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A Neutrosophic Extension of the MULTIMOORA Method

Dragisa STANUJKIC[1]*, Edmundas Kazimieras ZAVADSKAS[2], Florentin SMARANDACHE[3], Willem K.M. BRAUERS[4], Darjan KARABASEVIC[5]

[1] *Faculty of Management in Zajecar, John Naisbitt University, Belgrade, Serbia*
[2] *Research Institute of Smart Building Technologies*
Civil Engineering Faculty, Vilnius Gediminas Technical University, Vilnius, Lithuania
[3] *Department of Mathematics, University of New Mexico*
705 Gurley Avenue, Gallup, NM 87301, USA
[4] *Faculty of Applied Economics, University of Antwerp, Antwerp, Belgium*
[5] *Faculty for Applied Management, Economics and Finance, Business Academy University*
Jevrejska 24, Belgrade, Serbia
e-mail: dragisa.stanujkic@fmz.edu.rs, edmundas.zavadskas@vgtu.lt, fsmarandache@gmail.com, willem.brauers@uantwerpen.be, darjankarabasevic@gmail.com

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Abstract. The aim of this manuscript is to propose a new extension of the MULTIMOORA method adapted for usage with a neutrosophic set. By using single valued neutrosophic sets, the MULTIMOORA method can be more efficient for solving complex problems whose solving requires assessment and prediction, i.e. those problems associated with inaccurate and unreliable data. The suitability of the proposed approach is presented through an example.

Key words: neutrosophic set, single valued neutrosophic set, MULTIMOORA, MCDM.

1. Introduction

The MULTIMOORA (Multi-Objective Optimization by a Ratio Analysis plus the Full Multiplicative Form) was proposed by Brauers and Zavadskas (2010).

The ordinary MULTIMOORA method has been proposed for usage with crisp numbers. In order to enable its use in solving a larger number of complex decision-making problems, several extensions have been proposed, out of which only the most prominent are mentioned: Brauers *et al.* (2011) proposed a fuzzy extension of the MULTIMOORA method; Balezentis and Zeng (2013) proposed an interval-valued fuzzy extension; Balezentis *et al.* (2014) proposed an intuitionistic fuzzy extension and Zavadskas *et al.* (2015) proposed an interval-valued intuitionistic extension of the MULTIMOORA method.

*Corresponding author.

The MULTIMOORA method has been applied for the purpose of solving a wide range of problems.

As some of the most cited, the studies that consider different problems in economics (Brauers and Zavadskas, 2010, 2011; Brauers, 2010), personnel selection (Balezentis *et al.*, 2012a, 2012b), construction (Kracka *et al.*, 2015), risk management (Liu *et al.*, 2014a) and waste treatment (Liu *et al.*, 2014b) can be mentioned.

As some of the newest studies in which the MULTIMOORA method is used for solving various decision-making problems, the following ones can be mentioned: material selection (Hafezalkotob and Hafezalkotob, 2016; Hafezalkotob *et al.*, 2016) and the CNC machine tool evaluation (Sahu *et al.*, 2016).

A significant approach in solving complex decision-making problems was formed by adapting the multiple criteria decision-making methods for the purpose of using fuzzy numbers, proposed by Zadeh in the fuzzy set theory (Zadeh, 1965).

Based on the fuzzy set theory, some extensions are also proposed, such as: interval-valued fuzzy sets (Turksen, 1986), intuitionistic fuzzy sets (Atanassov, 1986) and interval-valued intuitionistic fuzzy sets (Atanassov and Gargov, 1989).

In addition to the membership function proposed in fuzzy sets, Atanassov (1986) introduced the non-membership function that expresses non-membership to a set, thus having created the basis for solving a much larger number of decision-making problems.

The intuitionistic fuzzy set is composed of membership (the so-called truth-membership) $T_A(x)$ and non-membership (the so-called falsity-membership) $F_A(x)$, which satisfies the conditions $T_A(x), F_A(x) \in [0, 1]$ and $0 \leqslant T_A(x) + F_A(x) \leqslant 1$. Therefore, intuitionistic fuzzy sets are capable of operating with incomplete pieces of information, but do not include intermediate and inconsistent information (Li *et al.*, 2016).

