An approach for analysing time dilation in the TSR (v3. 2017-08-20)

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Abstract. We investigate time dilation within the theory of special relativity (TSR); basing the argumentation on the Lorentz transformation. The location of the clocks used for time registrations is essential for the value of time dilation, and we investigate various observational principles. Three principles are in focus: A reference frame applying a single clock (SC); or applying multiple clocks (MC); or there can be a completely symmetric situation between reference frames. We specify two types of simultaneity. First, there can be a direct comparison of clock readings at identical positions ('basic' simultaneity). Further, we can consider simultaneity 'in the perspective' of any reference frame. We then introduce auxiliary reference frames, which combined with symmetry considerations, provides a means to define 'simultaneity at a distance'. We apply the approach to present a thorough discussion of the travelling twin example, giving a concise conclusion regarding the twins' ages.

Key words: Time dilation, Lorentz transformation; observational principle; symmetry; simultaneity; auxiliary reference frame; travelling twin.

1 Introduction

The present work explores the concept of time dilation within the theory of special relativity (TSR). Chapter 2 presents some background material, referring to some basic literature; essentially references that present the TSR for non-experts. We further introduce the rather fundamental questions discussed in this paper. Chapter 2 also provides basic assumptions and notation.

Chapter 3 starts out to give a somewhat modified version of the Lorentz transformation. We further pinpoint the importance of the observational principle, that is, the specification of which clocks to apply for the required time comparisons. The chapter provides a unified framework for the various observational principles, and points out the most interesting special cases.

Chapter 4 gives a thorough discussion on the common statement that the 'moving clock goes slower'. We should be very precise in interpreting this statement; the fundamental question obviously being which clock we single out to compare with time readings of the clocks passing on the 'other' reference frame. This distinction is fundamental also in the further approach.

Chapter 5 introduces the concept of *auxiliary reference frame*, which turns out to provide a useful tool. In particular, we investigate two special cases, both with some inherent symmetry. By utilizing this symmetry, we introduce the concept 'simultaneity at a distance'. This type of simultaneity applies in addition to 'basic simultaneity', which refer to events that occur at the same time and same place.

Overall, this work provides a tool for investigating time dilation within the framework of the TSR. In Chapter 6 we apply the approach to give a lengthy discussion on the 'travelling twin' example; claiming to give a logical conclusion regarding the twins' ages. Our result will deviate from the claim commonly given in the literature. In total, the given approach will challenge some of the prevailing views on time dilation and simultaneity in TSR.

Note that our main objective is to investigate the Lorentz transformation and its logical and philosophical implications. Thus, the focus is on the mathematical modelling rather than the physical.

2 Background and basic assumptions

We review some very basic literature, [1]-[6], and also a couple of web sites [7], [8]. Based on this we point out some questions that do not seem fully settled. We further present the basic assumptions of the current work.

2.1 Problem formulation

First, I find the literature somewhat ambiguous regarding the very interpretation of *time dilation*. For instance, how should we interpret the common statement: 'Moving clock goes slower'? Many authors apply the expression 'as seen' by the observer on the other reference system, perhaps indicating that it is an apparent effect, not a physical reality, but without elaborating on the interpretation of 'as seen'. Others stress that 'everything goes slower' on the 'moving system', not only the clocks; truly stating that the time dilation represents a physical reality also under the conditions of TSR (*i.e.* no gravitation *etc*.). On the other side Giulini [3] in Section 3.3 of his book states: 'Moving clocks slow down' is 'potentially misleading and should not be taken too literally'. However, the expression 'not be taken too literally' is not very precise. So in what sense – and under which precise conditions – is time dilation to be considered a true physical phenomenon?

Regarding the basic concept of *time dilation*, I in particular miss a more precise discussion of the multitude of (time) solutions offered by the Lorentz transformation. It is treated by some authors, *e.g.* in [4], but in my opinion not in sufficient depth. In the present work we will - rather than specifying *one* single time dilation formula — look at the total picture of *all* expressions for time dilation.

As pointed out *e.g.* by Pössel [7] the phenomenon of time dilation stems from the fact that clocks of the two systems have to be compared at least twice, so it cannot be the same two clocks being compared. Since movement is relative, however, an interesting question is how to decide which system (clock) is moving. Mermin [4] states that what 'moves' is decided by which clocks are chosen to be synchronized. This seems to be is in line with the views of the present work: the procedure of clock synchronization and clock comparison decides which reference system has the time moving faster/slower.

When reference frames are moving relative to each other, the definition of *simultaneity* becomes crucial. The convention seems to define simultaneity across reference frames by use of light rays, but this hardly maintains the symmetry, (*cf.* Chapter 6 of [14]). Now we can course always consider the 'basic' simultaneity; *i.e.* simultaneity of events occurring at the same location *and* time. In addition, we can consider the simultaneity of events from the 'perspective' of a certain reference frame: As all clocks on a specific reference frame are synchronized, the events that occur at any location, where the clock of this reference frame show the same time, are all simultaneous *in the perspective of this reference frame.* We want to explore the potential also of this type of simultaneity.

The question of *symmetry* is also essential. The TSR essentially describes a symmetric situation for the two systems/observers moving relative to each other, but it seems the literature does not seem to be completely consistent regarding this; some references describe situations apparently involving some asymmetry. For instance, when discussing the travelling twin paradox, Hamilton [8] clearly describes a symmetric situation, while Feynman [2] does not.

Actually, the so-called twins paradox, (*e.g.* [2], [4], [8]) is well suited to highlight the above dilemmas. And even if the main-stream conclusion is that the travelling twin actually ages less than the earthbound twin, there still seem to be some opponents. The so-called Dingle's question, [9] raises the important question of symmetry: "Which of the two clocks in uniform motion does the special theory require to work more slowly? McCausland [10] presents the full question and discusses it at length. I find this an important question, and find it rather surprising that according to [10] it has so far not been given a satisfactory answer.

Some authors also question the validity of the TSR and the Lorentz transformation, see for instance [11]-[13]. In particular, McCausland [11] reviews various controversies on the topic (related to H. Dingle) during several decades, and gives a lot of references. Further, Phipps [12] presents a harsh critique of the TSR modelling, claiming that this fails to include the causes of relative motion and thus to grasp the inherent asymmetry. However, the scope of the present work is to investigate logical and consistent consequences of the Lorentz transformation; thus, accepting the validity of the TSR as a premise.

