DEMONSTRATION OF THE RIEMANN HYPOTHESIS

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Summary: 158 years ago that in the complex analysis was raised Hypothesis, which was used in principle to demonstrate a The prime numbers, but, without any proof; With the passing of the years This hypothesis has become very important because it has multiple Applications to physics, to number theory, among others. In this article, Ass I present a demonstration that I consider is the one that has been elusive all This weather.

INTRODUCTION

In mathematics and in all sciences, although you have spent 2017 years after Christ, there are still some problems that have not been solved but that have provided the knowledge, infinite solid foundations that have allowed to create many inventions that, in the past Were very needy, but due to the circumstances, it was not possible to use the elements or instruments for what is needed.

This is the case of the Riemann hypothesis, which resulted in demonstrating an exercise on the prime numbers, but its veracity (at the time), was not important, however, with the passage of time this assertion was gaining strength.

This hypothesis states: " In pure mathematics, the Riemann hypothesis, first formulated by Bernhard Riemann in 1859, is a conjecture about the distribution of Zeros of the Riemann zeta function (Taken from Wikipedia); This conjecture at the time did not have much importance as it was Used to talk about prime numbers, but over time has Taken due to its number of applications, but it is A mathematical background that has been unproven 158 and that personal I hope to meet your demonstration someday.

At the time that I had to do the thesis to obtain my degree of I graduated in mathematics, I became interested in this subject and made a General approach of what it is in its application; with the Over the years this topic continued in my mete and I started full to work In his demonstration.

On this occasion I raise a possible demonstration which has already been reviewed by A teacher, who approved it and who is now raised to those who have Greater knowledge in the field of pure mathematics.

WHAT IS RIEMANN'S ZETA FUNCTION?

Riemann's zeta function named in honor of Bernhard Riemann is a function that is of significant importance in number theory because of its relation to the distribution of prime numbers. It also has applications in other areas such as physics, probability theory and statistics Applied.

DEFINITION

The zeta function of Riemann $\zeta(s)$ Is defined for the famous $p-$series in reals, for in the complex values with real part greater than one, by the series of Dirichlet:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

In the region $\{s \in \mathbb{C}/Re(s) > 1\}$, This infinite series converges and defines a function that is analytic in this region. Riemann observed that the zeta function can be extended in a unique way by analytical continuation to a meromorphic function in the whole complex plane with a single pole in $s = 1$. This is the function that is considered in the Riemann hypothesis.
For the complex with $Re < 1$, The values of the function must be calculated by means of its functional equation, obtained from the analytical continuation of the function.

SOME KNOWN IDENTITIES

- Relationship of zeta function and prime numbers

$$\frac{1}{\zeta(s)} = \prod_{n=1}^{\infty} \left(1 - p^{-s}\right)$$

- Extension to the whole plane

$$\zeta(s) = -\frac{\Gamma(1-s)}{2\pi i} \oint \frac{(-z)^{s-1}}{e^z - 1} dz$$

- Functional equation

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$$

- Laurent Series

$$\zeta(s) = \frac{1}{s-1} + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \gamma_n (s-1)^n$$

Where the constants γ_n, are called constants of Stieltjes and are defined as:

$$\gamma_n = \lim_{m \to \infty} \left(\left(\sum_{k=1}^{n} \frac{(\log k)^n}{k} \right) - \frac{(\log k)^{n+1}}{n+1} \right)$$

The constant γ_0 Corresponds to the constant of Euler-Mascheroni.

- Hadamard Product Representation

$$\zeta(s) = \frac{e^{\log(2\pi)-1-\gamma/2)s}}{2(s-1)\Gamma\left(\frac{s}{2}+1\right)} \prod_{\rho} \left(1 - \frac{s}{\rho}\right) e^{s/\rho}$$

In this time of work, I have devoted much of this to the study of $\zeta-$function, and, based on some of the existing identities, I managed to find two identities more to those already known; These identities are:

FUNCTIONAL EQUATION

In mathematics or its applications, a functional equation is an equation that is expressed through a combination of independent variables and unknown functions whose expression and value must be solved. It is possible to determine the properties of the functions by analyzing the types of functional equations that they satisfy. The term functional equation is usually reserved for equations that are not easily reducible to algebraic equations: this is because in many cases two or more known functions are replaced as arguments of an unknown function, which must be solved.