In intuitionistic fuzzy sets, indeterminacy $\pi_A(x)$ is $1 - T_A(x) - F_A(x)$ by default. Smarandache (1998, 1999) further extended intuitionistic fuzzy sets by proposing Neutrosophic, also introducing independent indeterminacy-membership.

Such a proposed neutrosophic set is composed of three independent membership functions named the truth-membership $T_A(x)$, the falsity-membership $F_A(x)$ and the indeterminacy-membership $I_A(x)$ functions.

Smarandache (1999) and Wang *et al.* (2010) further proposed a single valued neutrosophic set, by modifying the conditions $T_A(x)$, $I_A(x)$ and $F_A(x) \in [0, 1]$ and $0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3$, which are more suitable for solving scientific and engineering problems (Li *et al.*, 2016).

Compared with the fuzzy set and its extensions, the single valued neutrosophic set can be identified as more flexible, for which reason an extension of the MULTIMOORA method adapted for the purpose of using the single valued neutrosophic set is proposed in this approach.

Therefore, the rest of this paper is organized as follows: in Section 2, some basic definitions related to the single valued neutrosophic set are given. In Section 3, the ordinary MULTIMOORA method is presented, whereas in Section 4, the Single Valued Neutrosophic Extension of the MULTIMOORA method is proposed. In Section 5, an example is considered with the aim to explain in detail the proposed methodology. The conclusions are presented in the final section.

2. The Single Valued Neutrosophic Set

DEFINITION 1. (See Smarandache, 1999.) Let X be the universe of discourse, with a generic element in X denoted by x. Then, the Neutrosophic Set (NS) A in X is as follows:

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}, \tag{1}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively,

$$T_A, I_A, F_A : X \to]^-0, 1^+[\quad \text{and} \quad ^-0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3^+.$$

DEFINITION 2. (See Smarandache, 1999; Wang *et al.*, 2010.) Let X be the universe of discourse. The Single valued neutrosophic set (SVNS) A over X is an object having the following form:

$$A = \{x\langle T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}, \tag{2}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the intermediacy-membership function and the falsity-membership function, respectively,

$$T_A, I_A, F_A : X \to [0, 1] \quad \text{and} \quad 0 \leqslant T_A(x) + I_A(x) + F_A(x) \leqslant 3.$$

DEFINITION 3. (See Smarandache, 1999.) For an SVNS A in X, the triple $\langle t_A, i_A, f_A\rangle$ is called the single valued neutrosophic number (SVNN).

DEFINITION 4. Let $x_1 = \langle t_1, i_1, f_1\rangle$ and $x_2 = \langle t_2, i_2, f_2\rangle$ be two SVNNs and $\lambda > 0$; then the basic operations are defined as follows:

$$x_1 + x_2 = \langle t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2\rangle, \tag{3}$$

$$x_1 \cdot x_2 = \langle t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2\rangle, \tag{4}$$

$$\lambda x_1 = \langle 1 - (1 - t_1)^\lambda, i_1^\lambda, f_1^\lambda\rangle, \tag{5}$$

$$x_1^\lambda = \langle t_1^\lambda, i_1^\lambda, 1 - (1 - f_1)^\lambda\rangle. \tag{6}$$

DEFINITION 5. (See Sahin, 2014.) Let $x = \langle t_x, i_x, f_x\rangle$ be an SVNN; then the score function s_x of x can be as follows:

$$s_x = (1 + t_x - 2i_x - f_x)/2, \tag{7}$$

where $s_x \in [-1, 1]$.