2.2 Basic assumptions and some notation

The basis for the discussions is the standard theoretical experiment: Two co-ordinate systems (reference frames), K and K_v moving relative to each other at speed, v. We investigate the relation between space and time parameters, (x, t) on system K and the corresponding parameters (x_v, t_v) on the system K_v; thus, considering just one space co-ordinate, (x-axis). We will base the discussions on the Lorentz transformation, including the following specifications:

- There is a complete *symmetry* between the two co-ordinate systems, K and K_v; the systems being identical in all respects.
- On both reference frames there is an arbitrary number of identical, synchronized clocks, located at any positions where it is required to measure time.
- At time $t = t_v = 0$, clocks at the location $x = 0$ on K and location $x_v = 0$ on K_v are synchronized. This represents the defining starting point, from which all events are measured: the 'point of initiation'.
- We will choose the *perspective* of one of the systems, (here usually K), and refer to this as the *primary* system. The time on this 'primary' system is at any position, x given as a constant, $t(x) \equiv t$, independent of x, (all clocks being synchronized). In contrast, at a certain time, t on the primary system, the observed time, t_v on the other ('secondary') system(s), (here K_v), will depend on the location where the time reading is carried out. When there are several reference frames, we are free to choose any one as the primary.
- When we consider two different reference system, there can be a 'basic' *simultaneity* of events, meaning that the events occur at the same time *and* at the same location. So we verify this simultaneity by reading clocks on different reference frames at an instant when they are at the same location. We postulate that all reference frames agree on this basic simultaneity.
- In addition, we have the much weaker concept of 'simultaneity by perspective'. We will say that the events on a reference frame, which show the same time (t) on its synchronized clocks are simultaneous in the perspective of this frame. It is an essential feature of the Lorentz transformation, which in the perspective of the different reference frames there will be distinct differences in the definition of simultaneity.
- We can introduce auxiliary reference frame(s), and thus define simultaneity in the perspective of this additional reference frame. In particular, we find it useful to define an auxiliary reference frame exhibiting a certain symmetry, which leads us to introduce the concept 'simultaneity at a distance'; which is a specific version of 'simultaneity by perspective'.
- We use the notation that SC refers to a reference frame utilizing a 'single clock' (or 'same clock'), for the time comparisons with other reference frames; and similarly MC refers to a reference frame utilizing 'multiple clocks' (at various locations) for time comparisons.

3 The Lorentz transformation and special cases

We here present the Lorentz transformation, and further investigate a variant of this.

3.1 The standard formulation

The Lorentz transformation represents the fundament for our discussion of time dilation. Note that we introduce a change of the standard notation. Rather than the usual t' and x' we will write t_v and x_v. Then the Lorentz transformation takes the form

$$t_v = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - (\frac{v}{c})^2}} \tag{1}$$

$$x_v = \frac{x - vt}{\sqrt{1 - (\frac{v}{c})^2}} \tag{2}$$

Thus the position, x_v corresponds to (has the same location as) x when the clocks at this positions show time t and t_v, respectively. The formulas include the length contraction along the x-axis (Lorentz factor):

$$k_x = \sqrt{1 - (\tfrac{v}{c})^2} \tag{3}$$

So this transformation relates simultaneous time readings, t and t_v performed at identical locations x on K and x_v on K_v.

3.2 An alternative formulation

Taking the perspective of K, we may at any time t choose an 'observational position' equal to $x = wt$, (for an arbitrarily chosen w). By inserting $x = wt$ in (1) we directly get that time on K_v at this position equals:

$$t_v(w) = \frac{1-\frac{vw}{c^2}}{\sqrt{1-(\frac{v}{c})^2}}t \tag{4}$$

Thus to pinpoint the dependence on w we here -and when appropriate- write $t_v(w)$ rather than t_v. The new time dilation formula (4) will – for a given time, t, on the primary system, K - give the time, $t_v(w)$ on the secondary system, K_v, as a linear, decreasing function of w; *cf.* Fig. 1 at the end of the paper; (and a more complete figure in [14]). We can introduce

$$\gamma_v(w) = (1 - \tfrac{vw}{c^2}) / \sqrt{1 - (\tfrac{v}{c})^2} \tag{5}$$

as the 'generalized time dilation factor', valid for any location, (any $w=x/t$), *i.e.* any observational principle. Now we can write (4) as

$$t_v(w) = \gamma_v(w)\, t.$$

In summary the relation presented in Fig. 1 seem rather fundamental for the interpretation of relative time. Being a direct consequence of the Lorentz transformation, it is of course well-known. For instance Feynman [2], *p* 175, refer to this 'failure of simultaneity at a distance'. Further, Mermin [4] gives a thorough discussion on this relation, focusing on how the exact expression for $t_2' - t_1'$ depends on $x_2 - x_1$. But perhaps they do not fully utilize the potential of this general relation.

We note that the result (4), and the accompanying Fig. 1, is quite significant for our understanding of the concept 'simultaneity'. As all clocks on K are synchronized, we say that all events on K, (at any position x), occurring at the same time, t, are simultaneous *in the perspective of reference frame K*. So in the perspective of K also the event that time on K_v equals $t_v(w) = \gamma_v(w)t$ at the position corresponding to $x = wt$ on K, are simultaneous to these events, (*cf.* the 'basic' simultaneity). In the perspective of K_v we of course have quite another result, and so we clearly see the well-known result that the simultaneity in TSR is relative and depends on the perspective of the observer ('primary' reference frame).

Now, similarly to letting $x = wt$, we define w_v so that $x_v = w_v \cdot t_v = w_v \cdot t_v(w)$. By inserting both $x = wt$ and $x_v = w_v \cdot t_v$, in (2), we will after some manipulations obtain

$$w_v = \frac{x_v}{t_v(w)} = \frac{w-v}{1-\frac{w}{c}\cdot\frac{v}{c}} \tag{6}$$

So equations (4), (6) represent an alternative version of Lorentz transformation, here expressed by parameters (t, w) rather than (t, x). The equation (6) has a direct interpretation. According to standard results of TSR (*e.g.* [1]-[4]) the velocities v_1 and v_2 sums up to v, given by the formula

$$v = v_1 \oplus v_2 \stackrel{\text{def}}{=} \frac{v_1+v_2}{1+\frac{v_1}{c}\cdot\frac{v_2}{c}} \tag{7}$$

So by defining the operator $\oplus$ this way, eq. (6) actually says that $w_v = w \oplus (-v)$, or equivalently $w = w_v \oplus v$.

Note that we do not need to think of *w* as a velocity; rather as a way to specify a certain position $x = wt$ on *K,* representing the location of the clocks being applied at time *t*. However, we will later see that it can also be fruitful to interpret *w* as the velocity of a third observational reference frame.

3.3 Standard special cases

Focusing on time, (4) there are various interesting special cases. First, if a specific clock located at $x_v = 0$ on K_v is compared with the passing clocks on *K.* These clocks must have position $x = vt$, and thus we choose $w=v$ and get the relation

$$t_v(v) = t\sqrt{1-(v/c)^2} \tag{8}$$

Which equals the 'standard' time dilation formula. Further, when a specific clock at position $x = 0$, on *K* is used for comparisons with various clocks on K_v, we must choose $w=0$ and get

$$t_v(0) = t\,/\sqrt{1-(v/c)^2} \tag{9}$$

as the relation between *t* and t_v. We return to the special cases (8), (9) in Chapter 4.

Two other special cases are obtained by choosing $w = c$ and $w = -c$, respectively. First

$$t_v(c) \;= \frac{1-v/c}{\sqrt{1-(\frac{v}{c})^2}}t \;= \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}t \tag{10}$$

We could observe this by applying two clocks on both system: One at $x = 0$ and one at $x = ct$ on *K*; and similarly, one at $x_v = 0$ and one at $x_v = ct_v$ on K_v.

So eq. (10) is valid when the light ray is emitted in the positive direction ($x > 0$); *i.e. c* having the same direction as the velocity *v,* as seen from *K*. In the negative direction, (choosing $x = -ct$) we similarly get another well-known result:

$$t_v(-c) = \frac{1+v/c}{\sqrt{1-(\frac{v}{c})^2}}t \;= \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}t \tag{11}$$

The principles (10), (11) seems essentially to be applied for two ways light flashes ('round trips'), *e.g.* see [14]. A fifth special case is treated in Section 3.4.