FUNCTIONAL EQUATION OF THE ZEM FUNCTION OF RIEMANN

Riemann's zeta function can be extended analytically for every complex number except $s = 1$, by the following functional equation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

The equation has a simple pole at s = 1 with residue 1 and was shown by Bernhard Riemann in 1859 in his essay On the number of prime numbers smaller than a given quantity. An equivalent ratio was conjectured by Euler for the function $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$.

There is also a symmetric version of the functional equation under change

$$\zeta(s)\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}} = \zeta(1-s)\Gamma\left(\frac{1-s}{2}\right)\pi^{-\frac{1-s}{2}}$$

Where $\Gamma(s)$ is the gamma function.

Sometimes the function is defined

$$\xi(s) = \pi^{-s/2}\Gamma\left(\frac{s}{2}\right)\zeta(s)$$

With what

$$\xi(s) = \xi(1-s)$$

The functional equation also fulfills the following asymptotic limit:

$$\zeta(1-s) = \left(\frac{s}{2\pi e}\right)^s \sqrt{\frac{8\pi}{s}} \cos\left(\frac{\pi s}{2}\right)\left(1 + O\left(\frac{1}{s}\right)\right)$$

PROPOSITION 1

The logarithmic derivative of the $\zeta-$function evaluated in $s = 2k$ is constant and corresponds to $\log(2\pi)$.

Demonstration

Making use of the functional equation together with the extension to the whole plane; we have:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

$$-\frac{\Gamma(1-s)}{2\pi i} \oint \frac{(-z)^{s-1}}{e^z - 1} dz = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

$$-\frac{1}{2i} \oint \frac{(-z)^{s-1}}{e^z - 1} dz = (2\pi)^s \sin\left(\frac{\pi s}{2}\right) \zeta(1-s)$$

We derive both sides from s

$$-\frac{1}{2i} \oint \frac{(-z)^{s-1}\log(-z)}{e^z - 1} dz = \log(2\pi)(2\pi)^s \sin\left(\frac{\pi s}{2}\right) \zeta(1-s)\frac{\pi}{2}(2\pi)^s \cos\left(\frac{\pi s}{2}\right) \zeta(1-s) - (2\pi)^s \sin\left(\frac{\pi s}{2}\right) \zeta'(1-s)$$

We make s to $1-2k$, we have:

$$-\frac{1}{2i} \oint \frac{(-z)^{-2k}\log(-z)}{e^z - 1} dz = \log(2\pi)(2\pi)^{1-2k}(-1)^k\zeta(2k) - (2\pi)^{1-2k}(-1)^k\zeta'(2k)$$

$$(-1)^k(2\pi)^{1-2k}\zeta'(2k) = -\frac{1}{2i} \oint \frac{(-z)^{-2k}\log(-z)}{e^z - 1} dz + \log(2\pi)(2\pi)^{1-2k}(-1)^k\zeta(2k)$$

Taking a simple closed contour positively oriented that does not enclose the $z = \pm 2n\pi i$, we finally obtain

$$\frac{\zeta'(2k)}{\zeta(2k)} = \log(2k)$$

PROPOSITION 2

Sean ρ The non-trivial zeros of Riemann's zeta function, so:

$$\sum_\rho \left(\frac{1}{2k-\rho} + \frac{1}{\rho}\right) = \frac{2k}{2k-1} + \frac{1}{2}\sum_{n=1}^{k} \frac{1}{n}$$

DEMONSTRATION

Take the product representation of Hadamard and find its logarithmic derivative

$$\frac{\zeta'(s)}{\zeta(s)} = \log(2\pi) - 1 - \frac{\gamma}{2} - \frac{1}{s-1} - \frac{\Gamma'\left(\frac{s}{2}+1\right)}{2\Gamma\left(\frac{s}{2}+1\right)}$$

We evaluate in $s = 2k$

$$\frac{\zeta'(2k)}{\zeta(2k)} = \log(2\pi) - 1 - \frac{\gamma}{2} - \frac{1}{2k-1} - \frac{\Gamma'\left(k+1\right)}{2\Gamma\left(k+1\right)}$$

