The reference frames, K_1 and K_2 move relative to *K* at speed $-w^*$ and w^*, respectively. Thus, the lines $t_{-w^*}(w)$ and $t_{w^*}(w)$ give the time measured on clocks at K_1 and K_2, respectively; as a function of *w* (and time *t*) on *K*. We focus on three positions on *K*, *i.e.* *w* equal to $-w^*$, 0 and w^*, respectively. These three values correspond to the origins of the three reference frames, K_1, *K* and K_2, respectively.

First, the letter *a* in the figure indicates the simultaneous clock readings of reference frames K_1 and K_2, observed at the origin of *K*. At this position the clocks on K_1 and K_2 show the same time, and are simultaneously located at the same location, $w = 0$; so we are actually just referring to 'basic simultaneity'. For *these* measurements the reference frame *K* is a SC system, and its clock will appear slower than the corresponding clocks on K_1 and K_2: we observe the line *t* falling below the point *a*.

As stated, the two points marked with *b* correspond to the SC time readings at the origins of K_1 and K_2. Thus, using the *extended notation* introduced in Chapter 3, we have $t_{-w^*}^{SC} = t_{-w^*}(-w^*)$ and $t_{w^*}^{SC} = t_{w^*}(w^*)$. According to our result (13), these are identical. So the clock on K_1 at the position $-w^*$ and the clock on K_2 at the position w^* give identical time readings.

These origins have moved apart after time 0; and the evens that these two clock readings are equal are not simultaneous, neither in the perspective of K_1 nor in that of K_2, (clearly illustrated in Fig. 2). However, *eq.* (13) tells that *in the perspective of the auxiliary reference frame* we have two simultaneous events. Now simultaneity in the perspective of the auxiliary RF may seem a weak form of simultaneity. But, when we have this symmetry, the result becomes interesting, and not very surprising. Rather, I would postulate that this symmetric 'simultaneity at a distance represents a valid form of simultaneity.

[1] Alternatively we could let the auxiliary reference frame, *K* operate as SC; but would then just obtain the same result as given in Section 2.5, and this is therefore of limited interest.

This is not a strong assumption, considering this a consequence of the complete symmetry we have here. Claiming that the one of the two events *b* occur prior to the other would represent a contradiction.

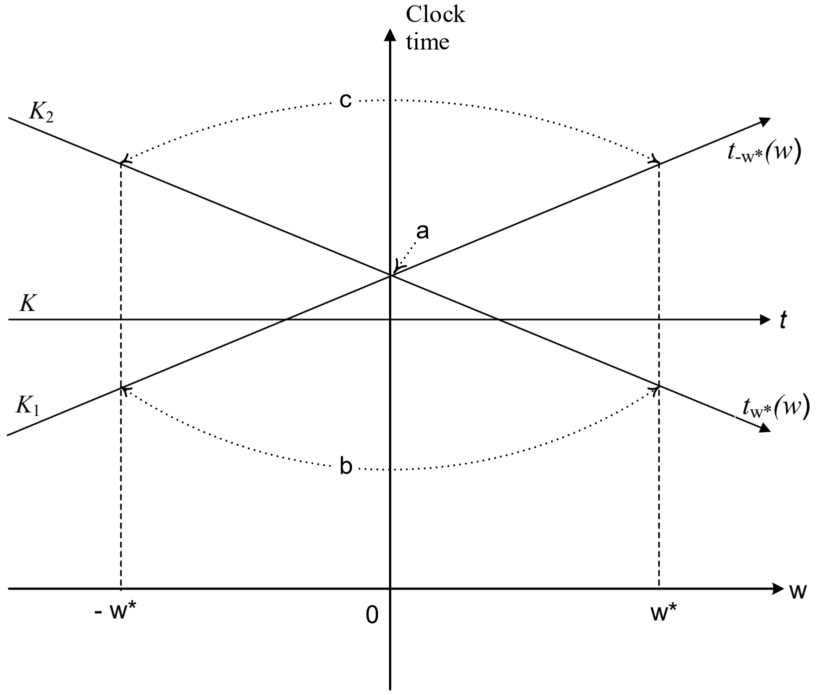

Figure 2. Clock measurements ('time') in the perspective of the auxiliary reference frame, *K,* where the reference frames K_1 and K_2 have velocity $-w^*$ and w^*, respectively, relative to *K.*

In conclusion, this is the most significant result obtained by using the auxiliary RF: We manage to establish a simultaneity of events at K_1 and K_2 'at a distance'. This is a key question in a proper handling of time dilation to achieve this. Also see the further discussion on simultaneity in Hokstad [13].