Definition 6. Let $x_1 = \langle t_1, i_1, f_1 \rangle$ and $x_2 = \langle t_2, i_2, f_2 \rangle$ be two SVNNs. Then the maximum distance between x_1 and x_2 is as follows:

$$d_{\max}(x_1, x_2) = \begin{cases} |t_1 - t_2|, & x_1, x_2 \in \Omega_{\max}, \\ |f_1 - f_2|, & x_1, x_2 \in \Omega_{\min}. \end{cases} \tag{8}$$

Definition 7. (See Sahin, 2014.) Let $A_j = \langle t_j, i_j, f_j \rangle$ be a collection of SVNSs and $W = (w_1, w_2, \dots, w_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Average (SVNWA) operator of A_j is as follows:

$$\begin{aligned} & SVNWA(A_1, A_2, \dots, A_n) \\ & \quad = \sum_{j=1}^{n} w_j A_j = \left(1 - \prod_{j=1}^{n} (1 - t_j)^{w_j}, \prod_{j=1}^{n} (i_j)^{w_j}, \prod_{j=1}^{n} (f_j)^{w_j}\right). \end{aligned} \tag{9}$$

where: w_j is the element j of the weighting vector, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 8. (See Sahin, 2014.) Let $A_j = \langle t_j, i_j, f_j \rangle$ be a collection of SVNSs and $W = (w_1, w_2, \dots, w_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Geometric (SVNWG) operator of A_j is as follows:

$$\begin{aligned} & SVNWG(A_1, A_2, \dots, A_n) \\ & \quad = \prod_{j=1}^{n} (A_j)^{w_j} = \left(\prod_{j=1}^{n} (t_j)^{w_j}, 1 - \prod_{j=1}^{n} (1 - i_j)^{w_j}, 1 - \prod_{j=1}^{n} (1 - f_j)^{w_j}\right). \end{aligned} \tag{10}$$

where: w_j is the element j of the weighting vector, $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

3. The MULTIMOORA Method

The MULTIMOORA method consists of three approaches named as follows: the Ratio System (RS) Approach, the Reference Point (RP) Approach and the Full Multiplicative Form (FMF).

The considered alternatives are ranked based on all three approaches and the final ranking order and the final decision is made based on the theory of dominance. In other words, the alternative with the highest number of appearances in the first positions on all ranking lists is the best-ranked alternative.

The ratio system approach. In this approach, the overall importance of the alternative i can be calculated as follows:

$$y_i = y_i^+ - y_i^-, \tag{11}$$

with:

$$y_i^+ = \sum_{j \in \Omega_{\max}} w_j r_{ij}, \quad \text{and} \tag{12}$$

$$y_i^- = \sum_{j \in \Omega_{\min}} w_j r_{ij}, \tag{13}$$

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \tag{14}$$

where: y_i denotes the overall importance of the alternative i, obtained on the basis of all the criteria; y_i^+ and y_i^- denote the overall importance of the alternative i, obtained on the basis of the benefit and cost criteria, respectively; r_{ij} denotes the normalized performance of the alternative i with respect to the criterion j; x_{ij} denotes the performance of the alternative i to the criterion j; $\Omega_{\max}$ and $\Omega_{\min}$ denote the sets of the benefit cost criteria, respectively; $i = 1, 2, \dots, m$; m is the number of the alternatives, $j = 1, 2, \dots, n$; n is the number of the criteria.

In this approach, the compared alternatives are ranked based on y_i in descending order and the alternative with the highest value of y_i is considered to be the best-ranked.

The reference point approach. The optimization based on this approach can be shown as follows:

$$d_i^{\max} = \max_j \left(w_j \left| r_j^* - r_{ij} \right| \right), \tag{15}$$

where: $d_i^{\max}$ denotes the maximum distance of the alternative i to the reference point and r_j^* denotes the coordinate j of the reference point as follows:

$$r_j^* = \begin{cases} \max\limits_i r_{ij}, & j \in \Omega_{\max}, \\ \min\limits_i r_{ij}, & j \in \Omega_{\min}. \end{cases} \tag{16}$$

In this approach, the compared alternatives are ranked based on $d_i^{\max}$ in ascending order and the alternative with the lowest value of $d_i^{\max}$ is considered the best-ranked.