Using in the immediately preceding result, we have

$$\sum_\rho \left(\frac{1}{2k-1} + \frac{1}{\rho}\right) = \log(2\pi) - \log(2\pi) + 1 + \frac{\gamma}{2} + \frac{1}{2k-1} + \frac{\Gamma'(k+1)}{2\Gamma(k+1)}$$

$$\sum_\rho \left(\frac{1}{2k-1} + \frac{1}{\rho}\right) = 1 + \frac{\gamma}{2} + \frac{1}{2k-1} + \frac{1}{2}\Psi(k+1)$$

Where $\Psi(k+1)$ Corresponds to the function digamma evaluated in $k+1$ and whose value is

$$\Psi(k+1) = -\gamma + \sum_{n=1}^{k} \frac{1}{n}$$

Replacing finally we get:

$$\sum_{\rho} \left(\frac{1}{2k-1} + \frac{1}{\rho} \right) = \frac{2k}{2k-1} + \frac{1}{2} \sum_{n=1}^{k} \frac{1}{n}$$

ZEROS OF THE FUNCTION

The value of the zeta function for negative even numbers is 0 (seeing the functional equation is obvious), so they are called trivial zeros. Apart from the trivial zeros, the function also cancels out in values of s that are within the range $\{s \in \mathbb{C} : 0 < Re(s) < 1\}$, And are called non-trivial zeros, because it is more difficult to prove the location of those zeros within the critical range. The study of the distribution of these ¡¡non-trivial zeros¿¿ is very important because it has profound implications for the distribution of prime numbers and questions related to number theory. The Riemann hypothesis, considered one of the greatest mathematical problems open today, ensures that any non-trivial zero has to fulfill $Re(s) = \frac{1}{2}$, Therefore, all zeros are aligned in the complex plane forming a line, called the critical line.

TRIVIAL ZEROS

The functional equation of the $\zeta-$function is given by:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

Let's do $s = -2k$

$$\begin{aligned} \zeta(-2k) &= 2^{-2k}\pi^{-2k-1} \sin\left(-\frac{2k\pi}{2}\right) \Gamma(1+2k)\zeta(1+2k) \\ \zeta(-2k) &= 2^{-2k}\pi^{-2k-1} \sin\left(-k\pi\right) \Gamma(1+2k)\zeta(1+2k) \end{aligned}$$

The sine function becomes zero when evaluated in $\pm\pi$, but Positive value, we find an inconsistency because, when defining the Functional equation of the $\zeta-$Function, we have that when we have $s > 1$, We work with the $p-$ series, but when we have $s < 1$, we work with the Functional equation, therefore, since this function is odd, we have:

$$\begin{aligned} \sin(-k\pi) &= -\sin(k\pi) \\ &= 0 \end{aligned}$$

The Gamma function, in this expression is being evaluated in a real, so Is not zero, since this function does not have zeros in the set of Real ($\mathbb{R}$); The zeta function evaluated in odd, is also different from zero, Thus:

$$\zeta(2k) = 0$$

HYPOTHESIS OF RIEMANN

Riemann mentioned the conjecture in 1859, which would be called the Riemann hypothesis, in his doctoral thesis on the prime numbers smaller than a given magnitude, by developing an explicit formula to calculate the number of cousins less than x. Since it was not essential to the central purpose of his article, he did not attempt to give a demonstration. He knew that the non-trivial zeros of the zeta function are distributed around the line $s = 1/2 + it$, And he also knew that all non-trivial zeros must be in the range $0 \leq Re(s) \leq 1$.

In 1896, Hadamard and de la Vallée-Poussin independently proved that no zero could be on the straight Re (s) = 1. Along with the other properties of non-trivial zeros demonstrated by Riemann, this showed that all nontrivial zeros Must be within the critical band 0 ¡Re (s) ¡1. This was a fundamental step for the first demonstrations of the prime number theorem.

In 1900, Hilbert included the Riemann hypothesis in his famous list of 23 unresolved problems, is part of Problem 8 on Hilbert's list along with Goldbach's conjecture. When asked what he would do if he woke up having slept five hundred years, Remarkably Hilbert replied that his first question would be whether Riemann's hypothesis had been proven. Riemann's hypothesis is the only problem proposed by Hilbert who is in the millennium award of the Clay Institute of Mathematics.

In 1914, Hardy showed that there exists an infinite number of zeros on the critical line $Re(s) = 1/2$. However it was still possible that an infinite (and possibly most) number of non-trivial zeros would be found elsewhere on the critical band. In later works by Hardy and Littlewood in 1921 and Selberg in 1942 estimates were given for the average density of zeros on the critical line.