Finally, in Fig. 2 we also see two clock readings corresponding to the letter *c.* These exhibit the same type of symmetric simultaneity as the points *b*, and the only difference is that the time readings at *c* will *not* correspond to the origins of the two main RFs themselves, but rather to the location on the other frame at the same position. We give a numerical example related to Fig. 2 in Appendix.

This concludes our discussion on the interpretation/handling of time dilation in TSR. The observant reader might realize that the experimental set-up given here is well suited for handling the travelling twin paradox; which we discuss in the next chapter.

5 Example: The travelling twin

We now utilize the framework provided in the previous chapters to analyze the so-called travelling twin example, which goes back to Langevin [14]. As stated for instance in Mermin [9] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

5.1 The problem

This paradox is indeed thoroughly discussed in the literature. Shuler [15] informs that about 200 per reviewed academic papers with *clock paradox* or *twin*(*s*) *paradox* in their title can be identified since 1911, most of them since 1955. He comments "Though the correct answer has never been in doubt the matter of *how to explain* the travelling twins appears be far from settled". He also refers to the following statement: "On the one hand, I think that the situation is well understood, and adequately explained in plenty of textbook. On the other hand… there are complementary explanations which take different points of view on the same underlying space-time geometry (though, alas, the authors don't always seem to realize this, which rather undermines my assertion that the effect is well enough understood)". The last part of this statement is close to my own impression. However – as may be evident from the discussion of the previous chapter – I seriously challenge the standard answer to the paradox.

Regarding the travelling twin example, Ref. [9] (in Chapter 10) gives the following numerical example: "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. Since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e.* under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all the whole argument relies on the Lorentz transformation! Thus, our discussion will restrict to the periods of constant velocity.

Throughout this chapter we avoid using the rather complex notation of the previous chapters and let

t_1 = time on the clock of the earthbound twin

t_2 = time on the clock of the travelling twin

Similarly, the distance between earth and the 'star' is denoted $x_1 = 3$ light years, and since the rocket has speed, $v = (3/5)c$, we get $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the reference frame of the earth/star, the rocket reaches the star at time, $t_1 = x_1/v = 5$ years. Further, the Lorentz transformation gives that at the arrival at the star this clock reads $t_2 = t_1 \cdot \sqrt{1-(v/c)^2} = 4$ years; so obviously, $t_2/t_1 = \sqrt{1-(3/5)^2} = 0.8$ at the star when the travelling twin arrives; (and the argument is valid also for the return travel).

So it does follow from the LT that the returning clock shows 8 years when he/she returns. However, recalling the discussion of Chapters 3-4 the case is not that straightforward, and since we have made no assumption of asymmetry regarding the periods of constant velocity, we have a true paradox.

Thus, we will not question the clock of the travelling twin, but take an overall look at the total situation. First, we observe that the above presentation describes the travelling twin as a 'SC system', and so in this description the earthbound twin is located on a 'MC system'. Therefore, we can just look at *eq.* (11) to obtain the above result. So the first question is how we can decide that this is the (only) correct observational principle. The second question is related to simultaneity. We should definitely ask: What event on earth is simultaneous with the arrival of the travelling twin of the star (or simply: What does the clock on the earth show 'at this instant').

We now elaborate on these questions, before referring to our results of the previous chapters. As we insist on the symmetry of the situation, we now simply assume that there is also a RF of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing the same time, t_2. Whether this is practically feasible is not relevant here. We are referring to the model of the TSR, and point out what this theory tells about clock readings, *if* we provide such an arrangement. Now we have established two RFs as required by the LT.

I think the core of the problem is that we actually do not fully know how to define the moment on the earth 'simultaneous to the arrival at the star of the traveling twin. The LT apparently does not give a definite answer regarding the simultaneity of events 'at a distance'. Therefore, we will now discuss various options regarding this simultaneity.