3.4 The symmetric case

Returning to our version of the Lorentz transformation, (4), (6), we may ask which value of *w* (and thus w_v) would results in $t_v(w) = t$. we easily derive that this equality is obtained by choosing

$$w = w' = \frac{c^2}{v}\left(1-\sqrt{1-\left(\frac{v}{c}\right)^2}\right) = \frac{v}{1+\sqrt{1-(v/c)^2}} \tag{12}$$

By this choice of *w* we further get $w_v = -w'$. This means that if we consistently consider the positions where simultaneously $x = w't$ and $x_v = -w't_v = -w't$, then no time dilation will be observed at these positions. In other words

$$t_v(w') \equiv t \tag{13}$$

At this position $x_v = -x$, and so we consider this to be the midpoint between the origins of the two reference frames; in total providing a nice symmetry. Observe that when we choose this observational principle, (12), then absolutely everything is symmetric, and it should be no surprise that this gives $t_v = t$.

Also note that we could give *w'* a nice interpretation. Recalling the definition of the operator $\oplus$ in eq. (7) for adding velocities in TSR, ($v = v_1 \oplus v_2$), it is easily verified that when *w'* is given by (12), then

$w' \oplus w' = v$. So when (the origins of) K and K_v have speed w' and $-w'$, respectively, relative to the point of observation, then the speed between K and K_v becomes v. Note that we could also link this observational point to an auxiliary reference frame, see Chapter 5.

4 "The moving clock". SC *vs*. MC

In this chapter we investigate the special cases (8) and (9) in more detail, focusing on the concepts SC (single clock) and MC (multiple clocks),

In (8), the specific clock at position $x_v \equiv 0$ on K_v passes the location $x = vt$ (on K) at time t. So on K_v we just apply a single clock (SC) for the time comparisons, so in this case we say that K_v operates as a SC system. So for these SC time readings, $t_v(v)$ of *eq.* (8) we now write $t_v(v) = t_v^{SC}$. Thus, *eq.* (8) becomes

$$t_v^{SC} = t \sqrt{1 - (\tfrac{v}{c})^2}$$

In (9) we follow a clock at $x \equiv 0$ on K, and at this position we make comparisons with various clocks on K_v as they pass along. Now let MC indicate a reference frame utilizing multiple clocks, and so we write $t_v(0) = t_v^{MC}$. Then *eq.* (18) equals:

$$t_v^{MC} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} t$$

But these two cases are closely linked. When K_v operates as SC then K becomes MC and *vice versa*. Thus the two symmetric results, (8), (9) could be presented in a compact form as

$$t^{SC} = t^{MC} \sqrt{1 - (v/c)^2} \tag{14}$$

Actually, this is much more than an effective way to write *eqs.* (8) and (9). By *eq.* (14) we stress that (8) and (9) actually represent the same result. So correctly understood (14) is much more informative than (8) and (9)! It points out which of the systems operates as SC, and which as MC. Note that we in (14) have dropped the subscript v. But we could of course add a subscript v at either t^{SC} or t^{MC}, to indicate which of the systems we see as the secondary system, (if that is of interest).

Inspired by *eq.* (14) we may introduce a new notation. Now t^{MC} is the time measured on a MC reference frame, denoted K^{MC}, and x^{MC} is the position of this time reading. Thus, there are two clocks on K^{MC}, located at $x^{MC} = 0$ and $x^{MC} = v\, t^{MC}$, respectively. Further, there is a SC reference frame, K^{SC} having time, t^{SC}, and we utilizes just one clock on its system, located at $x^{SC} = 0$. To a large extent we apply this notation in the following.

Some comments are relevant here. First we stress that observers on both reference frames agree on this result (14). Thus, I find it rather misleading to apply the phrase 'as seen' regarding the clock reading on 'the other' system; which is a formulation used by some authors. The time readings are objective, and all observers (observational equipment) on the location in question will 'see' the same thing. The main point is rather that observers at *different reference frames* will not agree regarding simultaneity of events.

Secondly, we have the formulation 'moving clock goes slower'. It is true that an observer on a reference frame (K^{MC}), observing a *specific clock* (on K^{SC}) passing by, will see this clock going slower when it is compared to his own clocks. So in a certain sense this confirms the standard phrase 'moving clock goes slower'. However, we could equally well take the perspective of the single clock, considering this to be at rest, implying that the clocks on K^{MC} are moving. The point is definitely not that clock(s) on K^{SC} are moving and clocks on K^{MC} are not. Rather, we could look at the symmetry of the situation: We are starting out with two clocks at origin, moving relative to each other. Then the decision on which of the two clocks we should compare with a clock on the other system, (a decision that can be interchanged at random!), will decide which of the two clocks comes out as the fast one!

So, first of all, none of the clocks are more moving than the other. Further, it is the observational principle that decides which of the two clocks initially at the origin, which we observe to move slower. Therefore I find the talk about the 'moving clock' rather misleading.

This choice on which reference frame shall apply just a single clock is obviously crucial, and it introduces an asymmetry between the two reference frames.

We note that the insight as provided by *eq.* (14) is in no way new. Our concepts SC and MC for instance correspond to the concepts 'proper' and 'improper' time used by Smith [5]. In particular, *eq*. (14) equals *eq.* (3-1) of that book. However, [5] has perhaps not fully utilized the potential of this relation.

We should further stress one fact. It is not required to point at one reference frame to be SC (having 'proper' time), and the other to be MC (having 'improper' time). Actually, we can specify any clock at one of the two reference frames; and if we decide to follow this clock, we will find that it goes slower than the passing clocks on the other reference frame. Thus, we may at the same time have clock(s) on *both* reference frames observed to 'go slower'. In this respect also *eq.* (14) could be misleading. We do not *have* to point at one reference frame to be SC and one to be MC. The equations just says that if we follow a specific clock, we will observe that this goes slower than the passing clocks on the other reference frame.

This point is essential, and in my opinion it gives an answer to the Dingle's question, [9], [10] (*cf.* Section 2.1). It is not the case that the clock(s) on *one* of the two reference frames go(es) slower than the clock(s) on the other, (as indicated by the Dingle's question). We could very well choose to follow *both* the two clocks being at the origin at time 0; which gives both reference frames to have a clock 'going slower'. (This is also most relevant when studying the travelling twin example; see Chapter 6 below.) So, the result on time dilation is actually fully symmetric with respect to the two reference frames! It is not a question of which reference frame has 'slower time', it is a question of which observational principle we choose. This actually demonstrates the inappropriateness of applying the statement 'moving clock goes slower'.

Actually Dingle in his later work claimed that the TSR was inconsistent; see thorough discussion by McCausland, [11]. However, I do not find this argumentation very convincing, as according to [11], Dingle seems to focus on the apparent inconsistency of our *eqs.* (8), (9), rather than discussing the interpretation of the more relevant *eq*. (14).

5 Using auxiliary reference frames

In this Chapter we elaborate further on the special cases represented by (8), (9), or rather (14), in combination with the use of an auxiliary reference frame. Two generalizations are given.