HYPOTHESIS

The real part of any non-trivial zero of Riemann's zeta function is $\frac{1}{2}$.

POSSIBLE DEMONSTRATION

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

Replacing s for $\frac{1}{2} + it$ You have:

$$\begin{aligned}
\zeta\left(\frac{1}{2}+it\right) &= 2^{\frac{1}{2}+it}\pi^{\frac{1}{2}+it-1}\sin\left(\frac{\pi((1/2)+it)}{2}\right)\Gamma\left(1-\frac{1}{2}-it\right)\zeta\left(1-\frac{1}{2}-it\right)\\
\zeta\left(\frac{1}{2}+it\right) &= 2^{\frac{1}{2}+it}\pi^{-\frac{1}{2}+it}\sin\left(\frac{\pi(1/2)+i\pi t}{2}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)\\
\zeta\left(\frac{1}{2}+it\right) &= \frac{\sqrt{2}}{\sqrt{\pi}}2^{it}\pi^{it}\sin\left(\frac{\pi}{4}+i\frac{\pi t}{2}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)\\
&= \sqrt{\frac{2}{\pi}}(2\pi)^{it}\left(\frac{e^{i(\frac{\pi}{4}+i\frac{\pi t}{2})}-e^{-i(\frac{\pi}{4}+i\frac{\pi t}{2})}}{2i}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)\\
&= \sqrt{\frac{2}{\pi}}(2\pi)^{it}\left(\frac{e^{i\frac{\pi}{4}-\frac{\pi t}{2}}-e^{-i\frac{\pi}{4}+\frac{\pi t}{2}}}{2i}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)
\end{aligned}$$

Let's see if $(2\pi)^{it}\left(\frac{e^{i\frac{\pi}{4}-\frac{\pi t}{2}}-e^{-i\frac{\pi}{4}+\frac{\pi t}{2}}}{2i}\right)$ Are bounded; For this we must See if it is the norm

$$\begin{aligned}
||(2\pi)^{it}|| &= ||e^{it\ln(2\pi)}||\\
&= ||\cos(t\ln(2\pi)) + i\sin(t\ln(2\pi))||\\
&= \sqrt{\cos^2(t\ln(2\pi)) + \sin^2(t\ln(2\pi))}\\
&= 1
\end{aligned}$$

Then it is bounded

$$\begin{aligned} \left\| \frac{e^{i\frac{\pi}{4}-\frac{\pi t}{2}} - e^{-i\frac{\pi}{4}+\frac{\pi t}{2}}}{2i} \right\| &= \left\| -\frac{i}{2} \right\| \left\| e^{i\frac{\pi}{4}-\frac{\pi t}{2}} - e^{-i\frac{\pi}{4}+\frac{\pi t}{2}} \right\| \\ &= \frac{1}{2} \left\| e^{-\frac{\pi t}{2}} \left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) - e^{\frac{\pi t}{2}} \left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right) \right\| \end{aligned}$$

Here, we will use a special case of triangular inequality and we have

$$\begin{aligned} &\leq \frac{1}{2} \left\| e^{-\frac{\pi t}{2}} \left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) \right\| + \frac{1}{2} \left\| e^{\frac{\pi t}{2}} \left(\cos\left(\frac{\pi}{4}\right) - i\sin\left(\frac{\pi}{4}\right)\right) \right\| \\ &\leq \frac{1}{2} \left[e^{-\frac{\pi t}{2}} \left(\cos^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{4}\right)\right) \right] + \frac{1}{2} \left[e^{\frac{\pi t}{2}} \left(\cos^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{4}\right)\right) \right] \\ &\leq \frac{1}{2}(e^{-\frac{\pi t}{2}} + e^{\frac{\pi t}{2}}) \\ &\leq \cosh\left(\frac{\pi t}{2}\right) \end{aligned}$$

This function is bounded.
Now, the asymptotic limit of the zeta function is of the form

$$\begin{aligned} \zeta(1-s) &= \left(\frac{s}{2\pi e}\right)^s \sqrt{\frac{8\pi}{s}} \cos\left(\frac{\pi s}{2}\right) \left(1 + O\left(\frac{1}{s}\right)\right) \\ \zeta\left(\frac{1}{2} - it\right) &= \left(\frac{(1/2)+it}{2\pi e}\right)^{\frac{1}{2}+it} \sqrt{\frac{8\pi}{(1/2)+it}} \cos\left(\frac{\pi((1/2)+it)}{2}\right) \left(1 + O\left(\frac{1}{(1/2)+it}\right)\right) \end{aligned}$$