To proceed we also introduce a symmetric auxiliary reference frame, K, with velocity $\pm w^*$, respectively, relative to the RFs of the two twins; (with w^* given in (9); also see Appendix). Then we can consider simultaneity in the perspective of each of these three RFs. Starting with the arrival at the star, where $t_2 = 4$, $t_1 = 5$, we now identify the simultaneous event on the earth from these three perspectives.

We present the result in Table 1. Note that on the earth it is the earthbound twins clock that acts as SC, that is here t_1 being a SC time reading, and t_2 a MC time reading. Thus, *eq.* (11) gives the result, t_2/t_1 = 1/0.8 = 1.25, for all observations on the earth, whatever instant we consider after departure. Therefore, at this location it is the clock on the earth that always 'goes slower'!

First, in the perspective of the travelling twin, the clock reading of his clock equals 4 years. So when the clock of his RF (showing 4 years) passes the earth, the clock on the earth just reads 0.8·4 = 3.2 years; see perspective 1 in Table 1.

Next, in the perspective of the earthbound twin, we have calculated that his clock located at the star, reads 5 years by the arrival of his twin. But when his own clock on earth shows 5 years, the passing clock of the travelling twin's reference frame then shows 5·1.25 = 6.25 years; see perspective 2. (If this is the relevant answer, we should expect the return of the twin brother after 12.5 years.)

Table 1. Various clock readings (light years) at/on the earth, *potentially* 'simultaneous to' the arrival of the travelling twin at the star; (so, at the star we have t_1 = 5, t_2 = 4).

Clock reading at earth	Perspective of		
	1.Travelling twin	2.Earthbound twin	3.Auxiliary (symmetric)
Earthbound twin system (t_1)	3.2	5	4
Travelling twin system (t_2)	4	6.25	5

The third possibility is to apply the perspective of the symmetric auxiliary reference frame. In this perspective, we treat both clocks belonging to the twins as SC. Then we get the following symmetric result regarding simultaneity: The arrival at the star occurs when both twins observe that their own clock shows 4 years, and the adjacent clock on the other RF shows 5 years; (closely related to length contraction). By these direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answer to the paradox. In addition to the symmetry, it is an important point here that in options 1 and 2 of Table 1 we directly follow the clock of just *one* twin, while we in option 3 follow *both* these clocks.

5.2 Conclusion and further discussion

Above we presented a rather lengthy discussion about simultaneity, which is the essential question in this paradox. However, we could directly apply the approach of Chapter 4 to provide our solution to the problem. Fig. 2 illustrates clock readings of the three reference frames, choosing the auxiliary frame, K as the primary one. We further associate K_1 with the earthbound twin and K_2 with the travelling twin. Further, the 'positions' w^* and $-w^*$ give the locations of the two twins. (From (9) we find that the twin has velocity $w^* = c/3$ relative to the auxiliary frame, K; see numerical example in Appendix.)

We let Fig. 2 illustrate the arrival at the star in the perspective of the RF, K. At that location t_2 = 4 years and t_1 =5 years (right side of figure); which is simultaneous to the event that clocks show t_1 = 4 years and t_2 = 5 years on the earth (left side of figure). Therefore, by following both twins, and thus considering *both* their systems as SC, the conclusion is that both twins have aged 4+4=8 years when they meet again; (we describe both twins as SC systems also on the return). Thus, we follow both clocks in a symmetric way, from the moment when they depart (having basic simultaneity) to the moment when they are again united (again basic simultaneity). So when it is often seen as paradoxical that we apparently have to 'choose' *one* twin to age more slowly, our solution is that they both age more slowly, as compared to 'passing clocks' on the other reference frame. This reduction in the clock time, compared to the result of 10 years, is related to the length contraction, experienced by SC systems.

So what does the literature say about this? Obviously quite a lot; as seen *e.g.* in the referred paper by Shuler [15]. Also Debs and Redhead [16] give a thorough discussion on this case. They refer to the two asymmetries that have been the basis for most of the standard explanations. The first group of arguments focuses on the effect of different standards of simultaneity, and secondly one can designate the acceleration as the main reason for the differential aging. However, regarding the last group of arguments they write "… since we are dealing with flat space-time, we regard the reference to general relativity in this context as decidedly misleading"; a statement in which we agree.