5.1 An auxiliary reference frame having a fixed point of observation ('SC system')

We go back to relations (8), (9), and the combined result, (14). These treat the case where we follow one clock, consistently comparing it with the adjacent clock on the other system (thus, applying several clocks, *i.e.* acting as a MC system). One way to write this result is:

$$t_v^{MC}\sqrt{1-(v/c)^2} \;=\; t^{SC}$$

(through the subscript v the notation here indicates that we see the SC reference frame as the 'primary system'). Now consider a slightly different situation. If we have two systems, K_1 and K_2 moving at relative speeds, v_1 and v_2 with respect to a new auxiliary reference frame denoted K^{SC}, then we similarly have

$$t_{v_i}^{MC}\sqrt{1-(v_i/c)^2} \;=\; t^{SC},\; i = 1, 2 \tag{15}$$

So as the notation indicates, here the auxiliary system, K^{SC} is SC, and K_1 and K_2 are MC, and so we specify a single point on the auxiliary reference frame, where we carry out all clock readings/comparisons at K_1 and K_2. Now we can of course eliminate t^{SC} (*i.e.* the time on the auxiliary system) from these two relations in (15), and then obtain

$$t_{v_2}^{MC} = \frac{\sqrt{1-(v_1/c)^2}}{\sqrt{1-(v_2/c)^2}} t_{v_1}^{MC} \tag{16}$$

In summary, v_1 and v_2 are the velocities of the two MC reference frames K_1 and K_2 relative to a common system, K^{SC}, and (16) now gives the relation between the times of these two reference frame measured at a fixed observational point on this common auxiliary system, K^{SC}. Here, the special case, $v_1 = 0$, reduces to the standard situations, (8). (When $v_1 = 0$ the observational point on K^{SC} is at rest with respect to K_1, and thus K_1 reduces to a SC system in this case.) Further, the special case $v_2 = 0$ reduces to the other standard situation, (9). However, we shall see that also the special case of symmetry (Section 3.4) comes out as a special case.

Of course the two times, $t_{v_i}^{MC}$ of (16) are identical when $v_1 = v_2$. But also by choosing $v_2 = - v_1$ we get the same time reading, *i.e.*

$$t_{v_1}^{MC} = t_{-v_1}^{MC} \tag{17}$$

In particular we can in *eq.* (17) choose $v_1 = - w'$ (which is given in (12)). In that case the relative velocity between K_1 and K_2 becomes $v = w' \oplus w'$, (see discussion in Section 3.4). So by choosing of $v_1 = - w'$ we see that the symmetric case treated in Section 3.4 comes out as a special case of (17); that is

$$t_{-w'}^{MC} = t_{w'}^{MC} \tag{18}$$

which is then actually another way to write (13). In addition, we also observe that *eq.* (15) provides the clock reading t^{SC} of the auxiliary reference frame:

$$t_{w'}^{MC} = t^{SC}/\sqrt{1-(w'/c)^2} \tag{19}$$

This time t^{SC} is however hardly of major interest.

Fig. 2a illustrates the clock readings of these three reference frames. This demonstrates that the clocks on K_1 and K_2, which show the same time, are those clocks, which simultaneously are located at the midpoint in between the origins of K_1 and K_2. (We have 'basic simultaneity, both same time and location.)

Fig. 3 is another illustration. It is a generalization of Fig. 1, presenting the time readings as a function of the position w on the auxiliary system (K). We indicates the simultaneous clock readings of the three reference frames (*cf.* (18), (19)) with a circle marked a. Note that we see K as the primary system.

5.2 The auxiliary reference frame being a MC system

In Section 5.1 we applied a fixed position on an auxiliary reference frame K^{SC} to observe time on the two reference frames K_1 and K_2, and thus obtain the general result (16), having the situations of both Section 3.4 and Chapter 4 as special cases; but seemingly without providing essential new results.

Of course we could also do it the other way. The two reference frames K_1 and K_2 could both be SC, and the auxiliary system would thus be MC. Now this means, that one is able to 'follow' single clocks on K_1 and K_2, and at any time compare these two clocks with clocks on K^{MC} (wherever they are located). This approach will actually provide us with an interesting new result.

In analogy with (15) and (16) we now get

$$t_{v_i}^{SC}/\sqrt{1-(v_i/c)^2} = t^{MC}, \; i = 1, 2 \tag{20}$$

$$t_{v_2}^{SC} = \frac{\sqrt{1-(v_2/c)^2}}{\sqrt{1-(v_1/c)^2}} t_{v_1}^{SC} \tag{21}$$

Again the observational principles (8) and (9) come out as special cases. Also a variant of the symmetric case appears by choosing $v_2 = -v_1$, and again we can choose $v_1 = -w'$ to achieve the relative velocity, v, between K_1 and K_2. Thus, the analogy to (18), (19) equals

$$t^{SC}_{-w\prime} = t^{SC}_{w\prime} = t^{MC}\sqrt{1-(w'/c)^2} \tag{22}$$

So here we specify one position on K_1 and one on K_2 (*e.g.* the origins of these systems), and all clock comparisons with the auxiliary system are carried out at these two locations, see Fig. 2.b, and circles marked *b* in Fig. 3.

Thus, the auxiliary reference frame is now a MC system with time, t^{MC}, and the result, (22) opens for a definition of 'simultaneity' at different – but symmetric – locations. Here $t^{SC}_{-w\prime}$ and $t^{SC}_{w\prime}$ are the times at the origin of K_1 and K_2, and as these origins have moved apart after time 0, and these clocks do not give the same time, neither in the perspective of K_1 nor K_2.

However, *eq.* (22) tells that *in the perspective of this auxiliary reference frame* we have two simultaneous events. When the clocks at the origin of K_1 nor K_2 are observed simultaneously at the auxiliary system, we also get identical time readings. Now simultaneity in the perspective of the auxiliary reference frame is a rather weak sense of simultaneity. However, when we also have this complete symmetry of the positions of the observations (with respect to the origin of the auxiliary system), the result becomes rather interesting, (and not very surprising).

Further, we could also mention that (22) also gives the clock reading at the time on the MC auxiliary system, t^{MC}. But again, this does not seem a very interesting result; here the real significance of the auxiliary reference frame is to establish this simultaneity of event at K_1 nor K_2 'at a distance' (*when they are no longer at the same position*). This is a key question in a proper handling of time dilation.

Finally recall that we illustrate the use of a auxiliary system in Figs. 2 and 3. Note that Fig. 3 is in the perspective of the auxiliary system. The position marked *a* corresponds to the origin of the auxiliary reference frames (see Section 5.1), and positions *b* correspond to the origins of the two main reference frames (treated in the current section). The observant reader might realize that this is an experimental set-up well suited for handling the travelling twin paradox; to be discussed in the following chapter.

6 The travelling twin

We now turn to an analysis of the so-called travelling twin example. As stated for instance in [4] the travelling twin paradox shall illustrate that two identical clocks, initially in the same place and reading the same time, can end up with different readings if they move apart from each other and then back together.