Let's see if these expressions are bounded

$$
\begin{aligned}
\left\|\left(\frac{(1/2)+it}{2\pi e}\right)^{\frac{1}{2}+it}\right\| &= \left\|\sqrt{\frac{(1/2)+it}{2\pi e}}e^{it\ln\left(\frac{(1/2)+it}{2\pi e}\right)}\right\| \\
&= \left\|\sqrt{\frac{(1/2)+it}{2\pi e}}\right\| \left\|\cos\left(t\ln\left(\frac{(1/2)+it}{2\pi e}\right)\right) + i\sin\left(t\ln\left(\frac{(1/2)+it}{2\pi e}\right)\right)\right\| \\
&= \left\|\sqrt{\frac{(1/2)+it}{2\pi e}}\right\| \left[\cos^2\left(t\ln\left(\frac{(1/2)+it}{2\pi e}\right)\right) + \sin^2\left(t\ln\left(\frac{(1/2)+it}{2\pi e}\right)\right)\right] \\
&= \left\|\sqrt{\frac{(1/2)+it}{2\pi e}}\right\| \\
&= \left\|e^{\frac{1}{2}\ln\left(\frac{(1/2)+it}{2\pi e}\right)}\right\| \\
&= \left\|e^{\left[\frac{1}{2}\ln\left\|\frac{(1/2)+it}{2\pi e}\right\|+iArgZ\right]}\right\|; \qquad Z = \frac{(1/2)+it}{2\pi e} \\
&= \left\|e^{\left[\frac{1}{2}\ln\left(\sqrt{\frac{1}{16\pi^2e^2}+\frac{t^2}{4\pi^2e^2}}\right)+iArgZ\right]}\right\| \\
&= \left\|e^{\left[\frac{1}{2}\ln\left(\frac{1}{2\pi e}\sqrt{\frac{1}{4}+t^2}\right)+iArgZ\right]}\right\| \\
&= \left\|e^{\left[\frac{1}{2}\ln\left(\frac{1}{2\pi e}\sqrt{\frac{1}{4}+t^2}\right)\right]}\right\| \, ||\cos(ArgZ) + i\sin(ArgZ)|| \\
&= \left\|e^{\left[\frac{1}{2}\ln\left(\frac{1}{2\pi e}\sqrt{\frac{1}{4}+t^2}\right)\right]}\right\| (\cos^2(ArgZ) + \sin^2(ArgZ)) \\
&= e^{\left[\frac{1}{2}\ln\left(\frac{1}{2\pi e}\sqrt{\frac{1}{4}+t^2}\right)\right]}
\end{aligned}
$$

So, $\left(\frac{(1/2)+it}{2\pi e}\right)^{\frac{1}{2}+it}$ is Bounded

$$
\begin{aligned}
\left\|\sqrt{\frac{8\pi}{(1/2)+it}}\right\| &= \left\|\sqrt{\frac{8\pi}{(1/2)+it}\cdot\frac{(1/2)-it}{(1/2)-it}}\right\| \\
&= \left\|\frac{1}{(1/4)+t^2}\right\| \|4\pi - i8\pi t\| \\
&= \frac{1}{1/4+t^2}\sqrt{16\pi^2+64\pi^2t^2} \\
&= \frac{4\pi\sqrt{1+4t^2}}{(1/4)+t^2} \\
&= \frac{16\pi\sqrt{1+4t^2}}{1+4t^2}
\end{aligned}
$$