The full multiplicative form. In the FMF, the overall utility of the alternative i can be determined in the following manner:

$$u_i = \frac{a_i}{b_i}, \tag{17}$$

with:

$$a_i = \prod_{j \in \Omega_{\max}} w_j r_{ij}, \tag{18}$$

$$b_i = \prod_{j\in\Omega_{\min}} w_j r_{ij}, \tag{19}$$

where: u_i denotes the overall utility of the alternative i, a_i denotes the product of the weighted performance ratings of the benefit criteria and b_i denotes the product of the weighted performance ratings of the cost criteria of the alternative i.

As in the RSA, the compared alternatives are ranked based on their u_i in descending order and the alternative with the highest value of u_i is considered the best-ranked.

The final ranking of alternatives based on the MULTIMOORA method. As a result of evaluation by applying the MULTIMOORA method, three ranking lists of the considered alternatives are obtained. Based on Brauers and Zavadskas (2011), the final ranking order of the alternatives is determined based on the theory of dominance.

4. An Extension of the MULTIMOORA Method Based on Single Valued Neutrosophic Numbers

For an MCDM problem involving m alternatives and n criteria, whereby the performances of the alternatives are expressed by using SVNS, the calculation procedure of the extended MULTIMOORA method can be expressed as follows:

Step 1. Determine the ranking order of the alternatives based on the RS approach. The ranking of the alternatives and the selection of the best one based on this approach in the proposed extension of the MULTIMOORA method can be expressed through the following sub steps:

Step 1.1. *Calculate* Y_i^+ and Y_i^- by using the SVNWA operator, as follows:

$$Y_i^+ = \left(1 - \prod_{j\in\Omega_{\max}} (1-t_j)^{w_j}, \prod_{j\in\Omega_{\max}} (i_j)^{w_j}, \prod_{j\in\Omega_{\max}} (f_j)^{w_j}\right), \tag{20}$$

$$Y_i^- = \left(1 - \prod_{j\in\Omega_{\min}} (1-t_j)^{w_j}, \prod_{j\in\Omega_{\min}} (i_j)^{w_j}, \prod_{j\in\Omega_{\min}} (f_j)^{w_j}\right), \tag{21}$$

where: Y_i^+ and Y_i^- denote the importance of the alternative i obtained based on the benefit and cost criteria, respectively; Y_i^+ and Y_i^- are SVNNs.

Step 1.2. *Calculate* y_i^+ and y_i^- by using the Score Function, as follows:

$$y_i^+ = s(Y_i^+), \tag{22}$$

$$y_i^- = s(Y_i^-). \tag{23}$$

Step 1.3. *Calculate the overall importance* for each alternative, as follows:

$$y_i = y_i^+ - y_i^-. \tag{24}$$

Step 1.4. *Rank the alternatives and select the best one*. The ranking of the alternatives can be performed in the same way as in the RS approach of the ordinary MULTIMOORA method.

Step 2. Determine the ranking order of the alternatives based on the RP approach. The ranking of the alternatives and the selection of the best one, based on the RP approach, can be expressed through the following substeps:

Step 2.1. *Determine the reference point*. In this approach, each coordinate of the reference point $r^* = \{r_1^*, r_2^*, \dots, r_n^*\}$ is an SVNN, $r_j^* = \langle t_j^*, i_j^*, f_j^*\rangle$, whose values are determined as follows:

$$r_j^* = \begin{cases} \langle \max_i t_{ij}, \min_i i_{ij}, \min_i f_{ij}\rangle, & j \in \Omega_{\max}, \\ \langle \min_i t_{ij}, \min_i i_{ij}, \max_i f_{ij}\rangle, & j \in \Omega_{\min}, \end{cases} \tag{25}$$

where: r_j^* denotes the coordinate j of the reference point.
For the sake of simplicity, r_j^* could be determined as follows:

$$r_j^* = \begin{cases} \langle 1, 0, 0\rangle, & j \in \Omega_{\max}, \\ \langle 0, 0, 1\rangle, & j \in \Omega_{\min}. \end{cases} \tag{25a}$$

Step 2.2. *Determine the maximum distance from each alternative to all the coordinates of the reference point* as follows:

$$d_{ij}^{\max} = d_{\max}\big(r_{ij}, r_j^*\big) w_j, \tag{26}$$

where $d_{ij}^{\max}$ denotes the maximum distance of the alternative i obtained based on the criterion j determined by Eq. (8).