6.1 The numerical example

Reference [4] gives the following numerical example, (in his Chapter 10): "If one twin goes to a star 3 light years away in a super rocket that travels at 3/5 the speed of light, the journeys out and back each takes 5 years in the frame of the earth. But since the slowing-down factor is $\sqrt{1-(3/5)^2} = 4/5$ the twin on the rocket will age only 4 years on the outward journey, and another 4 years on the return journey. When she gets back home, she will be 2 years younger than her stay-at-home sister, who has aged the full 10 years." So the claim is that the referred difference in ageing occurs during the periods of the journey with a constant speed; *i.e.* under the conditions of the TSR, (ignoring the acceleration/deceleration periods). After all the whole argument relies on the Lorentz transformation! Thus, our discussion will fully restrict to the periods of constant velocity. Now throughout this chapter we let

t = time on clock of earthbound twin

t_v = time on clock of travelling twin

The distance between earth and the 'star' equals $x = 3$ light years, and the rocket has speed, $v = (3/5)c$, giving $\sqrt{1-(v/c)^2} = 4/5$. It follows that in the reference frame of the earth/star, the rocket reaches the star at time, $t = x/v = 5$ years. And the clock on the rocket is then located on $x_v = 0$, corresponding to $x = vt$, and thus the Lorentz transformation gives that at the arrival at the star this clock reads $t_v = t \cdot \sqrt{1-(v/c)^2} = 4$ years; so obviously, $t_v/t = 0.8$ at the star (travelling twin); and the argument is also valid for the return travel.

This is a rather convincing argument. It does follow from the Lorentz transformation that the returning clock shows 8 years when he/she returns. However, recalling the discussion of Chapters 3-5 the case is perhaps not that straightforward, and since we have made no assumption of asymmetry regarding the periods of constant velocity, we seem to have a true paradox.

Thus, we will not question the clock of the travelling twin, but take a new look at the clock of the earthbound twin, trying to look at the total situation. First, we observe - following the notation of Chapter 3 - that the above presentation describes the travelling twin as a 'SC system', and so the earthbound twin is located on a 'MC system'. So, actually we could just look at *eq.* (14) to obtain the above result. And as observed in Appendix A.1 this is related to the length contraction: Seen from the perspective of the travelling twin, the distance between earth and the star does not equal $x = 3$ light years but just $x_v = 3 \cdot 0.8 = 2.4$ light years; fully 'explaining' the reduction in travelling time.

Now the question is: Could we not similarly describe the situation as the travelling twin being located on a MC system, and the earthbound twin on a SC system (which would then have one clock located on the earth). If we insist on the symmetry of the situation, the answer must be yes. Thus, we simply assume that there is also a reference frame of the travelling twin with the required number of clocks. Say, he is equipped with rockets at appropriate and fixed distances from his own rocket, all moving with constant speed in the same direction as himself, and all equipped with a synchronized clock showing the same time, t_v. Whether this is practically feasible is not relevant here. We are referring to the model of the TSR, and point out what this theory tells about clock readings, *if* we provide such an arrangement.

By making this assumption, we could consider the earthbound twin as travelling back and forth along the reference frame of the travelling twin. This will now give that the one way 'travelling time' of the earthbound twin is 4 years; while the time passed for the travelling twin equals 5 years. Due to this symmetry of results, we find it required to proceed with the discussion. Now there is both a lengthy and a short argument on this paradox. We first take the lengthy. But those fully familiar with the discussion of Chapter 5, might skip Section 6.2 and go directly to Section 6.3.

6.2 The lengthy argument

We include this section to contemplate on the phenomenon with an open mind, without directly utilizing the simultaneity result of the previous chapter. We first follow up on the possibility of treating the earthbound twin as a SC system. Thus, he just apples his clock at the earth for time comparisons with the various travelling clocks; (therefore the reference frame of the travelling twin is equipped with several clocks at various locations). Under this assumption *eq.* (14) gives the result, $t_v / t = 1/0.8 = 1.25$ (for his observations on the earth) whatever instant we consider after departure.

We now want him to make an observation of the clocks exactly at the instant when his traveling twin arrives at the 'star'. Then the problem is that we actually do not know which moment on the earth that corresponds to this. The Lorentz transformation does not seem to give a definite answer regarding the simultaneity of events 'at a distance'. So now let us consider various options regarding the moment at which he should observe the clocks; (both his own clock positioned on the earth, and the one passing by):

1. *Perspective of the travelling twin*. At the moment when the travelling twin arrives to the star, the clock on his rocket shows 4 years. So all clocks on the reference frame of the travelling twin show

time, $t_v = 4$ years. This is also the case for the clock which as this moment is passing the earth, *i.e.* at $x = 0$. Thus, the clock of the earthbound twin at this instant shows time $t = t_v \cdot 0.8 = 3.2$ years.

2. *Perspective of the earth/star system*. At the instant when the twin arrives at the star, the time of the earthbound system at this location equals $t = 5$ years. The earthbound twin could verify this by also installing a clock at the 'star'. When he performs a clock comparison at the earth ($x = 0$) at this moment, it gives that $t_v = t \cdot 1.25 = 6.25$ years for the clock which passes the earth at this moment.
3. *Perspective of the auxiliary system* (*'Symmetric solution', cf.* Section 5.2) The above two cases demonstrate that the two twins completely disagree about which event at the earth is simultaneous with the travelling twin's arrival at the star; (which of course is obvious also from *e.g*. Fig. 1). Now consider the moment when the clock on the earth shows $t = 4$ years and the passing 'travelling clock' shows $t_v = t \cdot 1.25 = 5$ years. This instant obviously occurs in between the previous two moments, and represents a moment being completely symmetric to the event of the twin's arrival at the star (regarding clock readings). And more important: It is the instant when the earthbound twin have carried out a 'travel' equivalent to the distance of the travelling twin. There is a complete symmetry!

In summary, when we now let the earthbound twin represent a 'SC system', and thus, carry out the clock comparison at the earth, we always get $t_v/t = 1.25$; *i.e.* it is the clock on the earth that 'goes slower'. We summarize the findings in Table 1. These three 'perspectives' in some way all 'correspond to' the arrival of the travelling twin at the star, and thus, demonstrate the problem we have to define the 'simultaneous' event on the earth. Note that we have skipped the case of having a single clock located in the midpoint between the two twins (Section 5.1). Our objective is to follow the two clocks, and in this option we follow neither of the clocks.

Table 1. Various clock readings (light years) at/on the earth, *potentially* 'simultaneous to' the arrival of the travelling twin at the star.

Location of time reading	'Perspective'		
	1.Travelling twin	2.Earthbound twin	3.Auxiliary system
Travelling twin system (***t_v***)	4	6.25	5
Earth/star system (***t***)	3.2	5	4

So how should we conclude regarding the time (clock readings) at the turning of the rocket? When now the information of Table 1 is available, let us assume that the earthbound twin is in charge. He could control his twin's travel by sending a light signal to the star, which on arrival initiates the return of his travelling twin. How should he do this to be sure the signal arrives at the right moment? One possibility that he might consider is to send a signal that arrives at the star when his earthbound clock shows 5 years (*i.e.* perspective 2 in Table 1). The problem is that at this moment the clock on the travelling twin system passing the earth shows 6.25 years. Thus, one could suspect the travelling twin when he returns have aged 12.5 years (and not 8). The reason being that if he turns when his twin's signal reaches the rocket, he may have travelled a longer distance than the intended 3 light years. A similar objection applies to using perspective 1.

Actually, if the earthbound twin should be in charge, I guess the following strategy should be the most ingenious. Knowing about the length contraction, he will know that the travelling twin will observe a travelling distance to the star that equals just 2.4 light years. So the earthbound twin will adopt option 3: he sends a signal ordering to turn, such that the travelling twin will receive this signal when the clock on his own earthbound system shows 4 years, (*cf.* 'perspective 3' of Table 1).