So, $\sqrt{\frac{8\pi}{1/2+it}}$ is Bounded

$$\begin{aligned}
\left\|\cos\left(\frac{\pi((1/2)+it)}{2}\right)\right\| &= \left\|\frac{e^{i(\frac{\pi}{4}+i\frac{\pi t}{2})}+e^{-i(\frac{\pi}{4}+i\frac{\pi t}{2})}}{2}\right\| \\
&= \left\|\frac{e^{i\frac{\pi}{4}-\frac{\pi t}{2}}+e^{-i\frac{\pi}{4}+\frac{\pi t}{2}}}{2}\right\| \\
&= \left\|\frac{e^{-\frac{\pi t}{4}}(\cos(\pi/4)+i\sin(\pi/4))+e^{\pi t/4}(\cos(\pi/4)-i\sin(\pi/4))}{2}\right\| \\
&\leq \left\|\frac{e^{-\pi t/4}(\cos(\pi/4)+i\sin(\pi/4))}{2}\right\|+\left\|\frac{e^{\pi t/4}(\cos(\pi/4)-i\sin(\pi/4))}{2}\right\| \\
&\leq \frac{e^{-\frac{\pi t}{2}}+e^{\frac{\pi t}{2}}}{2} \\
&\leq \cosh\left(\frac{\pi t}{2}\right)
\end{aligned}$$

So, $\cos\left(\frac{\pi(1/2+it)}{2}\right)$ is Bounded.

This O, He is indicating an upper bound. As the expressions that Accompany, are also quotas; You have to $\zeta(1/2-it)$ is Bounded.

The hypothesis does not present any restriction for the imaginary part, therefore So we must take limit when t Tends to infinity, to find the value of $\zeta(1/2+it)$

$$\zeta(1/2+it)=\lim_{t\to\infty}\frac{\sqrt{2}}{\sqrt{\pi}}2^{it}\pi^{it}\sin\left(\frac{\pi}{4}+i\frac{\pi t}{2}\right)\Gamma\left(\frac{1}{2}-it\right)\zeta\left(\frac{1}{2}-it\right)$$

In this expression, we have to find the value of $\Gamma\left(\frac{1}{2}-it\right)$,Which we will find next to the limit

Solution 1

$$\begin{aligned}
\int_0^\infty T^{\frac{1}{2}-it-1}e^{-T}dT &= \int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT \\
T &= z^2 \\
dT &= 2zdz \\
\text{Si } T &= 0 \to z = 0 \\
\text{Si } T &= \infty \to z = \infty \\
\int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT &= \int_0^\infty (z^2)^{\frac{-1}{2}-it}e^{-z^2}2zdz \\
&= \int_0^\infty z^{-1-it}e^{-z^2}2zdz \\
&= \int_0^\infty 2z^{-it}e^{-z^2}dz \\
&= \int_0^\infty e^{\ln(2z^{-it})}e^{-z^2}dz \\
&= \int_0^\infty e^{-it\ln(2z)}e^{-z^2}dz \\
&= \int_0^\infty (\cos(t\ln(2z)) - i\sin(t\ln(2z)))e^{-z^2}dz \\
&= \int_0^\infty \cos(t\ln(2z))e^{-z^2}dz - i\int_0^\infty \sin(t\ln(2z))e^{-z^2}dz
\end{aligned}$$

Here we can see that:

$$\begin{aligned}
-\cos(2tz)e^{-tz} &\leq \cos(t\ln(2z))e^{-z^2} \leq \cos(2tz)e^{-tz} \\
-\int_0^\infty \cos(2tz)e^{-tz}dz &\leq \int_0^\infty \cos(t\ln(2z))e^{-z^2}dz \leq \int_0^\infty \cos(2tz)e^{-tz}dz
\end{aligned}$$

These quotas would remain

$$\begin{aligned}
&\int_0^\infty \cos(2tz)e^{-tz}dz \\
&\quad u = \cos(2tz) \qquad dv = e^{-tz}dz \\
&\quad du = -\tfrac{1}{2t}\sin(2tz)dz \quad v = -\tfrac{1}{t}e^{-tz} \\
&= -\frac{1}{t}\cos(2tz)e^{-tz}|_0^\infty - \frac{1}{2t^2}\int \sin(2tz)e^{-z^2}dz \\
&\quad u = \sin(2tz) \qquad dv = e^{-tz}dz \\
&\quad du = \tfrac{1}{2t}\cos(2tz)dz \quad v = -\tfrac{1}{t}e^{-tz} \\
&= -\frac{1}{t}\cos(2tz)e^{-tz}|_0^\infty - \frac{1}{2t^2}\left[-\frac{1}{t}\sin(2tz) + \frac{1}{2t^2}\int \cos(2tz)e^{-tz}dz\right]
\end{aligned}$$

