Evidence for quantum-interference phenomena in the femtometer scale of baryons.

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Abstract
In a series of papers we have shown that through the imposition of gauge invariance conditions to the wavefunctions representing each particle, it is possible to relate rest energy to magnetic moment for the baryons. A key point of this model is the requirement that the magnetic flux linked through the region covered by the particle be quantized in units of $\frac{hc}{e}$, which converges to the inverse dependence of mass with the fine structure constant alpha, as reported in the literature. Our most accurate results however display deviations from the strict integer numbers of flux quanta, which requires an explanation. The objective of the present paper is to show that such deviations can be precisely associated to the flux dependence of the phase differences of interfering currents flowing through Josephson Junctions in the DC mode. In the same way as in macroscopic Josephson Junctions between superconductors, quantum interference between the constituents of baryons takes place when constituents superpose, which gives rise to squared sinusoidal undulations observed in a plot of the flux confined for each baryon against the respective magnetic moments.
1. Introduction.

In a series of papers we have shown that through the imposition of gauge invariance conditions to the wavefunctions representing each particle, it is possible to relate rest energy to magnetic moment for the baryons. A key point of this model is the requirement that the magnetic flux linked through the region covered by the particle be quantized in units of $hc/e$, which converges to the inverse dependence of mass with the fine structure constant alpha, as reported in the literature. In our most recent work [1] we (following E. Post [2]) called this situation the “limit” realization of the Aharonov-Bohm Effect (A-B) [3]. Our most accurate results however display deviations from the strict integer numbers of flux quanta, which requires an explanation. The objective of the present paper is to show that such deviations can be precisely traced back to the oscillations observed in the phase differences of interfering currents flowing through Josephson Junctions [4] in the DC mode. In the same way as in macroscopic Josephson Junctions, quantum interference between the constituents of baryons take place when constituents superpose, and thus phase differences vary from one baryon to another, which gives rise to the squared sinusoidal undulations observed in a plot of flux for each baryon against the respective magnetic moments.

2. Theory.

Isolated current-loops containing a single quantum of flux of value $\phi_0/2$ are well known from type-II superconductivity [4]. The formation of superconductor current loops is a many-body effect, though. In a series of papers we have investigated if there might exist single-particle systems confining flux in a similar manner. It is essential that such proposal be
quantitatively supported by experimental data. Let’s consider the actual case of particles of the baryon octet. In spite of their rather short mean-lives all the eight particles have well-established rest masses and magnetic moments. E.J. Post [2] considered how to associate these latter two variables in a tentative model for the electron. Post showed that the magnetic moment for the electron could be obtained up to the first-order correction (from QED) with the equation:

\[ mc^2 = \frac{\phi i}{c} + eV \]  

(1)

Here the left side is the rest energy of the electron, which from the right side is considered as fully describable by electromagnetic terms. The first one on the right side is the magnetic energy of an equivalent (hypothetic) current ring of value \( i \) linking an amount of flux \( \phi \), that should occur in a number \( n \) of flux quanta \( \phi_0 \). The second (electrostatic energy) term is much smaller than the first (it will be neglected hereafter) and accounts for the radiation-reaction correction for the magnetic moment which is proportional to the fine structure constant \( \alpha \), as is well known[2]. Post associates the current with the magnetic moment \( \mu \) and the size \( R \) of the ring with the equation:

\[ \mu = \pi R^2 i/c \]  

(2)

It must be pointed out that albeit useful, the parameters in these equations cannot be strictly interpreted as their macroscopic counterparts do. For instance, Barut developed a full theory for the leptons[5-7] in which he shows that radiation-reaction terms from QED can be translated in the simple picture of a conventional interaction between the self magnetic field of a lepton and an anomalous magnetic moment, characterized by a \( g \)-factor which is introduced to make the bridge between the two pictures.
In the present case we are interested in assessing a sufficiently large group of particles in order that flux quantization can be properly demonstrated, as predicted by A-B, and the baryons form such a group. The parameter $R$ must be determined for substitution into (2). In the case of nucleons, experimental determinations of the radius of a proton have been undertaken since the 1950s, and the most recent value is of about 0.85 fm. Miller [8] has carried out detailed theoretical calculations of the charge distribution around nucleons. His plot of charge distribution [8] points towards an averaged radius of 0.6 fm, which will be adopted as discussed below.

The model by Post was devised to fit a single fundamental particle, the electron. There is however consistent evidence that the constituents inside baryons form a topologically individualized structure, resembling a correlated Cooper pair in superconductors, so that a “single particle-model” may be applicable (as evidence, the said structure is not observed outside the baryon they belong to, and the constituents are confined by strong inward forces). One then inserts equation (2) into equation (1) (without the electrostatic small term) and thus eliminates the current. The parameter $R$ has been calculated/measured for the nucleons only, but it remains part of the final expression for all baryons obtained after the combination of (1) and (2). We may conveniently eliminate $R$ from this treatment by adopting for all baryons an expression which is valid for the leptons (for $R= \lambda$, the Compton wavelength), namely:

$$\mu = eR/2$$

which is certainly valid for the proton for $R= 0.6$ fm from [8] (cf. Table 1).