Following this option, we conclude that at the local time when each of the twins now consider to be the turning of the rocket, the twins will agree on the following facts: Their own clock shows 4 years, and the adjacent clock on the other system shows 5 years. So by the direct measurements, they observe that the other twin at this moment apparently has aged more than himself by a factor 1.25. This gives a completely symmetric and consistent answer to the paradox.

Following this argument, the clocks of both twins show 4 years at the point of return. The same argument applies for the return travel, and we should conclude that by the reunion *both* clocks show 8 years.

Another way to put it. We can choose between three options (all in agreement with the Lorentz transformation):

1. Either the travelling twin being on a SC system giving travelling times 8 years for him and 10 for the earthbound, *eq.* (14), or
2. The earthbound twin is located on a SC system, giving 8 years for him and 10 for the travelling *eq* (14);
3. The symmetric solution, *eq.* (22), treating both systems as SC, giving a total duration of 8 years for both twins.

To me this choice is easy. In addition to the symmetry, it is a point that options 1 and 2 just follow the clock of *one* twin, while option 3 follows *both.*

6.3 The short argument and further comments

The short argument utilize the symmetry of the situation and directly applies the simultaneity in the perspective of the auxiliary system; that is 'simultaneity at a distance', *cf.* Section 5.2 and Figs. 2-3. According to this result, the event (t_v = 4 years, t =5 years) at the star is 'simultaneous' to the event (t_v = 5 years, t = 4 years) at the earth. So by considering *both* systems as SC, the conclusion is that both twins have aged 8 years when they meet again. We follow both clocks in a symmetric way, from the moment when they depart (having basic simultaneity) to the moment when they are again united (again basic simultaneity). The reduced time is explained by SC observers experiencing a length contraction, *cf.* Appendix.

To arrive at this conclusion we also assume that the return travels for both twins have the same duration as their travel before turning. They are passing exactly the same distance, at exactly the same speed, (giving same length contraction), and so their clocks should measure the same time duration for the outwards and return travels.

One might still raise the question: since the simultaneity by perspective obviously differ so wildly, why should we trust the perspective of the auxiliary reference frame introduced here, and not the result of any of the other reference frames that we could come up with? Our answer is simply that this is the only perspective exhibiting the desired symmetry between the two reference frames. Can we come up with an asymmetry here, requiring a certain asymmetry; we would choose an auxiliary system to accommodate for this. So far we have seen no such asymmetry. Actually, when we – as in the present work - focus on the full symmetry of the situation, it would be rather meaningless to claim that one ages slower than the other.

Some authors (*e.g.* [2], [5]) actually argue that the acceleration required for the travelling twin when he turns, is the reason why we can distinguish between the two twins, introducing an asymmetry, and therefore infer that one of them ages slower. Of course, this acceleration introduces an asymmetry between them, but the difference in ageing is always explained by the Lorentz transformation, which actually exhibits symmetry (and does not treat acceleration periods). Therefore, I do not find these explanations convincing.

So what is then wrong with the standard argument. It is obviously about simultaneity. The standard narrative seems implicitly to assume that the arrival of the twin at the star occurs 'simultaneously' with the earthbound twin having aged 5 years. I disagree with this claim. The Lorentz transformation tells that the clock of the earthbound system, *which is located at the 'star'*, shows 5 years by the arrival of the traveling twin. However, that does not imply that we can say that the earthbound twin has aged 5 years 'at the same time'. In my understanding, one cannot infer such simultaneity from the TSR.

A similar objection applies to the argumentation given in Chapter 6 of [5], which also claims that the returning twin has actually aged just 8 years and not 10, (when we apply the present numerical example). However, the various arguments starts out with statements like, 'He [the earthbound twin] thinks the whole trip took T seconds', (where T corresponds to 5 years in our case), 'the earth twin knows the outbound trip took $T/2$ seconds', 'the whole trip takes a time T'. So to me it seems he starts out by making an assumption that must lead to the desired conclusion, and we have a circle argument. A crucial question is the totality of information in principle available for the earthbound twin, as discussed in the previous section; *cf.* Table 1. Then we should not take for granted 'what he thinks', and he will not necessarily 'know' that the whole trip takes 10 years. As many authors discussing this he seems to lack a more holistic view, considering the various, apparently contradictory observations we have even on the same reference system. That there is an observation of 5 years at one location of his reference frame, does not imply that the earthbound twin concludes the event 'occurs at time 5 years'. As we know, time depends on location! Thus, the argumentation of [5] is too simplistic.

Now I have to admit, that also the solution presented here is somewhat paradoxical. (It is hard for me to follow those claiming there is no paradox at all here.) It involves apparently contradictory observations for events 'at a distance'. However, this is a paradox seemingly inherent in the Lorentz transformation. By accepting this as a model for how the world is 'working', I find the solution presented here consistent and logical.

6.4 Some final numerical considerations

Now to familiarize a little further with the argument regarding the 'simultaneity at a distance' (in the perspective of the auxiliary reference frame) we elaborate on the above numerical example. As an illustration, we use Fig. 3, and consider this to represent the situation when the travelling twin has reached his point of destination. From the above numerical values, we will from (12) get $w' = c/3$ ($= 5v/9$), also giving $\sqrt{1-(w'/c)^2} = \frac{2}{3}\sqrt{2} \approx 0.94$. Now Fig. 3 illustrates time readings of the three reference frames: 1)Auxiliary system, 2)Travelling twin, moving at speed w' with respect to this, 3)Earthbound twin, moving at speed $-w'$ relative to the same system. The time readings are now given by the perspectives of:

1. The auxiliary reference frame (primary): horizontal line; value t.
2. Time of travelling twin as a function of w (=position at auxiliary system). According to *eq.* (4) the function equals $(\sqrt{2}/4) \cdot (3 - w/c)t$.
3. Time of earthbound twin as a function of w (=position at auxiliary system). According to *eq.* (4) the function equals $(\sqrt{2}/4) \cdot (3 + w/c)t$.

In Fig. 3 we now let the lower point of the observational point b have value 4 years, (*i.e.* the clock readings of the travelling twins at this instant). Then the clock reading at on the auxiliary system according to *eq.* (22) equals $t = 4/\sqrt{1-(w'/c)^2} = 3\sqrt{2} \approx 4.24$ years.

Now inserting $t = 3\sqrt{2}$ in the above time expressions for the two twins, we get $1.5 \cdot (3 - w/c)$ and $1.5 \cdot (3 + w/c)$, respectively. In order to get the the clock readings at the 'point of return', we choose w equal to the two 'locations' of the standard time dilation formula, see (8), (9) (summarized in (14) of Chapter 4). First $w = 0$, corresponds to the origin of the auxiliary system, giving time 4.5 years; *cf.* point a in Fig. 3. Next $w = w' = c/3$, corresponds to the origin of travelling twin, giving 4 and 5 years respectively. Similarly, $w = -c/3$ corresponds to the origin of the earthbound twin; see point b in Fig. 3. So all values are in full agreement with those used in the example.