$$\begin{aligned}
\int_0^\infty \cos(2tz)e^{-tz}dz &= -\frac{1}{t}\cos(2tz)e^{-tz}|_0^\infty + \frac{1}{2t^3}\sin(2tz)|_0^\infty - \frac{1}{4t^4}\int \cos(2tz)e^{-tz}dz \\
\int_0^\infty \cos(2tz)e^{-tz}dz + \frac{1}{4t^4}\int \cos(2tz)e^{-tz}dz &= -\frac{1}{t}\cos(2tz)e^{-tz}|_0^\infty + \frac{1}{2t^3}\sin(2tz)|_0^\infty \\
\frac{4t^4+1}{4t^4}\int \cos(2tz)e^{-tz}dz &= -\frac{1}{t}\cos(2tz)e^{-tz}|_0^\infty + \frac{1}{2t^3}\sin(2tz)|_0^\infty \\
\int \cos(2tz)e^{-tz}dz &= -\frac{4t^3}{4t^4+1}\cos(2tz)e^{-tz}|_0^\infty + \frac{4t}{4t^4+1}\sin(2tz)|_0^\infty
\end{aligned}$$

The value of this integrals does not exist but they are bounded, therefore we can Find the limit when t tends to infinity

$$\lim_{t\to\infty}\left[-\frac{4t^3}{4t^4+1}\cos(2tz)e^{-tz}|_0^\infty+\frac{4t}{4t^4+1}\sin(2tz)|_0^\infty=0\right]$$

Therefore we have

$$0\leq\int_0^\infty\cos(t\ln(2z))e^{-z^2}dz\leq 0$$

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$$-\int_0^\infty\cos(2tz)e^{-tz}dz=-0=0$$

As $\int_0^\infty\cos(t\ln(2z))e^{-z^2}dz$ This upper bound And inferiorly by 0, we finally have

$$\lim_{t\to\infty}\Gamma\left(\frac{1}{2}-it\right)=0$$

Solution 2

$$\begin{aligned}\lim_{t\to\infty}\Gamma\left(\frac{1}{2}-it\right) &= \lim_{t\to\infty}\int_0^\infty T^{\frac{1}{2}-it-1}e^{-T}dT\\ &= \lim_{t\to\infty}\int_0^\infty T^{-\frac{1}{2}-it}e^{-T}dT\end{aligned}$$

Let us see if this expression converges uniformly; If he does, we can Exchange the limit with the integral.

$$\begin{aligned}\left\|T^{\frac{1}{2}-it-1}e^{-T}\right\| &= \left\|T^{-1/2}e^{it\ln T}e^{-T}\right\|\\ &= \left\|T^{-1/2}e^{-T}(\cos(t\ln T)+i\sin(t\ln T))\right\|\\ &= ||T^{-1/2}e^{-T}||(\cos^2(t\ln T)+\sin^2(t\ln T))\\ &= T^{-1/2}e^{-T}\\ &\leq e^{-T}\end{aligned}$$

Then, this function is bounded ie converges evenly; Thus

$$\begin{aligned}\lim_{t\to\infty}\left[\int_0^\infty T^{\frac{1}{2}-it-1}e^{-T}dT\right] &= \int_0^\infty\left[\lim_{t\to\infty}T^{-\frac{1}{2}-it}e^{-T}\right]dT\\ &= \int_0^\infty\left[\lim_{t\to\infty}\frac{1}{\sqrt{T}}T^{-it}e^{-T}\right]dT\\ &= \int_0^\infty\left[\lim_{t\to\infty}\frac{1}{\sqrt{T}}T^{-\infty}e^{-T}\right]dT\\ &= \int_0^\infty 0dT\\ &= k-k\\ &= 0\end{aligned}$$

Therefore, as this zero multiplies all other functions (which do not They exist but they are bounded), we have to:

$$\zeta(1/2+it)=0$$

CONCLUTION

Riemann's hypothesis is a statement Riemann enunciated to show the number of prime numbers less than a given quantity, but the fact that this claim was true was not important; The veracity of this hypothesis was becoming more valuable with the passage of time since this was appearing when making statements such as the Casimir effect or the sum of positive integers (this in the field of physics and number theory).

This function has been very useful in many fields, but here it is doubtful. Is this hypothesis true? Today 158 years after its postulation I present to the world a demonstration of the Riemann hypothesis, which verifies to me the approaches in which it was used in previous years.

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