Step 2.3. *Determine the maximum distance of each alternative*, as follows:

$$d_i^{\max} = \max_j d_{ij}^{\max}. \tag{27}$$

Step 2.4. *Rank the alternatives and select the best one*. At this step, the ranking of the alternatives can be done in the same way as in the RPA of the ordinary MULTIMOORA method.

Step 3. Determine the ranking order of the alternatives and select the best one based on the FMF. The ranking of the alternatives and the selection of the best one can be expressed through the following sub steps:

Step 3.1. *Calculate* A_i and B_i as follows:

$$A_i = \left(\prod_{j\in\Omega_{\min}} (t_j)^{w_j}, 1 - \prod_{j\in\Omega_{\max}} (1 - i_j)^{w_j}, 1 - \prod_{j\in\Omega_{\max}} (1 - f_j)^{w_j} \right), \tag{28}$$

$$B_i = \left(\prod_{j\in\Omega_{\min}} (t_j)^{w_j}, 1 - \prod_{j\in\Omega_{\min}} (1 - i_j)^{w_j}, 1 - \prod_{j\in\Omega_{\min}} (1 - f_j)^{w_j} \right), \tag{29}$$

where: $A_i = \langle t_{Ai}, i_{Ai}, f_{Ai}\rangle$ and $B_i = \langle t_{Bi}, i_{Bi}, f_{Bi}\rangle$ are SVNNs.

Step 3.2. *Calculate* a_i and b_i by using the Score Function as follows:

$$a_i = s(A_i), \tag{30}$$

$$b_i = s(B_i). \tag{31}$$

Step 3.3. *Determine the overall utility for each alternative* as follows:

$$u_i = \frac{a_i}{b_i}. \tag{32}$$

Sep 3.4. *Rank the alternatives and select the best one*. The ranking of the alternatives can be performed in the same way as in the FMF of the ordinary MULTIMOORA method.

Step 4. Determine the final ranking order of the alternatives. The final ranking order of the alternatives can be determined as in the case of the ordinary MULTIMOORA method, i.e. based on the dominance theory.

5. A Numerical Example

In order to demonstrate the applicability and efficiency of the proposed approach, an example has been adopted from Stanujkic *et al.* (2015). In order to briefly demonstrate the advantages of the proposed methodology, this example has been slightly modified.

Suppose that a mining and smelting company has to build a new flotation plant, for which reason an expert has been engaged to evaluate the three Comminution Circuit Designs (CCDs) listed below:

– A_1, the CCDs based on the combined use of rod mills and ball mills;
– A_2, the CCDs based on the use of ball mills; and
– A_3, the CCDs based on the use of semi-autogenous mills.

For the purpose of conducting an evaluation, the following criteria have been chosen:

– C_1, Grinding efficiency;
– C_2, Economic efficiency;
– C_3, Technological reliability;
– C_4, Capital investment costs; and
– C_5, Environmental impact.

The ratings obtained from the expert are shown in Table 1.

The ranking based on the RS approach. The ranking results and the ranking order of the alternatives obtained based on the RS approach, i.e. by applying Eqs. (19) to (23), are accounted for in Table 2.

The ranking based on the RPA. The ranking of the alternatives based on the RP approach begins by determining the reference point, as it is shown in Table 3.

Table 1
The ratings of the three generic CCDs obtained from an expert.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.7, 0.2, 0.3\rangle$	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.9, 0.1, 0.2\rangle$
A_2	$\langle 0.8, 0.1, 0.3\rangle$	$\langle 0.8, 0.1, 0.3\rangle$	$\langle 0.8, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.8, 0.1, 0.3\rangle$
A_3	$\langle 1.0, 0.1, 0.3\rangle$	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.9, 0.1, 0.2\rangle$	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.7, 0.2, 0.3\rangle$

Table 2
The ranking orders of the alternatives obtained on the basis of the RS approach.