This numerical example illustrates that the relation between the clock readings of the auxiliary system and any of the twins' system is equal to the relation between clock readings the reference frames of the two twins; we just replace the 'slowing down' factor $\sqrt{1-(v/c)^2}$ with $\sqrt{1-(w'/c)^2}$.

We might observe that in the perspective of the auxiliary system, the time back and forth equals $2 \cdot 3\sqrt{2} = 6\sqrt{2} \approx 8.5$ years. However, this is not so relevant. It is similar to the above result that in the perspective of earthbound twin the total time equals 10 years. (We could successively introduce new auxiliary reference frames, whose measured time quite obviously would approach 8 years.) The role of the auxiliary system is essentially to establish 'simultaneity at a distance', and allowing us to treat the reference frames of both twins as SC.

7 Summary and conclusions

We discuss how to analyse time dilation between two reference frames moving relative to each other at constant speed v. Starting out from the Lorentz transformation, we make the following specifications:

- There is a complete *symmetry* between the two reference frames.
- Each reference frame is equipped with a number of synchronized clocks, (at any required position).
- We may take the perspective of one reference frame and specify this to be the *primary* one.
- *Basic simultaneity* of events at different reference frames will refer to events at the same location at the same time; 'simultaneity by location'.
- We also introduce the concept simultaneity in the perspective of a reference frame. By introducing an *auxiliary reference frame* we can use this in combination with symmetry to define 'simultaneity at a distance', (or equivalently 'simultaneity by symmetry').
- We always specify the applied *observational principle*, which means that we specify the location of the clocks that are used for time comparisons between the reference frames. Investigations of time dilation should clearly account for the effect of the observational principle.
- We stress the distinct difference between 'single clock' (SC) observations – where the same clock is used for time comparisons, and 'multiple clock' (MC) observations – where several clocks along the x- axis of a reference frame are applied. In more complex situations (*cf.* travelling twin), we may consider both type of observations on both frames.
- We do not use the expression '*as seen'* (from the other reference frame). Observers (observational equipment) on both reference frames agree on the time readings; as they are carried out 'on location'.
- We do not describe time dilation by the expression 'moving clock goes slower'. It is the observational principle that matters.

It is an important fact that at a given time, t on K, (in the perspective of K), the time, t_v observed on a clock at K_v will depend on the position, x on K. So we focus on a variant of the Lorentz transformation showing how t_v/t depends on $w = x/t$.

From this general time dilation formula, we get the 'standard' time dilation result, here written as

$$t^{SC} = t^{MC}\sqrt{1-(v/c)^2}$$

This tells that when we follow a single clock (SC) on a reference frame, this is seen to go slower than the passing clocks on the other frame by a factor $\sqrt{1-(v/c)^2}$. This formulation is much more informative than the potentially misleading phrase 'moving clock goes slower'. There is no reason to see the single clock to be moving and the other clocks not. Movement is relative. In general the specification of which system(s) act as SC and which act as MC is a crucial element of the approach.

Another special case of the general time dilation formula is to permanently choose the midpoint between $x = 0$ and $x_v = 0$ as the location for time comparisons. Here we will observe $t_v = t$. Therefore, this choice represents an observational principle being symmetric with respect to the two reference frames. So when we otherwise observe $t \neq t_v$, in our symmetric situation, we can claim that this is caused by the asymmetry of the chosen observational principle.

We apply the approach to the travelling twin paradox. As a standard numerical example goes, the travelling twin will – at a speed of $0.6 \cdot c$ – travel a distance of 3 light years both ways, and age only 8 years during his round trip, as opposed to the 10 years passed for the twin on the earth.

By treating this example in a completely symmetric manner, we come to a different result. Both twins have their single clock (SC) reference frame, which are moving relative to each other. Both twins will thus observe a length contraction and a time dilation relative to the *reference frame* of the other twin passing by. So they will both observe that only 4 years has passed when they shall turn. By introducing an auxiliary reference frame located 'in between' the twins' frames, we will establish a 'simultaneity at a distance' for the turning events of the twins. My claim is that the problem with the standard arguments on the travelling twin paradox relates to this simultaneity.

Further, under conditions of complete symmetry it is rather meaningless to claim a 'true' time dilation, causing different ageing at the two systems. Therefore, it could be interesting to identify conditions – *e.g.* departures from symmetry - that could cause time dilation to represent a physical reality. I cannot see that anybody has truly identified such conditions in the travelling twin case.

A general comment regarding our findings: An observer moving relative to a reference frame where an event takes place could be a rather unreliable observer. Various observational principles will provide him with different results. Thus, one should be careful to let such an 'outside' observer define the phenomenon, without taking an overall view and properly consider his own position.

We finally note that the results given here are a rather direct consequence of the Lorentz transformation, and are not necessarily new. However, in total the suggested approach for investigating time dilation has some distinct differences, compared to current narratives on the topic.

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Appendix A A note on length contraction

The interpretation of x and x_v in (1) and (2) is rather straightforward. The position x_v on K_v corresponds exactly to the position x on K at an instant where the clock located at x_v shows time t_v and the clock at x shows time t.

However, x and x_v could also have a slightly different interpretation. Consider again a 'SC system', K_v moving at relative speed along a system, K. Now let a distance, x be marked out on K in the same direction as this movement. As known, the time measured on K_v for its single clock to pass this distance will imply that – as measured from K_v – the length of the distance x equals

$$x_v^{SC} = x\sqrt{1-(\frac{v}{c})^2} \tag{A.1}$$

So now x_v^{SC} equals the length of x on K 'as seen from' K_v. (In order to utilize the clock reading to observe x_v^{SC} one first have to establish the relative speed, v, between the reference frames.) Therefore, this length contraction (A.1) corresponds exactly to the time dilation observed for a single clock moving relative to a fixed distance on the other reference frame. Thus, anyone on K_v observes the distance travelled to be shorter, and so the time required to travel this distance will be observed to be shorter (both on K and K_v). Therefore, the length contraction and time dilation are indeed two aspects of the same phenomenon.