Thus, [16] follows up on discussing the simultaneity, and in particular argue for the *conventionality of simultaneity.* This implies that when establishing simultaneity at a distance by the use of light signals, the definition of simultaneity is essentially a matter on convention; (we could consider any time in a certain interval to be simultaneous with a distant event).

I am uncertain about the interpretation of this. We might apply a similar argument to the above approach of using auxiliary reference frames: Various reference frames at different speeds would give different results concerning simultaneity at the two 'main' frames. This might correspond to various degrees of asymmetry, (in addition to the *symmetric* solution discussed above). However, we should not interpret this to mean that all solutions are equally valid. I would rather say that we should choose the auxiliary reference frame, which corresponds to the situation we want to model. If we want to model a symmetric situation, there should also be a symmetric reference frame, as we have chosen here. Even if there are several possible definitions for simultaneity at a distance, this does not mean that all are equally valid.

Several standard arguments regarding the twins' ageing seems to have a problem in handling simultaneity. Often the standard narrative seems to implicitly assume that the arrival of the twin at the star occurs 'simultaneously' with the earthbound twin having aged 5 years (in the present example). It is true that the Lorentz transformation tells that the clock of the earthbound system, *which is located at the 'star'*, shows 5 years when the traveling twin arrives (with a clock reading 4 years). However, that does not imply that the clock on the earth reads 5 years 'at the same time'. In my understanding, one cannot infer such simultaneity from the TSR.

This objection applies for instance to the argumentation given in Ref. [10], Chapter 6, which also claims that the returning twin has aged less. However, the various arguments given there starts out with statements like, 'He [the earthbound twin] thinks the whole trip took *T* seconds', (where *T* corresponds to 5 years in our case), 'the earth twin knows the outbound trip took *T*/2 seconds', and 'the whole trip takes a time *T*'. However, we should take a more holistic view, considering the various, apparently, contradictory observations we might have, *cf.* Table 1, and so the argumentation *e.g.* of Ref. [10] appears too simplistic.

Thus, we link our claim regarding simultaneity to arguments of symmetry. After all, we calculate the assumed ageing by use of the LT, which truly exhibits symmetry. I find it hard to defend an asymmetric solution to this symmetric mathematical framework; apparently by adding some ad hoc assumptions outside the scope of the chosen mathematical framework.

Now one could say that also the solution presented here is somewhat paradoxical as it involves apparently contradictory observations for events 'at a distance'. However, this paradox is seemingly inherent in the Lorentz transformation. By accepting this as a model for how the world is 'working', I find the solution presented here consistent and logical.

6 Summary and conclusions

Starting out from the Lorentz transformation (LT) we present an approach for analyzing time dilation and simultaneity in the theory of special relativity. The main elements of the approach are as follows:

- We *reformulate the LT*, in a way that facilitates a *graphical presentation* of the clock time of both reference frames (RFs) in the same diagram. We can also directly read out the effect that the observational principle has on the observed time dilation.
- As an initial step we should decide which RF is chosen as the *primary* one. This means that we will take the perspective of this RF; and simultaneity *in the perspective* of this frame implies that we have the same clock reading at any location on this frame.

- We should also consider the inclusion of an *auxiliary RF*. If so, we usually take this additional RF to be the primary one. The objective could be to provide a sensible definition of simultaneity at a distance.
- We further specify the *observational principle*; that is, we specify the location of the clocks that are used for time comparisons between the reference frames. The graphical presentation is helpful to illustrate this.
- All results obtained will be relative to a chosen *point of initiation*. This is time 0 (on all RFs), where the clocks at the origin of the RFs are at the same location; then being synchronized.