The combination of equations (1)-(3) with $\phi = n(hc/e)$ can therefore be cast in the final form (inserting $\alpha = e^2/hc$):

$$n = (2e^2 \alpha/e^3) \mu m.$$
3. Analysis.

Equation (4) is the main result of this work. It has been derived from the assumption that a femtometer-scale particle (baryon) can be represented by a loop of current, based upon an analogy from superconductivity and from Post’s model for the electron. A plot of $n$ against $\mu$ should be confined to a straight diagonal line provided the mass $m$ does not depend on $n$ and $\mu$. The continuity of the wavefunction around the loop requires $n$ to be an integer, which would constitute the limiting case of the A-B effect. All the parameters on the right side are known for the eight baryons of the octet, and are listed in Table 1 (data from [10]). Figure 1 shows the plot of the calculated $n$ against the magnetic moment for each particle. There is indeed a tendency to form Shapiro-like steps at integer numbers of flux quanta but there are also clear undulating deviations for all values of the moment.

As stressed earlier, it was shown by Barut that radiation terms from QED can be treated in the rather simple classical picture of the interaction between the anomalous magnetic moment with the particles self-field. A classical electromagnetic calculation carried out in ref. [9] considers the cyclotron extra rotations produced by the effect of the magnetic field due to an electron’s spin magnetic moment. The calculation is used to predict that one quantum of flux across the area covered by the particle is associated with each (Bohr- or) nuclear-magneton of magnetic moment. As shown in Table 1, some values of $n$ calculated to simultaneously fit mass and magnetic moment follow such classical result quite closely. However, the plot in Figure 1 displays clear deviations from the classical result.

Clarification of the origin of such deviations requires further investigation. The theory of transport across Josephson Junctions between superconductors provides a proper explanation, which is now described[4].
Up to now we have assumed that there is no break in coherence in the flow of the “currents” inside the particle. It would be consistent with the picture proposed many years ago by Herbert Jehle[11] that the electrically charged quark constituents individually move and superpose, interfering with each other in this process. Quantum interference of the kind considered in the theory of Junctions might then be applied. In the DC case it can readily be shown that the current is determined by the (wave functions) phase difference $\theta$ across a contact between constituents:

$$I = I_c \sin(\theta)$$  \hspace{1cm} (5)

Where $I_c$ is a critical current across the contact. We might rewrite the magnetic energy term in (1) adding to the phase-independent $i\phi/c$ the work to establish the phase difference $\theta$ given by the integration in $\theta$ of the current in (5). The deviation $\Delta m$ from the formerly calculated mass should be proportional to such integrated energy, given by:

$$\Delta m = m_1 (1 - \cos(\theta)) = m_1 (2 \sin^2(\theta/2))$$  \hspace{1cm} (6a)

$$\Delta n = (2c^2 \alpha/e^3) \mu \Delta m$$  \hspace{1cm} (6b)

Here $m_1$ is a parameter (to be adjusted by fit) which would include the unknown details of the tunneling process across the junctions, including the critical current. Expression (6b) has the expected form to justify undulations deviating from the straight line in the ploy for $n$ in Figure 1. The mass $m$ now acquires a squared sine correction dependent on phase. Such phase is proportional to the number of flux quanta $n$, which to first order should be proportional to the moments $\mu$. Figure 2 displays two fits of the data in Table 1 including a squared sine dependence with phase for $m$, with the phase taken as proportional to $\mu$. It is possible to see that even a
simple consideration of such femtometer-scale quantum interference phenomena can lead to fully quantitative corrections to the previous model.


In conclusion, we have analyzed well-known tabulated data for the masses and magnetic moments of the baryon octet particles. The model associates magnetic energy with the rest energy, and gauge invariance implies flux quantization within the area covered by the particles charge motion. However, Figure 1 displays a tendency for flux to adopt quantized values, but with deviations associated with quantum interference effects akin to those described by the theory of the DC Josephson Effect. The data are successfully fit when such interference is accounted for by means of the predicted squared sinusoidal dependence upon the phase difference between interfering wavefunctions.

References

Table 1: Data for the baryon octet (moments $\mu$ from ref. [10]). According to equation (4) in gaussian units, $n = 1.16 \times 10^{47} \, \mu \, m$. The plot of $n$ against $\mu$ (n.m.) is shown in Figure 1.

<table>
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<th>abs $\mu$ (n.m.)</th>
<th>$\mu$ (erg/G) x $10^{23}$</th>
<th>$m$ (Mev/c$^2$) x $10^{24}$</th>
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Figure 1: Plot of $n$ against the magnetic moment following eq (4) and Table 1. The diagonal line is the classical prediction of one flux quantum per nuclear magneton (n.m.)[9]. Horizontal steps at integer values of $n$ are shown. The data display a tendency to reach for the steps (traced line as guide), but deviations are obvious.
Figure 2: Corrections to the model due to quantum interference between charged constituents inside baryons. Jehle [11] proposed a complete theory for the topology of quarks constituents, which should move and superpose inside baryons. This may result in interference effects like the undulations in Figure 1. The solid line includes a pure squared sine curve multiplied by $\mu$ to transform mass into $n$ according to (4) and (6b). The traced line includes an extra constant phase factor in the argument of the sine function to better fit the data. The result is an additional undulating $\Delta m$ (and $\Delta n$) above the diagonal solid line.