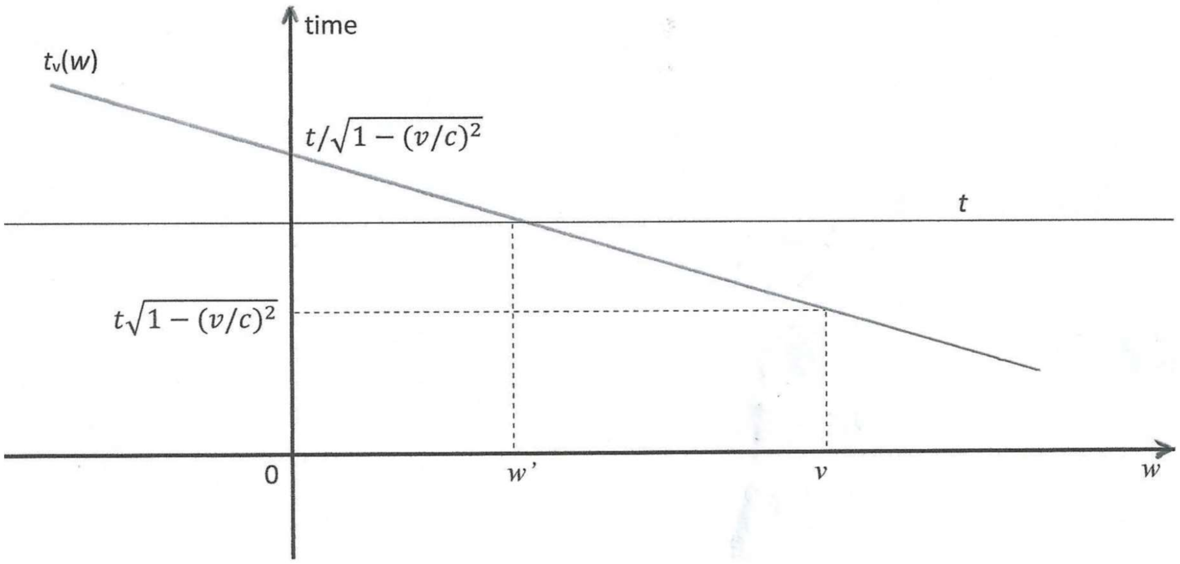
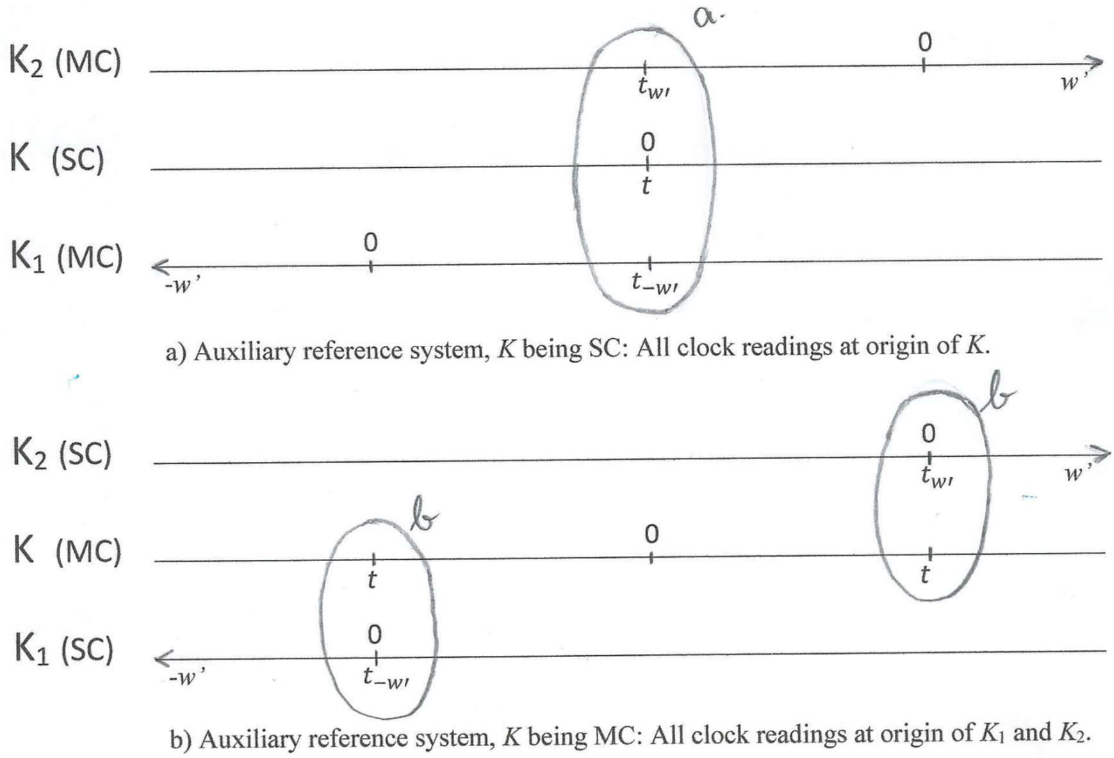


Figure 1. Time, $t_v(w)$,on K_v as a function of w. Here we have the perspective of K: The time all over K equals, t. Further, $w=x/t$ gives the position on K at which we observe the time on K_v. The three special cases of *eqs.* (9), (13) and (8) are inserted; corresponding to $w = 0$, w' and v, respectively.



a) Auxiliary reference system, K being SC: All clock readings at origin of K.

b) Auxiliary reference system, K being MC: All clock readings at origin of K_1 and K_2.

Figure 2. Time observations on reference frames K_1 and K_2, having relative speed -*w'* and *w'*, respectively, relative to the auxiliary system, K. Here K serves as the 'primary system'; so time equals t all over K. Note that time readings at these positions on K_1 and K_2 are identical; *i.e.* $t_{w'} = t_{-w'}$; for case *a* see *eq.* (18), and for case *b* see *eq.* (22. Observe that case *b* refers to two different locations.

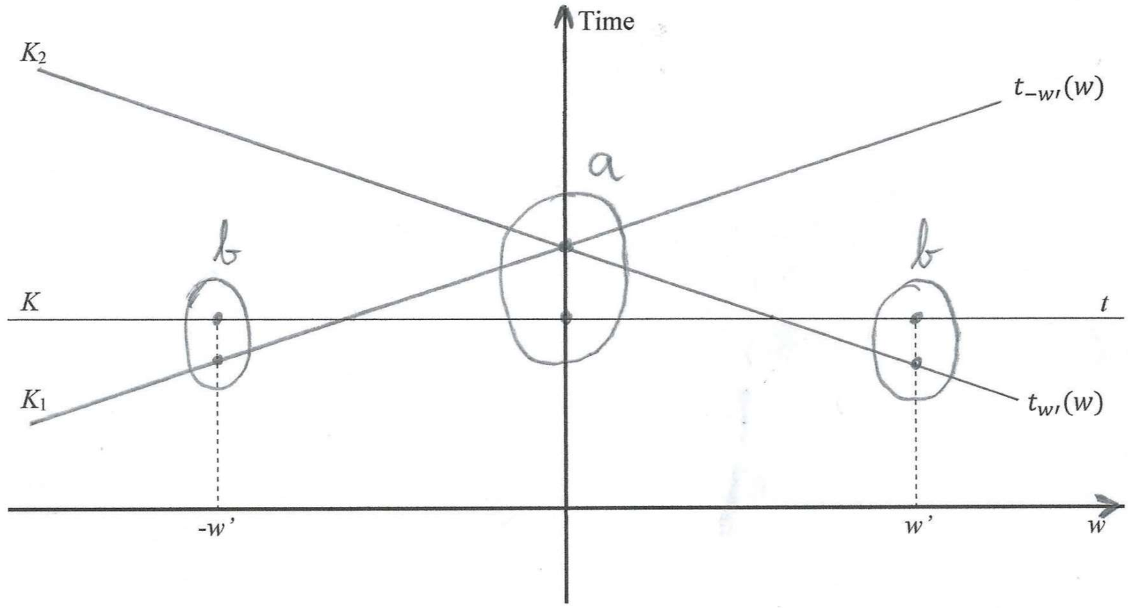


Figure 3. Time measurements in the perspective of the auxiliary reference frame, K. So time equals t all over K. Times $t_{-w'}(w)$ on K_1 and $t_{w'}(w)$ on K_2, given as a function of the 'position' w on K. So the reference frames K_1 and K_2 have speed ±*w'* relative to the auxiliary system K. Simultaneous time readings (in the perspective of (K) for cases *a* and *b* are marked with circles as in Fig. 2. Again observe that we have $t_{w'} = t_{-w'}$ in both cases.