	Y_i^+	Y_i^-	y_i^+	y_i^-	y_i	Rank
A_1	$\langle 0.73, 0.25, 0.38\rangle$	$\langle 0.55, 0.45, 0.57\rangle$	0.425	0.045	0.380	2
A_2	$\langle 0.65, 0.22, 0.46\rangle$	$\langle 0.51, 0.45, 0.60\rangle$	0.372	0.006	0.366	3
A_3	$\langle 1.0, 0.22, 0.39\rangle$	$\langle 0.34, 0.57, 0.73\rangle$	0.583	−0.263	0.845	1

Table 3
The reference point.

	C_1	C_2	C_3	C_4	C_5
r_j^*	$\langle 1.0, 0.1, 0.3\rangle$	$\langle 0.9, 0.2, 0.3\rangle$	$\langle 0.9, 0.1, 0.3\rangle$	$\langle 0.7, 0.1, 0.2\rangle$	$\langle 0.7, 0.1, 0.2\rangle$

Table 4
The ranking order of the alternatives obtained based on the RP approach.

I	II	III	IV	V	VI	VII	VI
	r_1^*	r_2^*	r_3^*	r_4^*	r_5^*	$d_i^{\max}$	Rank
A_1	0.02	0.03	0.00	0.00	0.00	0.034	1
A_2	0.05	0.02	0.02	0.00	0.01	0.048	2
A_3	0.00	0.00	0.00	0.06	0.01	0.063	3

Table 5
The ranking order of the alternatives obtained based on the FMF.

	A_i	B_i	a_i	b_i	u_i	Rank
A_1	$\langle 0.89, 0.25, 0.15\rangle$	$\langle 0.96, 0.45, 0.08\rangle$	0.618	0.498	1.242	3
A_2	$\langle 0.86, 0.22, 0.21\rangle$	$\langle 0.95, 0.45, 0.09\rangle$	0.605	0.481	1.258	2
A_3	$\langle 0.96, 0.22, 0.16\rangle$	$\langle 0.88, 0.57, 0.18\rangle$	0.674	0.283	2.379	1

The maximum distances from each alternative to the coordinate j of the reference point obtained by using Eq. (25) and the maximum distance of each alternative obtained by using Eq. (26) are presented in Table 4. The ranking order of the alternatives is also presented in Table 4.

The ranking based on the FMF. The ranking results and the ranking order of the alternatives obtained on the basis of the FMF approach, i.e. by applying Eqs. (27) to (31), are demonstrated in Table 5.

Table 6
The final ranking order of the alternatives according to the MULTIMOORA method.

	RS	RP	FMF	Rank
A_1	2	1	3	3
A_2	3	2	2	2
A_3	1	3	1	1

The final ranking order of the alternatives which summarizes the three different ranks provided by the respective parts of the MULTIMOORA method is shown in Table 6.

As it can be seen from Table 6, all three approaches, integrated in the MULTIMOORA, have resulted in different ranking orders, for which reason the final ranking order is determined based on the dominance theory.

6. Conclusion

The MULTIMOORA method has been proven in solving different decision-making problems. In order to enable its application in the solving of a larger number of complex decision-making problems, numerous extensions have been proposed for the MULTIMOORA method.

Compared to crisp, fuzzy, interval-valued and intuitionistic fuzzy numbers, the neutrosophic set provides significantly greater flexibility, which can be conducive to solving decision-making problems associated with uncertainty, estimations and predictions.

Therefore, an extension of the MULTIMOORA method enabling the use of single valued neutrosophic numbers is proposed in this paper.

The usability and efficiency of the proposed extension is presented in the example of the comminution circuit design selection.

Finally, it should be noted that the proposed extension of the MULTIMOORA method can be used for solving a much larger number of complex decision-making problems. A number of real-world decision making problems which have to be solved is based on the data acquired from respondents can be identified as one of the areas where the proposed extension of the MULTIMOORA method can reach its advantages.

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D. Stanujkic

E.K. Zavadskas

F. Smarandache

W.K.M. Brauers

D. Karabasevic