One of the challenges is to balance (account for both) the inherent symmetry of the two RFs and the need to take the perspective of one RF. We claim to achieve this here. In particular, we stress the usefulness of writing the standard time dilation formula as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2}$$

This gives a mutual relation between the two RFs, and tells that when we follow a 'single clock' (SC) on one of the reference frames, this clock goes slower than the passing 'multiple clocks' (MC) on the other frame by a factor $\sqrt{1-(v/c)^2}$. We find this formulation more informative than the potentially misleading phrase 'moving clock goes slower'. There is no reason to see the single clock to be moving and the other clocks not. In general the specification of which system(s) act as SC and which act as MC is crucial for how to comprehend this phenomenon. We illustrate that in a more complex case, one can have both types of observations on both frames. Actually multiple clocks on any RFs will observe that a single clock on the 'other system' goes slower, (and a search for the one RF where time goes slower is indeed in vain).

This 'standard time dilation formula' actually illustrates a specific observational principle. We get another observational principle if we permanently perform clock comparisons at the midpoint between the origins of the two reference frames. This will give identical clock readings at the two frames. So when we apply this observational principle - being symmetric with respect to the two RFs - we also get a symmetric result! Time dilation is indeed caused by an asymmetry in the observational principle.

A main element of the approach presented here is the method for (in some cases) to decide simultaneity at a distance. We introduce an *auxiliary reference frame,* and its position is at any instant completely symmetric with respect to the two main RFs. Following a SC on each of the main RFs it follows by symmetry that identical clock reading corresponds to simultaneity at a distance. We would see an asymmetric solution here as a contradiction.

We further apply the given framework to analyze the travelling twin paradox; here seen as a thought experiment, (disregarding the acceleration periods), presenting a symmetric observational set-up, where we follow the single clocks of *both* twins. We arrive at the conclusion that the age of *both* twins is reduced by a factor $\sqrt{1-(v/c)^2}$, as compared to the travelling time calculated at a stationary reference frame. From any SC perspective one will actually observe a length contraction, and thus also a time reduction. By specifying a RF for each twin, we see that they will both observe this phenomenon.

Actually, one should identify conditions (departures from symmetry) that could cause time dilation to represent a physical reality. I cannot see that such conditions are identified in the travelling twin example for those arguing within the TSR. Thus, the problem with some standard arguments on the travelling twin paradox seems to be a failure to handle properly the simultaneity at a distance.

We finally comment that an observer moving relative to a RF where the event takes place could be a rather unreliable observer. Different observational principles will give different results. Thus, such an outside observer should be careful to define the phenomenon, without taking an overall, holistic view.

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Appendix Example of a numerical calculation

Now we elaborate on the numerical example on the travelling twin in Ch. 6. We use Fig. 2 as an illustration, and this represents the situation when the travelling twin has reached his point of destination. The numerical values, of Ch. 6 gives $w^* = c/3$ (*eq.* (9)), being the speed between any twin and the auxiliary RF. Further, $\sqrt{1-(w^*/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$. Next, we apply *eq.* (4), here inserting w^* for *v,* and then obtain the following clock readings of the three RFs:

i. The auxiliary reference frame (primary). Time is constant, *t*, (see horizontal line in Fig. 2).
ii. Earthbound twin. Time as function of *w*: $t_{-w^*}(w) = (\sqrt{2}/4)\cdot(3+w/c)t$.
iii. Travelling twin. Time as function of *w*: $t_{w^*}(w) = (\sqrt{2}/4)\cdot(3-w/c)t$.

In Fig. 2 we now let the observational point *b,* correspond to $w^* = c/3$; Further let time for the travelling twin equal 4 years, (*i.e.* his arrival). Thus $t_{w^*}(w^*) = 4$, which gives $t = 3\sqrt{2} \approx 4.24$ years, and then we have completely specified the clock readings by the arrival, in the perspective of the auxiliary RF.

So using this $t = 3\sqrt{2}$ in the expressions $t_{w^*}(w)$ and $t_{-w^*}(w)$ above, we will – referring to Fig.2 - get that the point *a* corresponds to 4.5 years and the point *c* corresponds to 5 years; in full agreement with the given example.

The clock reading, $t = 3\sqrt{2} \approx 4.24$ years of the auxiliary (primary) system at this instant seems less relevant. The main role of the auxiliary RF is to allow us to treat the RFs of *both* twins as SC, and thus to establish 'simultaneity at a distance' with respect to these.